

NAME : _____

STUDENT ID : _____

INSTRUCTIONS:

1. Write your name and student ID number in the space provided above. Also write your name at the top of each page.
2. There are 8 problems on 6 pages. You must show sufficient work to justify all answers. Correct answers with inconsistent work may not be given credit. Write your answer in the box provided on each problem.
3. No books, notes or calculators are allowed.
4. The exam is self-explanatory. Please do not ask the instructor to interpret any of the exam questions.

Page number :	1	2	3	4	5	6	TOTAL
Possible max. :	12	24	22	14	14	14	100
Your score :							

-
1. (12 points) Find the general solution of the given differential equation.

$$y' - \frac{2}{t}y = 2t^3 + t^2 - 1$$

Answer Box 1

$y(t) =$

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Place your answers in the spaces provided. **You must show work to receive any credit.**

2. (12 points) For what value of λ is $y = x^\lambda \ln x$ a solution of the differential equation $x^2 y'' - 3xy' + 4y = 0$?

Answer Box 2

$\lambda =$

3. (12 points) Solve the following initial value problem. Express y in terms of x explicitly.

$$x \frac{dy}{dx} = y^2 - 2y - 3, \quad y(-1) = 1$$

Answer Box 3

$y(x) =$

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Place your answers in the spaces provided. **You must show work to receive any credit.**

4. (10 points) Determine (without solving the problem) an interval in which the solution of the given initial value problem is certain to exist. (Justify your answer by explaining how you decided on the interval with one sentence!)

$$y' - \frac{1}{t \cos t} y = t^2, \quad y(1) = 3$$

Answer Box 4

5. (12 points) Use the formula $y = x^2v$ to express the given differential equation as a differential equation in terms of x and $v(x)$. Then, find $y(x)$.

$$\frac{dy}{dx} = \frac{2y^2 + x^3}{xy}$$

Answer Box 5

$y(x) =$

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Place your answers in the spaces provided. **You must show work to receive any credit.**

6. (14 points) Find the value of m for which the given equation is exact, and then solve it using this value of m .

$$-2xy^{(m+1)} + y^m(x^2 + 1) \frac{dy}{dx} = 0$$

<i>Answer Box 6</i>

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Place your answers in the spaces provided. **You must show work to receive any credit.**

7. (14 points) A motorboat weighs 1280 lb. The thrust of the motor after t seconds in the direction of motion is given by $f(t) = 24e^{-0.25t}$ lb. If the resistance of water to the motion is equal numerically to twice the speed in feet per second, and if the boat is initially at rest, find the velocity v of the boat at time t .

Answer Box 7

$v(t) =$

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Place your answers in the spaces provided. **You must show work to receive any credit.**

8. (14 points) Solve the given initial value problem.

$$\frac{dy}{dx} = \frac{2y^3 + x^3}{xy^2}, \quad y(1) = 1$$

<p><i>Answer Box 8</i></p>
