MA 262 Spring 2000
FINAL EXAM INSTRUCTIONS

NAME ______________________ INSTRUCTOR ______________________

INSTRUCTIONS:

1. You must use a #2 pencil on the mark–sense sheet (answer sheet).

2. On the mark–sense sheet, fill in the instructor's name and the course number.

3. Fill in your name and student identification number and blacken in the appropriate spaces.

4. Mark in the section number, the division and section number of your class. For example, for division 02, section 03, fill in 0203 and blacken the corresponding circles, including the circles for the zeros. (If you do not know your division and section number ask your instructor.)

5. Sign the mark–sense sheet.

6. Fill in your name and your instructor's name above.

7. There are 25 questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25. Do all your work on the question sheets. Turn in both the mark–sense sheets and the question sheets when you are finished.

8. No partial credit will be given, but if you show your work on the question sheets it may be considered if your grade is on the borderline.

9. NO CALCULATORS, BOOKS OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

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1. If $y(x)$ is the solution of the initial value problem

$$y' = e^y x, \qquad y(0) = 0,$$

then $y(1) =$

A. $1/4$

B. $\ln(2)$

C. $-\ln 3$

D. $\ln(4/3)$

E. $3/4$

2. Which of the following is the **general** solution of the differential equation

$$y' + \frac{y}{x} = \frac{1}{x^3} \quad , \quad x > 0 \quad ?$$

A. $y = \frac{c}{x^2} - \frac{1}{x}$

B. $y = e^x + \frac{c}{x^2}$

C. $y = -\frac{1}{x^2} + \frac{c}{x}$

D. $xy - \frac{1}{x} = c$

E. $y = \frac{c}{x} - \frac{1}{2x^3}$

3. The following differential equation is exact. Find the general solution.

$$(e^x - y)dx - xdy = 0.$$

A. $e^x + \frac{x^2}{2} - \frac{y^2}{2} = C$

B. $e^x - 7xy = C$

C. $e^x + \frac{x^2}{2} - \frac{y^2}{2} = C$

D. $e^x - xy = C$

E. $e^x + xy = C$

4. Solve the initial value problem:

$$y' = \frac{x}{2y} + \frac{y}{x}, \quad y(1) = 1.$$

(Note that the equation is homogeneous of degree zero.)

A. $y = x\sqrt{\ln x + 1}$

B. $y = x\sqrt{\ln x} + 1$

C. $y = \sqrt{x \ln x + 1}$

D. $y = \sqrt{\ln x + 1}$

E. $y = \sqrt{\ln x} + 1$

5. A body with initial temperature $32^o F$ is placed in a refrigerator whose temperature is a constant $0^o F$. An hour later the temperature of the body is $16^o F$. What will its temperature be three hours after it is placed in the refrigerator?

A. $1^o F$

B. $2^o F$

C. $3^o F$

D. $4^o F$

E. $8^o F$

6. What is the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 \end{bmatrix}$?

A. 1

B. 2

C. 3

D. 4

E. 5

7. The homogeneous linear system $A\mathbf{x} = 0$, with coefficient matrix A has only the trivial solution. Which of the following statements must be true?

A. A is a square matrix and $\det A \neq 0$.

B. A is a square matrix and $\det A = 0$.

C. The rank of A equals the number of columns of A.

D. The rank of A equals the number of rows of A.

E. The reduced row–echelon form of A is 0.

8. Find all the values of k for which the system

$$\begin{aligned} kx + y \quad &+ z = 1 \\ 3x + (k+2)y - &z = 5 \\ 2x + 2y \quad &+ 2z = k+1 \end{aligned}$$

has no solutions. You may use the fact that $\det \begin{bmatrix} k & 1 & 1 \\ 3 & k+2 & -1 \\ 2 & 2 & 2 \end{bmatrix} = (k-1)(k+3)$.

A. $k = 0, 1, -3$

B. $k = 1, -3$

C. $k \neq 1, -3$

D. $k \neq 1$

E. $k = -3$

9. Let A be a 3×3 matrix and let I be the identity 3×3 matrix. Suppose that the 3×6 matrix $[A \,|\, I]$ can be transformed to

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 2 & -1 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{array}\right]$$

by elementary row operations. Which statement is correct.

A. A is nonsingular and $A^{-1} = \begin{bmatrix} 0 & 2 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

B. A is singular and $A = \begin{bmatrix} 1 & -2 & -4 \\ 0 & 1 & 3 \\ -1 & 2 & 4 \end{bmatrix}$

C. A is nonsingular and $A = \begin{bmatrix} 1 & -2 & -4 \\ 0 & 1 & 3 \\ -1 & 2 & 4 \end{bmatrix}$

D. A is singular but cannot be determined from the above information.

E. A is nonsingular but cannot be determined from the above information.

10. A and B are 3×3 matrices. The equality $(AB)^2 = A^2B^2$

A. always holds

B. only holds when A and B are diagonal matrices

C. holds only if $A = 0$ or $B = 0$

D. holds if $AB = BA$

E. never holds

11. The vectors $[0,0,1]$, $[0,2,3]$ and $[4,5,6]$.

A. are linearly independent and do not span $\mathbf{R}^3$

B. are linearly dependent and span $\mathbf{R}^3$

C. are linearly independent and span $\mathbf{R}^3$

D. are linearly dependent and do not span $\mathbf{R}^3$

E. are linearly dependent and form a basis for $\mathbf{R}^3$

12. Let $T: \mathbf{R}^4 \to \mathbf{R}^2$ be the linear transformation given by $T(\mathbf{x}) = A\mathbf{x}$, where

$$A = \begin{bmatrix} 2 & -3 & 4 & -5 \\ 6 & -7 & 8 & -9 \end{bmatrix}.$$

Find the dimension of the kernel of T.

A. 0

B. 1

C. 2

D. 3

E. 4

13. If A and B are 3×3 matrices and $\det A = 2$, $\det B = 3$, then $\det(2A^{-1}B^3) =$

A. $\frac{27}{2}$

B. 27

C. 54

D. 108

E. 216

14. Which of the following are vector spaces?

i) the set of all singular 3×3 matrices

ii) the set of all polynomials $p(x)$ with $p(0) = 0$

iii) the set of all vectors of the form $(r + s, r, r - s)$, $r, s \in \mathbf{R}$

A. (i) and (ii)

B. (i) and (iii)

C. (ii) and (iii)

D. only (i)

E. only (ii)

15. For how many values of k are the vectors $[k, 1, -1]$, $[1, k, -1]$, $[1, -1, k]$ linearly **dependent** ?

A. no values

B. one value

C. two values

D. three values

E. all values

16. The **sum** of the eigenvalues of the matrix $\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$ is

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B. 2

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17. The eigenvalues of the matrix $\begin{bmatrix} 5 & 1 & -1 \\ -3 & 1 & 3 \\ 0 & 0 & 4 \end{bmatrix}$ are 2 and 4. One of the two eigenspaces has dimension one. This eigenspace has a basis consisting of

A. $\begin{bmatrix} 2 \\ -4 \\ 0 \end{bmatrix}$

B. $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

D. $\begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}$

E. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

18. Let L be the differential operator $Ly = y' - x^2 y$. Then $\text{Ker}(L) =$

A. span $\{e^{x^3/3}\}$

B. span $\{e^{-x^3/3}, e^{x^3/3}\}$

C. span $\{e^{-x^2}, e^{x^2}\}$

D. span $\{e^{-x^3}\}$

E. span $\{e^{x^2}\}$

22. The oscillation of a spring–mass system is governed by the differential equation

$$\frac{d^2x}{dt^2} + 9x = 0$$

with initial conditions $x(0) = 1$ and $\frac{dx}{dt}(0) = 3$. Then $x(t) =$

A. $2\cos(t - \pi/2)$

B. $\cos(t - \pi/3)$

C. $2\cos(3t - \pi/2)$

D. $\cos(3t - \pi/6)$

E. $\sqrt{2}\cos(3t - \pi/4)$

23. Find the general solution of the system $\mathbf{x}' = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \mathbf{x}$.

A. $c_1 \begin{bmatrix} e^{2t} \\ -e^{2t} \end{bmatrix} + c_2 \begin{bmatrix} e^{4t} \\ e^{4t} \end{bmatrix}$

B. $c_1 \begin{bmatrix} e^{t} \\ e^{t} \end{bmatrix} + c_2 \begin{bmatrix} e^{4t} \\ -e^{4t} \end{bmatrix}$

C. $c_1 \begin{bmatrix} e^{3t} \\ e^{t} \end{bmatrix} + c_2 \begin{bmatrix} e^{t} \\ e^{3t} \end{bmatrix}$

D. $c_1 \begin{bmatrix} e^{2t} \\ e^{2t} \end{bmatrix} + c_2 \begin{bmatrix} e^{4t} \\ -e^{4t} \end{bmatrix}$

E. $c_1 \begin{bmatrix} e^{t} \\ -e^{t} \end{bmatrix} + c_2 \begin{bmatrix} e^{3t} \\ e^{3t} \end{bmatrix}$

24. The 2×2 real matrix A has eigenvalues $-1+2i$ and $-1-2i$, and corresponding eigenvectors $\begin{bmatrix} 1 \\ i \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -i \end{bmatrix}$ respectively. Which of the following is a **real** solution of the system $\mathbf{x}' = A\mathbf{x}$.

A. $e^{-2t} \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$

B. $e^{-t} \begin{bmatrix} \sin 2t \\ -\cos 2t \end{bmatrix}$

C. $e^{-t} \begin{bmatrix} \cos 2t \\ -\sin 2t \end{bmatrix}$

D. $e^{t} \begin{bmatrix} \cos 2t \\ \sin 2t \end{bmatrix}$

E. $e^{-2t} \begin{bmatrix} \cos t \\ -\sin t \end{bmatrix}$

25. The system

$$\mathbf{x}'(t) = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} e^t \\ 7e^t \end{bmatrix}$$

has fundamental matrix

$$\mathbf{X}(t) = \begin{bmatrix} e^t & e^{5t} \\ -e^t & e^{5t} \end{bmatrix}, \quad \text{and} \quad \mathbf{X}(t)^{-1} = \frac{1}{2} \begin{bmatrix} e^{-t} & -e^{-t} \\ e^{-5t} & e^{-5t} \end{bmatrix}.$$

Use the method of variation of parameters to find a particular solution.

A. $\begin{bmatrix} (-2t+1)\,e^t \\ (t-1)\,e^t \end{bmatrix}$

B. $\begin{bmatrix} (-3t-1)\,e^t \\ (3t-1)\,e^t \end{bmatrix}$

C. $\begin{bmatrix} (4t-3)\,e^t \\ (t+1)\,e^t \end{bmatrix}$

D. $\begin{bmatrix} (3t+1)\,e^t \\ (t+5)\,e^t \end{bmatrix}$

E. $\begin{bmatrix} (3t+2)\,e^t \\ (t-1)\,e^t \end{bmatrix}$