July 16, 2001 MA 266 Name:_____

McQuinn ID#:_____

# Exam II

**Instructions**:

- Write your name on each page.
- You have 60 minutes to complete all problems.
- Write your final answers in the boxes provided, where appropriate.
- Do all work to be graded on the FRONT of the exam sheets. Use the back of the exam sheets for scratch paper, and do not use any other scratch paper. No work on the back of the exam sheets will be graded.
- **No credit will be given for any problem without work being shown.**
- No calculators, no notes, no books, no formula sheets may be used.
- It is advisable to check your work carefully before turning in the exam.
- This exam has 7 pages plus this cover sheet and a table of Laplace transforms; please ensure that you have all pages.

| | | |
|---|---|---|
| Page 1 | | 16 |
| Page 2 | | 20 |
| Page 3 | | 12 |
| Page 4 | | 12 |
| Page 5 | | 10 |
| Page 6 | | 12 |
| Page 7 | | 18 |
| Total | | 100 |

**Write your answers in the boxes provided. Be sure to show all work.**

(8) 1. Find the general solution (explicitly).

$$y^{(4)} - y''' - 3y'' + y' + 2y = 0$$

$y =$

(8) 2. Find the inverse Laplace transform of $F(s) = \dfrac{s-1}{s^2 - 4s + 5}$.

$f(t) =$

(12) 3. For $L[y] = t^2y'' + ty' - y$, a fundamental set of solutions of $L[y] = 0$ is given by $y_1(t) = t$ and $y_2(t) = \frac{1}{t}$. Find (explicitly) the general solution of $L[y] = 15t^4$.

$y =$

(8) 4. Give the general form of a particular solution $Y$ that is suitable for use with the method of undetermined coefficients. *Do not solve for the coefficients!*

$$y^{(5)} - 3y^{(4)} + 2y''' = t + te^t$$

$Y =$

(12) 5. Solve the initial value problem for $y$ (explicitly)

$$y'' - 3y' + 2y = 10\cos t \qquad \text{and} \qquad y(0) = y'(0) = 1$$

$y =$

(12) 6. One solution of the equation is $y_1(t) = t$. Find another solution $y_2$ that is not a constant multiple of $y_1$.

$$t^2y'' - ty' + y = 0 \qquad (t > 0)$$

$y_2(t) =$

(10) 7. On the interval $(-\infty,\infty)$, three solutions of the third-order linear homogeneous differential equation are given. Decide whether these three functions form a *fundamental set of solutions* and justify your answer in one sentence. You might find it helpful to use the fact that $y_2'(t) = \sin 2t$ and $y_3'(t) = -\sin 2t$.

$$y'''(t) + 4y'(t) = 0$$

$$y_1(t) = 1 \qquad y_2(t) = \sin^2 t \qquad y_3(t) = \cos^2 t$$

(12) 8. Use Laplace transforms to solve explicitly:

$$y'' - 2y' + y = 0 \qquad y(0) = 2 \quad y'(0) = 0$$

$y(t) =$

9. A weight of 16 pounds stretches a spring 1.6 feet. The spring is suspended in a liquid that offers a resistance of 8 pounds when speed is 2 feet per second. The spring is set in motion from the equilibrium position with a downward initial velocity of 6 feet per second.

(14) (a) Set up and solve an initial value problem for the position $u$ of the spring at time $t$

$u(t) =$

(4) (b) What is the first time that the mass returns to the equilibrium position? $t =$

BONUS (Movie Quotes): Name the movie in which the quote is spoken.

"It means Luca Brasi sleeps with the fishes."

"Copper, you're my very best friend."