

NAME : \_\_\_\_\_

STUDENT ID : \_\_\_\_\_

## INSTRUCTIONS:

1. Write your name and student ID number in the space provided above. Also write your name at the top of each page.
2. There are 8 problems on 6 pages. You must show sufficient work to justify all answers. Correct answers with inconsistent work may not be given credit. Write your answer in the box provided on each problem.
3. No books, notes or calculators are allowed.
4. The exam is self-explanatory. Please do not ask the instructor to interpret any of the exam questions.

Page number :	1	2	3	4	5	6	TOTAL
Possible max. :	12	24	14	24	14	12	100
Your score :							

- 
1. (6 points each) Find the general solution of the given differential equation.

a)  $y'' + 4y' - 21y = 0$

*Answer Box 1a*

$$y(t) =$$

b)  $y'' + 4y' + 20y = 0$

*Answer Box 1b*

$$y(t) =$$

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Place your answers in the spaces provided. **You must show work to receive any credit.**

2. (12 points) Solve the following initial value problem.

$$4y''' - 4y'' + y' = 0, \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = -2$$

*Answer Box 2*

$y(t) =$
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3. (12 points) Find the general solution of the following differential equation.

$$y'' - 9y = t - 12e^{3t}$$

*Answer Box 3*

$y(t) =$
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Place your answers in the spaces provided. **You must show work to receive any credit.**

4. (14 points) Given that  $y_1(t) = t^3$  is a solution of

$$t^2 y'' + 2ty' - 12y = 0, \quad t > 0,$$

use the method of reduction of order to find a second solution  $y_2$  such that  $y_1$  and  $y_2$  will form a fundamental set of solutions of the given differential equation.

*Answer Box 4*

$y_2(t) =$

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Place your answers in the spaces provided. **You must show work to receive any credit.**

5. (12 points) Consider the following nonhomogeneous differential equation,

$$y'' - \frac{x}{x-1} y' + \frac{1}{x-1} y = \frac{x-1}{x}, \quad y > 1$$

Find a particular solution if  $y_1 = x$  and  $y_2 = e^x$  are solutions of the corresponding homogeneous equation.*Answer Box 5*

6. (12 points) Determine a suitable form for the particular solution of the given differential equation if the method of undetermined coefficients is to be used. Do not evaluate the constants.

$$y^{iv} - 2y'' + y = te^t + \cos t$$

*Answer Box 6*

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Place your answers in the spaces provided. **You must show work to receive any credit.**

7. (*14 points*) A spring is stretched 6 in. by a mass that weighs 8 lb. The system is acted on by an external force of  $6 \sin 2t$  lb. If the mass is set in motion from its equilibrium position with a downward velocity of 12 in./sec, find its position  $u$  at any time  $t$ . Assume that there is no air resistance.

*Answer Box 7*

$u(t) =$

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Place your answers in the spaces provided. **You must show work to receive any credit.**

8. (12 points) Use the Laplace transform to solve the given initial value problem.

$$y'' - y' - 12y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

*Answer Box 8*

$y(t) =$
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**Elementary Laplace Transforms:**

$f(t)$	$F(s)$
1	$\frac{1}{s}$
$t^n$	$\frac{n!}{s^{n+1}}$
$e^{at}$	$\frac{1}{s - a}$
$\sin at$	$\frac{a}{s^2 + a^2}$
$\cos at$	$\frac{s}{s^2 + a^2}$
$e^{at} \sin bt$	$\frac{b}{(s - a)^2 + b^2}$
$e^{at} \cos bt$	$\frac{s - a}{(s - a)^2 + b^2}$