MA 162 Exam 3 Spring 2000

NAME SOLUTION KEY

STUDENT ID

REC. INSTR. REC. TIME.

INSTRUCTOR

INSTRUCTIONS:

1. Make sure that you have all 7 test pages.
2. Fill in the information requested above and on the answer sheet.
3. Mark the letter of your response for each question on the mark–sense answer sheet.
4. There are 11 problems worth 9 points each.
5. No books or notes or calculators may be used.

Some infinite series:

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

$$(1+x)^s = \sum_{n=0}^{\infty} \binom{s}{n} x^n$$

1. For which x does the series $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^2 3^{n+1}}$ converge?

use Ratio Test:

$$\left|a_{n+1} \cdot \frac{1}{a_n}\right| = \left|\frac{x^{n+1}}{(n+1)^2 3^{n+2}} \cdot \frac{n^2 3^{n+1}}{x^n}\right|$$

$$= \frac{n^2}{(n+1)^2} \cdot \frac{1}{3} \cdot |x| \to \frac{1}{3}|x| \text{ as } n \to \infty$$

Series converges for $\frac{1}{3}|x| < 1 \to |x| < 3$

$\to -3 < x < 3$

End points: $x = 3 \to \sum^{\infty} \frac{(-1)^n (3)^n}{n^2 3^{n+1}} = \sum^{\infty} (-1)^n \cdot \frac{1}{3} \cdot \frac{1}{n^2}$ converges

$x = -3 \to \sum^{\infty} \frac{(-1)^n (-3)^n}{n^2 3^{n+1}} = \sum^{\infty} \frac{1}{3} \cdot \frac{1}{n^2}$ converges

A. $-\frac{1}{3} < x < \frac{1}{3}$

B. $-\frac{1}{3} \leq x \leq \frac{1}{3}$

C. $-3 \leq x < 3$

D. $-3 < x \leq 3$

E. $-3 \leq x \leq 3$

2. If the power series of $f(x)$ is $\sum_{n=1}^{\infty} \frac{2^{n-1} x^n}{n+2}$, what is $f^{(4)}(0)$?

$$\frac{f^{(4)}(0)}{4!} = \frac{2^{4-1}}{4+2}$$

$$\to f^{(4)}(0) = \frac{2^3}{6} \cdot 4!$$

$$= \frac{2^3 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot 1}{\cancel{6}_{1}} = 32$$

A. $\frac{2}{9}$

B. $\frac{4}{3}$

C. 32

D. $\frac{1}{3}$

E. $\frac{16}{3}$

3. Find the first three nonzero terms for the power series of $f(x) = \frac{x}{1+3x^2}$.

Start with geometric series

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots, \quad |x| < 1$$

$$\rightarrow \frac{1}{1+x} = \frac{1}{1-(-x)} = 1 + (-x) + (-x)^2 + (-x)^3 + \cdots, \quad |x| < 1$$

$$= 1 - x + x^2 - x^3 + \cdots, \quad |x| < 1$$

$$\rightarrow \frac{1}{1+3x^2} = 1 - (3x^2) + (3x^2)^2 - (3x^2)^3 + \cdots, \quad |3x^2| < 1$$

$$\rightarrow \frac{x}{1+3x^2} = x - 3x^3 + 9x^5 - 27x^7 + \cdots, \quad |3x^2| < 1$$

A. $x - 3x^3 + 9x^5$

B. $2x + 9x^3 + 81x^5$

C. $\frac{1}{3}x - \frac{1}{9}x^2 - \frac{1}{27}x^3$

D. $\frac{1}{3}x - \frac{1}{9}x^3 + \frac{1}{27}x^5$

E. $1 - 3x^2 + 9x^4$

4. Find the first three terms of the power series for $f(x) = \frac{1}{\sqrt{1+2x}}$.

$$f(x) = (1+2x)^{-1/2} \quad \text{use binomial series}$$

$$= 1 + \left(-\tfrac{1}{2}\right)(2x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(2x)^2 + \cdots$$

$$= 1 - x + \tfrac{3}{2}x^2 + \cdots$$

A. $1 - \frac{x}{2} + \frac{3x^2}{8}$

B. $1 + x - \frac{x^2}{2}$

C. $1 + \frac{x}{2} - \frac{x^2}{8}$

D. $1 - x + \frac{3x^2}{2}$

E. $1 - x - 3x^2$

5. Find the Taylor polynomial $p_3(x)$ about $a = 1$ for the function $f(x) = x^4 - 3x^2 + x + 1$.

$f(x) = x^4 - 3x^2 + x + 1$ $\quad f(1) = 0$

$f'(x) = 4x^3 - 6x + 1$ $\quad f'(1) = -1$

$f''(x) = 12x^2 - 6$ $\quad f''(1) = 6$

$f'''(x) = 24x$ $\quad f'''(1) = 24$

A. $1 + (x-1) - 2(x-1)^2$

B. $-(x-1) + 6(x-1) + 24(x-1)^3$

C. $1 + (x-1) - 4(x-1)^2$

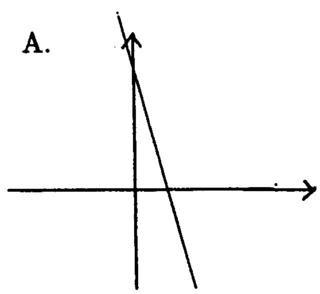
D. $-(x-1) + 3(x-1)^2 + 4(x-1)^3$

E. $-(x-1) + 3(x-1)^2 + 8(x-1)^3$

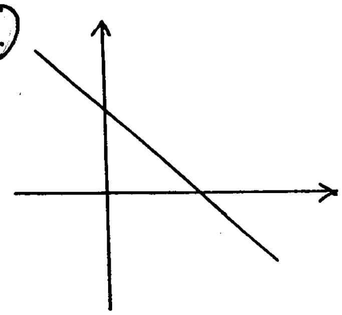
$$p_3(x) = 0 - 1(x-1) + \frac{6}{2!}(x-1)^2 + \frac{24}{3!}(x-1)^3$$
$$= -(x-1) + 3(x-1)^2 + 4(x-1)^3$$

6. Which of the curves below is described by the parameter equations $x = 1 + 2t$, $y = 2 - 2t$.

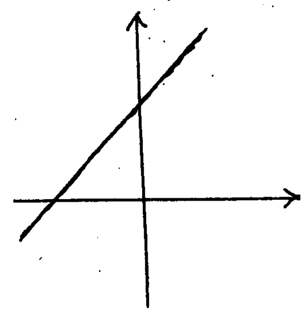
A.



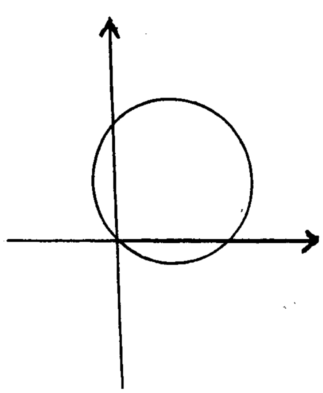
B.



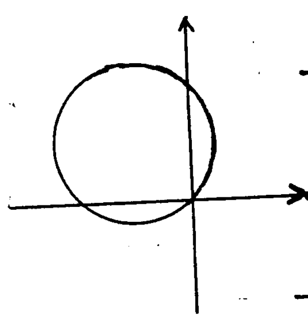
C.



D.



E.



$x = 1 + 2t \rightarrow t = \frac{x-1}{2}$

$\rightarrow y = 2 - 2\left(\frac{x-1}{2}\right)$
$= 2 - (x-1)$
$= 2 - x + 1$
$= -x + 3$

$\rightarrow$ choice B is best.

7. An object is moving in the plane. If its position at time t is $(1-\cos^2 t, \sin 2t)$, then its velocity when $t=\frac{\pi}{4}$ is

A. $\frac{\sqrt{5}}{2}$

B. $\sqrt{\frac{3}{2}}$

C. $\frac{1}{2}$

D. $\frac{\sqrt{2}}{2}$

E. 1

$$r(t) = (1-\cos^2 t)\,i + (\sin 2t)\,j$$

$$v(t) = r'(t) = (-2\cos t(-\sin t))\,i + (2\cos 2t)\,j$$

$$v\left(\tfrac{\pi}{4}\right) = \left(2\cos\tfrac{\pi}{4}\sin\tfrac{\pi}{4}\right)i + \left(2\cos\tfrac{\pi}{2}\right)j$$

$$= \left(2\cdot\tfrac{\sqrt{2}}{2}\cdot\tfrac{\sqrt{2}}{2}\right)i + (2\cdot 0)\,j$$

$$= i + 0j$$

$$\left\|v\left(\tfrac{\pi}{4}\right)\right\| = 1$$

8. The length of the curve $x=\frac{t^3}{3}-t$, $y=t^2$, $1\le t\le 2$, is

A. $\frac{10}{3}$

B. $\frac{14}{3}$

C. $\frac{16}{5}$

D. $\frac{19}{5}$

E. 4

$$L = \int_1^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$= \int_1^2 \sqrt{(t^2-1)^2 + (2t)^2}\,dt$$

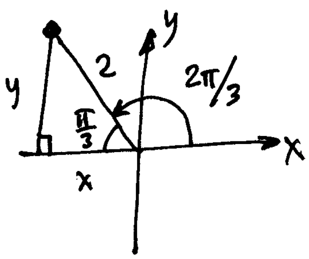
$$= \int_1^2 \sqrt{(t^4-2t^2+1)+4t^2}\,dt$$

$$= \int_1^2 \sqrt{t^4+2t^2+1}\,dt$$

$$= \int_1^2 \sqrt{(t^2+1)^2}\,dt = \int_1^2 (t^2+1)\,dt$$

$$= \left(\tfrac{1}{3}t^3+t\right)\Big|_1^2 = \left(\tfrac{8}{3}+2\right)-\left(\tfrac{1}{3}+1\right) = \tfrac{14}{3}-\tfrac{4}{3} = \tfrac{10}{3}$$

9. If the polar coordinates of a point are $\left(2, \frac{2\pi}{3}\right)$, then its Cartesian coordinates are



$x = -1$, $y = \sqrt{3}$

A. $(\sqrt{3}, 2\sqrt{3})$
B. $(-\sqrt{3}, 2)$
C. $(-1, \sqrt{3})$
D. $\left(\frac{1}{2}, \frac{3}{2}\right)$
E. $\left(1, -\frac{\sqrt{3}}{2}\right)$

OR

$x = r\cos\theta = 2\cos\frac{2\pi}{3} = 2\left(-\frac{1}{2}\right) = -1$

$y = r\sin\theta = 2\sin\frac{2\pi}{3} = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$

10. If the polar equation of a curve is $2r\cos\theta = \tan\theta$, what is its Cartesian equation?

A. $x^2 + y = 1$
B. $2x^2y^2 = 1$
C. $xy = y^2 + 1$
D. $2x^2y = 1$
E. $y = 2x^2$

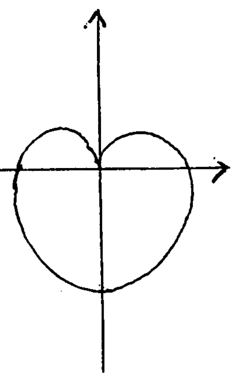
$2(r\cos\theta) = \tan\theta$

$\longrightarrow 2x = \frac{y}{x}$

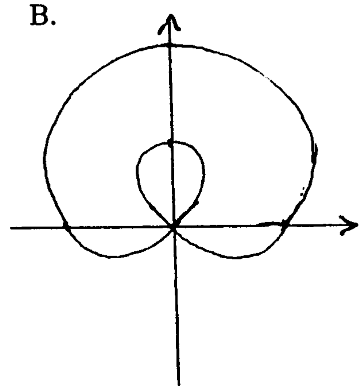
$\longrightarrow 2x^2 = y$

11. The equation $r = 3 + \sin\theta$ describes the curve

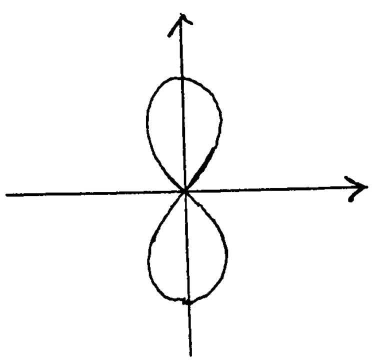
A.



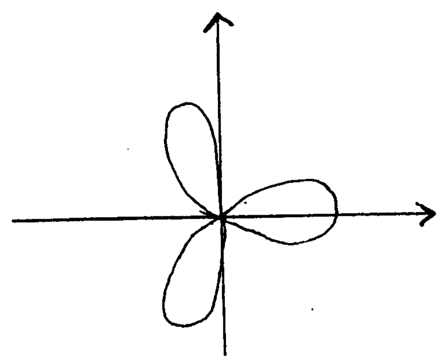
B.



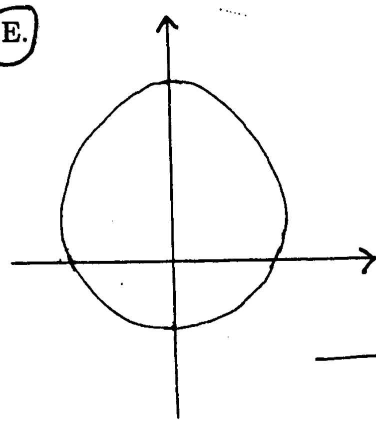
C.



D.

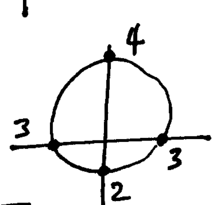


E.



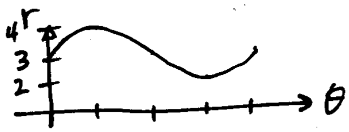
variation of r with θ

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$3+\sin\theta$	$3 \nearrow$	$4 \searrow$	$3 \searrow$	$2 \nearrow$	3



→ answer is E.

OR $r = 3+\sin\theta$ on cartesian coord:



then wrap around the pole in cartesian coord.