MATH 261 - SPRING 2000

Name ______________________________ Instructor ______________________________

Signature ________________________ Recitation Instructor ________________________

Div. Sect. No. ______________________________

FINAL EXAM INSTRUCTIONS

1. You must use a #2 pencil on the mark-sense sheet (answer sheet).
2. If you have test 01, mark 01 and blacken the corresponding circles under test/quiz number.
   If you have test 02, mark 02 and blacken the corresponding circles under test/quiz number.
3. On the mark-sense sheet, fill in the instructor's name and the course number.
4. Fill in your name and student identification number and blacken in the appropriate spaces.
5. Mark in your division and section number of your class. For example, for division 02, section 03, fill in 0203 and blacken the corresponding circles, including the circles for the zeros. (If you do not know your division and section number ask your instructor.)
6. Sign the mark-sense sheet.
7. Fill in your name and your instructor's name above and on the first page of the question sheets.
8. There are 20 questions, each worth 10 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–20. Do all your work on the question sheets. Turn in both the mark-sense sheets and the question sheets when you are finished.
9. No partial credit will be given, but if you show your work on the question sheets it may be considered if your grade is on the borderline.
10. Calculators are not allowed. NO BOOKS OR PAPERS ARE ALLOWED. Use the back of the test pages for scratch paper.

Name ______________________________

1. Parametric equations for the tangent line to the curve $\overrightarrow{r(t)} = t\vec{i} + t^2\vec{j} + t^3\vec{k}$ at the point $(-2, 4, -8)$ are:

A. $\dfrac{x-1}{-2} = \dfrac{y+4}{4} = \dfrac{z-12}{-8}$

B. $x + 2 = \dfrac{y-4}{-4} = \dfrac{z+8}{12}$

C. $\overrightarrow{r(t)} = (-2+t)\vec{i} + (4-4t)\vec{j} + (-8+12t)\vec{k}$

D. $x = -2+t,\ y = 4-4t,\ z = -8+12t$

E. $x = 1-2t,\ y = -4+4t,\ z = 12-8t$

2. The area of the triangle with vertices $(1,1,1)$, $(2,2,2)$, and $(1,2,3)$ is:

A. 6

B. $\sqrt{6}$

C. 3

D. $\sqrt{3}$

E. $\sqrt{6}/2$

3. A particle passes through the point $(2,1,1)$ when $t = 1$ and moves with velocity $\overrightarrow{v(t)} = \vec{i} + 2t\vec{j} + \vec{k}$. Where is it when $t = 2$?

A. $\vec{i} + 4\vec{j} + \vec{k}$

B. $2\vec{i} + 4\vec{j} + \vec{k}$

C. $3\vec{i} + 4\vec{j} + 2\vec{k}$

D. $2\vec{i} + 4\vec{j} + 2\vec{k}$

E. $4\vec{i} + 5\vec{j} + 3\vec{k}$

4. The arclength of the curve $\overrightarrow{r(t)} = e^{2t}\vec{i} + \sin t\vec{j},\ \pi \le t \le 2\pi$, is given by:

A. $\int_{\pi}^{2\pi} \sqrt{e^{4t} + \sin^2 t}\ dt$

B. $\int_{\pi}^{2\pi} \sqrt{e^{4t} + \cos^2 t}\ dt$

C. $\int_{\pi}^{2\pi} \sqrt{4e^{4t} + \cos^2 t}\ dt$

D. $\int_{\pi}^{2\pi} \sqrt{1 + e^{4t} + \sin^2 t}\ dt$

E. $\int_{\pi}^{2\pi} \sqrt{1 + 4e^{4t} + \cos^2 t}\ dt$

5. If $f(u,v) = u + v + uv,\ u = g(s,t),\ v = h(s,t)$ where $g(1,2) = 3,\ h(1,2) = 2$, $\frac{\partial g}{\partial s}(1,2) = -2,\ \frac{\partial h}{\partial s}(1,2) = 1,\ \frac{\partial g}{dt}(1,2) = 0,\ \frac{\partial h}{\partial t}(1,2) = 10$, what is $\frac{\partial}{\partial s}(f(g(s,t),h(s,t)))$ when $s = 1$ and $t = 2$?

A. -4

B. -2

C. 0

D. 38

E. 40

6. An equation for the tangent plane to the surface $x^2 + xy^3 + z^2 = 4$ at the point $(-1,1,2)$ is:

A. $-x - 3y + 4z = 6$

B. $-x + 3y + 4z = 12$

C. $x - y + 4z = 6$

D. $-2x + 3y + 4z = 13$

E. $-2x - 3y + 4z = 7$

7. If $f(x,y) = x^2 + 3y^2$, for which unit vector $\vec{u}$ does the directional derivative $(D_{\vec{u}}f)(2,1)$ have a minimum value?

A. $4\vec{i} + 6\vec{j}$

B. $-4\vec{i} - 6\vec{j}$

C. $\frac{2}{\sqrt{13}}\,\vec{i} + \frac{3}{\sqrt{13}}\,\vec{j}$

D. $\frac{-2}{\sqrt{13}}\,\vec{i} - \frac{3}{\sqrt{13}}\,\vec{j}$

E. $\frac{2}{\sqrt{5}}\,\vec{i} + \frac{1}{\sqrt{5}}\,\vec{j}$

8. The function $f(x,y) = x^2y - 2xy + 2y^2 - 15y$ has critical points $(1,4)$, $(-3,0)$, and $(5,0)$. At these points f has:

A. 1 relative minimum and 2 saddle points

B. 1 relative minimum, 1 relative maximum, and 1 saddle point

C. 2 relative minima and 1 saddle point

D. 2 relative maxima and 1 saddle point

E. 2 relative minima and 1 relative maximum

9. The largest area of a rectangle inscribed in the ellipse $4x^2 + y^2 = 4$ is:

A. 1

B. 2

C. 3

D. 4

E. 5

10. Evaluate $\iint\limits_R e^{x^2+y^2}\,dA$ where R is the region bounded by the circle $x^2+y^2=9$:

A. e^9-1

B. $\frac{\pi}{3}(e^9-1)$

C. πe^9

D. $\pi(e^9-1)$

E. $2\pi(e^9-1)$

11. The volume of the region inside the cylinder $x^2+y^2=1$ and bounded by $z=2$ and $z=1-x^2-y^2$ is:

A. $\frac{5\pi}{4}$

B. $\frac{3\pi}{2}$

C. $\frac{7\pi}{4}$

D. 2π

E. 3π

12. The triple integral $\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{x^2+y^2}}(x^2+y^2)\,dzdydx$ when converted to cylindrical coordinates becomes:

A. $\int_0^{\pi}\int_0^4\int_0^r z^2 r\,dzdrd\theta$

B. $\int_0^{\pi}\int_0^2\int_0^r z^2 r\,dzdrd\theta$

C. $\int_0^{\pi}\int_0^2\int_0^r r^3\,dzdrd\theta$

D. $\int_0^{2\pi}\int_0^2\int_0^r r^3\,dzdrd\theta$

E. $\int_0^{2\pi}\int_0^4\int_0^r r^3\,dzdrd\theta$

13. An object occupies the solid region between the graphs of $z = 2-x^2-y^2$ and $z = x^2+y^2$. The mass density of the object at each point is the distance from the point to the x axis. Which of the following gives the total mass of the object?

A. $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{2-x^2-y^2} x\,dzdydx$

B. $\displaystyle\int_{-1}^{1}\int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}}\int_{2-x^2-y^2}^{x^2+y^2} \sqrt{y^2+z^2}\,dzdydx$

C. $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{2-x^2-y^2} \sqrt{y^2+z^2}\,dzdydx$

D. $\displaystyle\int_{-1}^{1}\int_{-1}^{1}\int_{x^2+y^2}^{2-x^2-y^2} \sqrt{x^2+y^2}\,dzdydx$

E. $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{2-x^2-y^2}^{x^2+y^2} \sqrt{x^2+z^2}\,dzdydx$

14. Let S be that part of the surface $z = \dfrac{y^2}{4}$ which lies over the triangle with vertices $(0,0)$, $(0,1)$, $(1,1)$ in the xy plane. Which of the following is the surface area of S? (Hint: Describe the region of integration as a horizontally simple region.)

A. 1

B. $\dfrac{1}{2}$

C. $\dfrac{5^{3/2}-8}{6}$

D. $\dfrac{5}{2}$

E. $\dfrac{5^{3/2}-1}{6}$

15. Let D be the solid region in space given by $z \geq 0$ and $x^2 + y^2 + z^2 \leq 4$. In spherical coordinates, $\iiint\limits_D z dv$ is given by:

A. $\int_0^{2\pi} \int_0^{\pi/2} \int_0^2 \rho \cos \phi d\rho d\phi d\theta$

B. $\int_0^{2\pi} \int_0^{\pi} \int_0^2 \rho \cos \phi d\rho d\phi d\theta$

C. $\int_0^{2\pi} \int_0^{\pi/2} \int_0^2 \rho^3 \sin \phi \cdot \cos \phi d\rho d\phi d\theta$

D. $\int_0^{2\pi} \int_0^{\pi} \int_0^2 \rho^3 \sin \phi \cos \phi d\rho d\phi d\theta$

E. $\int_0^{2\pi} \int_0^{\pi/2} \int_0^2 \rho^3 \sin^2 \phi d\rho d\phi d\theta$

16. Let $\vec{F}(x, y, z) = 2y\vec{i} - x\vec{j} + 3\vec{k}$. Which of the following statements is true?

(i) Curl $\vec{F} = \vec{0}$.

(ii) $\vec{F}$ is a gradient of some function f.

(iii) Line integrals, $\int_C \vec{F} \cdot d\vec{r}$, over a curve C from a point P to a point Q, are path independent.

(iv) $\int_C \vec{F} \cdot d\vec{r} = 0$ for all smooth closed curves C.

A. (i) only

B. (ii) only

C. (ii) and (iii) only

D. (ii), (iii) and (iv) only

E. None of the above

17. Suppose $\vec{F}(x, y, z)$ is a vector field and $g(x, y, z)$ is a real–valued function. Exactly one of the expressions below is meaningless. Which one?

A. curl(grad g)

B. div(curl $\vec{F}$)

C. curl (curl $\vec{F}$)

D. curl (div $\vec{F}$)

E. div($g\vec{F}$)

18. Let $\vec{F}(x, y) = 3x^2\vec{\imath} - \vec{\jmath}$. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is the top half of the circle of radius 2 centered at $(0, 0)$, starting at $(2, 0)$ and ending at $(-2, 0)$.

A. 16

B. 8

C. 0

D. −8

E. −16

19. Let R be the rectangle with corners $(0,0)$, $(2,0)$, $(2,1)$, and $(0,1)$ and let C be the curve consisting of the boundary of R in the counterclockwise direction starting at $(0,0)$ and ending at $(0,0)$. Then $\int_C ydx + xydy$ equals:

A. $\int_0^2 \int_0^1 (y-1)dydx$

B. $\int_0^2 \int_0^1 (y+1)dydx$

C. $\int_0^2 \int_0^1 (1-y)dydx$

D. $\int_0^2 \int_0^1 ydydx$

E. $\int_0^2 \int_0^1 (1+x)dydx$

20. Evaluate $\iint_\Sigma \vec{F} \cdot \vec{n} dS$ if Σ is the top half of the surface of the sphere of radius 1 about the origin, $\vec{n}$ is the upward unit normal vector, and $\vec{F}(x,y,z) = y\vec{i} - x\vec{j} + 2\vec{k}$:

A. 0

B. $\pi/2$

C. π

D. $3\pi/2$

E. 2π