Review 2 Math 266 Summer 2001

APPLICATIONS

You should be able to set up and solve up and solve problems of the types:

- Rate of change of y is proportional to y, $y' = ky$.
- Newton's Law of Cooling, $T' = k(T - T_e)$, where T_e is the temperature of the surrounding environment.
- Problems in mechanics, $F = ma$ or $m\dfrac{dv}{dt} =$ (Sum of external forces).

Forces on an object that is moving vertically near the surface of the earth are gravity, $w = mg$, and a force proportional to velocity, so $m\dfrac{dv}{dt} = \pm mg - kv$.

For rocket problems, the force due to gravity is $F = -\dfrac{mgR^2}{(x+R)^2}$.

Use $\dfrac{dv}{dt} = v\dfrac{dv}{dx}$ and find the velocity as a function of the height above the surface of the earth.

- Mixing problems. Let $Q(t)$ be the amount of whatever is in solution and let $V(t)$ be the amount of solution.

The concentration of Q at time t is then $c(t) = \dfrac{Q(t)}{V(t)}$, so $Q(t) = c(t)V(t)$.

Rate of Q coming in is (Concentration in)(Rate of solution in).

Rate of Q going out is (Concentration at time t)(Rate of solution out).

$\dfrac{dQ}{dt} =$ (Rate of Q coming in) $-$ (Rate of Q going out).

Caution: if (Rate of solution in) $\neq$ (Rate of solution out) then you must solve for $V(t)$ from $\dfrac{dV}{dt} =$ (Rate of solution in) $-$ (Rate of solution out). If the in rates and out rates are constant, then $V(t) = V(0) + kt$, where k is the difference of the rates.

MA 266 SPR 01 REVIEW 2 PRACTICE QUESTIONS

1. Suppose y' is proportional to y, $y(0) = 4$, and $y(2) = 2$. Find y in terms of t. For what value of t does $y(t) = 3$?

2. A thermometer reads 36° when it is moved into a 70° room. Five minutes later the thermometer reads 50°. Find the thermometer reading t minutes after it is moved into the room. What will it read ten minutes after it is moved into the room?

3. Determine the vertical velocity of a 128–lb parachutist t seconds after jumping from an airplane that is flying slowly and horizontally at an altitude of 5000 feet. Assume that air resistance is eight times the speed and ignore horizontal motion.

4. A rocket is launched from earth with initial vertical velocity $\sqrt{Rg}$. Find a formula that relates the velocity of the rocket and its height. What is the maximum height of the rocket? Assume that gravity is the only force acting on the rocket and that the mass of the rocket is constant.

5. A 500–gal tank contains 200 gal of brine with salt concentration of 2 oz/gal. Pure water flows into the tank at a rate of 10 gal/min, while the mixture flows out of the tank at a rate of 5 gal/min. Find the salt concentration in the tank at the time the tank becomes completely filled.

6. Consider the differential equation $\dfrac{dy}{dt} = y(y-2),\ t \geq 0,$. Hint: draw a rough graph of $F(y) = y(y-2)$ to analyse where y' is positive and negative.

(a) Sketch the graph of the solution of the differential equation for $t \geq 0$ with each of the initial values $y(0) = -2/3,\ y(0) = 0,\ y(0) = 2/3,\ y = 4/3,\ y(0) = 2,\ y(0) = 8/3$.

(b) What are the equilibrium solutions?

(c) Which equilibrium solutions are stable?

(d) For which intervals of y is the graph of $y(t)$ increasing?

(e) For which intervals of y is the graph of $y(t)$ concave upward?

7. Solve the initial value problem $y' + 2y = 0,\ y(0) = 3$.

8. Solve the initial value problem $y' = 2(y-1),\ y(0) = 3$.

9. Solve the initial value problem $y' = -\dfrac{1}{(t+1)^2},\ y(0) = 1$. What is the largest open interval on which the solution is valid?

10. Solve the initial value problem $y' = y(y-1),\ y(0) = 2$. What is the largest open interval on which the solution is valid?

11. Solve the initial value problem $y'' = y'/x,\ y(1) = 4,\ y'(1) = 2$.

12. Solve the initial value problem $y'' = (y')^2/y,\ y(0) = e,\ y'(0) = 1$.

13. For each of the initial value problems determine the largest interval for which a unique solution is guaranteed:

(a) $y' - \dfrac{2}{t}y = \dfrac{1}{t},\ y(1) = 0$

(b) $y' + (\tan t)y = \sec t,\ y(0) = 0$

(c) $y' + \dfrac{t}{t^2-9}y = \dfrac{1}{t-2},\ y(0) = 1$

(d) $(t+4)y' - ty = \dfrac{1}{t},\ y(-2) = 1$

14. For each of the initial value problems determine all initial points (t_0, y_0) for which a unique solution is guaranteed in some interval $t_0 - h < t < t_0 + h$:

(a) $y' = t^2 + y^2,\ y(t_0) = y_0$

(b) $y' = \sqrt{t^2 + y^2},\ y(t_0) = y_0$

(c) $y' = t/y,\ y(t_0) = y_0$

(d) $y' = t^{1/3}y^{2/3},\ y(t_0) = y_0$

MA 266 SPR 01 REVIEW 2 PRACTICE QUESTION ANSWERS

1. $y = 4e^{(\ln 0.5)t/2}$, $t = \dfrac{2\ln 0.75}{\ln 0.5} \approx 0.83.$

2. $T = 70 - 34e^{(\ln(10/17))t/5}$, $T(10) \approx 58.2°$.

3. $v(t) = -16 + 16e^{-2t}$

4. $v^2 = \dfrac{2R^2 g}{R + x} - Rg$, $x_{\max} = R$ when $v = 0$.

5. $c(60) = 0.32$ oz/gal

6. $y' > 0$ if either $y < 0$ or $y > 2$. $y' < 0$ if $0 < y < 2$.

(a) For $y(0) = -2/3$, $y(t)$ increases asympt. to line $y = 0$. For $y(0) = 0$, stays on $y = 0$ line. For $y(0) = 2/3$, or $4/3$, $y(t)$ decreases asympt. to $y = 0$ line. For $y(0) = 2$, stays on $y = 2$ line. For $y(0) = 8/3$, decreases asympt. to line $y = 2$. (b) $y = 0$, $y = 2$

(c) $y = 0$

(d) $y < 0$, $y > 2$

(e) $0 < y < 1$, $y > 2$

7. $y = 3e^{-2t}$

8. $y = 1 + 2e^{2t}$

9. $y = \dfrac{1}{t+1}$, $t > -1$

10. $y = \dfrac{2}{2 - e^t}$, $t < \ln 2$

11. $y = 3 + x^2$

12. $y = e \cdot e^{x/e}$

13. (a) $t > 0$, (b) $-\dfrac{\pi}{2} < t < \dfrac{\pi}{2}$, (c) $-3 < t < 2$, (d) $-4 < t < 0$

14. (a) All (t_0, y_0). (b) All (t_0, y_0) except $(0,0)$. (c) All (t_0, y_0) with $y_0 \neq 0$. (d) All (t_0, y_0) with $y_0 \neq 0$.