

1. Find the solution of the initial value problem  $y' = 2y - 1$ ,  $y(0) = 1$ .  
 $\phi(t) =$  \_\_\_\_\_

Find the approximate value of the solution of the initial value problem  $y' = 2y - 1$ ,  $y(0) = 1$ , where  $t = 0.4$  using:

- the Euler method (eul) with  $h = 0.1$ \_\_\_\_\_
- the Euler method (eul) with  $h = 0.05$ \_\_\_\_\_
- the Euler method (eul) with  $h = 0.025$ \_\_\_\_\_
- the improved Euler method (rk2) with  $h = 0.1$ \_\_\_\_\_
- the Runge–Kutta method (rk4) with  $h = 0.1$ \_\_\_\_\_
- the solution  $\phi(t)$ \_\_\_\_\_

2. Find the approximate value of the solution of the initial value problem  $y' = \sqrt{t+y}$ ,  $y(1) = 3$ , where  $t = 2$  using :

- the Euler method (eul) with  $h = 0.025$ \_\_\_\_\_
- the Euler method (eul) with  $h = 0.0125$ \_\_\_\_\_
- the improved Euler method (rk2) with  $h = 0.1$ \_\_\_\_\_
- the improved Euler method (rk2) with  $h = 0.05$ \_\_\_\_\_
- the Runge–Kutta method (rk4) with  $h = 0.2$ \_\_\_\_\_
- the Runge–Kutta method (rk4) with  $h = 0.1$ \_\_\_\_\_

3. Give reasons why the Euler tangent line method with  $h = 0.1$  does not give a good approximation of the value of the solution of the initial value problem where  $t = 1$ .

(a)  $y' = (y + 1.25)^2$ ,  $y(0) = 0$ ,

solution  $y = \frac{25t}{4(4 - 5t)}$ .

(b)  $y' = \frac{50t}{64(1 - 2y)}$ ,  $y(0) = 0$ ,

solution  $y = \frac{1 - \sqrt{1 - 25t^2/16}}{2}$ .

(c)  $y' = 2(ty)^{1/3}$ ,  $y(0) = 0$ ,

solution  $y = t^2$ .

(d)  $y' = 4e^{-t} - 3(1 - y)$ ,  $y(0) = 0$ ,

solution  $y = 1 - e^{-t}$ .