

NAME : _____

STUDENT ID : _____

INSTRUCTIONS:

1. Write your name and student ID number in the space provided above. Also write your name at the top of each page.
2. There are 8 problems on 6 pages. You must show sufficient work to justify all answers. Correct answers with inconsistent work may not be given credit. Write your answer in the box provided on each problem.
3. No books, notes or calculators are allowed.
4. The exam is self-explanatory. Please do not ask the instructor to interpret any of the exam questions.

Page number :	1	2	3	4	5	6	TOTAL
Possible max. :	12	12	24	14	24	14	100
Your score :							

-
- 1.** (12 points) Find the general solution of the given differential equation. Express y in terms of x explicitly.

$$(x^2 + 4)\frac{dy}{dx} - xy = 0$$

Answer Box 1

$y(x) =$

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Place your answers in the spaces provided. **You must show work to receive any credit.****2.** (12 points) Find the direction field that best represents the given differential equation.

i) $y' = -y^2/t$ Circle one: A B C D None

ii) $y' = -yt$ Circle one: A B C D None

iii) $y' = y/t$ Circle one: A B C D None

iv) $y' = y^2/t$ Circle one: A B C D None

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Place your answers in the spaces provided. **You must show work to receive any credit.****3.** (14 points) Solve the following initial value problem. Express y in terms of t explicitly.

$$y'' - \frac{2}{t} y' = t^2 - 1, \quad y(0) = 1 \quad y'(0) = \frac{1}{2}$$

Answer Box 3

$y(t) =$

4. (10 points) Determine (without solving the problem) an interval in which the solution of the given initial value problem is certain to exist. (*Justify your answer by explaining how you decided on the interval with one sentence!*)

$$y' - (\cot t) y = \frac{1}{t-2}, \quad y(3) = 3$$

Answer Box 4

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Place your answers in the spaces provided. **You must show work to receive any credit.**

5. (14 points) Use the formula $v = 1/y$ to express the given differential equation as a differential equation in terms of x and $v(x)$. Then, find $y(x)$.

$$\frac{dy}{dx} - y = xy^2$$

Answer Box 5

$y(x) =$

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Place your answers in the spaces provided. **You must show work to receive any credit.**

6. (12 points) For what value of k the following equation becomes exact? (*Don't solve the equation.*)

$$-y(x^2 + y^2)^k dx + x(x^2 + y^2)^k dy = 0$$

Answer Box 6

$k =$

7. (12 points) Solve the following equation.

$$x^2 \frac{dy}{dx} = x^2 - xy + y^2$$

Answer Box 6

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Place your answers in the spaces provided. **You must show work to receive any credit.**

8. (*14 points*) A 400 liters tank is full of brine (water containing salt) of concentration 0.8 grams per liter. Brine of concentration 0.2 grams per liter flow into the tank at 2 liters per minute and the well-mixed solution is drawn off at the same rate. How many minutes does it take for the concentration to reach 0.5 grams per liter?

Answer Box 8