Mathematical Analysis of Novel Advanced Materials: Epitaxy and Quantum Dots

Plenary Speaker: Irene Fonseca

Abstract

Quantum dots are man-made nanocrystals of semiconducting materials. Their formation and assembly patterns play a central role in nanotechnology, and in particular in the optoelectronic properties of semiconductors. Changing the dots' size and shape gives rise to many applications that permeate our daily lives, such as the new Samsung QLED TV monitor that uses quantum dots to turn "light into perfect color"!

Quantum dots are obtained via the deposition of a crystalline overlayer (epitaxial film) on a crystalline substrate. When the thickness of the film reaches a critical value, the profile of the film becomes corrugated and islands (quantum dots) form. As the creation of quantum dots evolves with time, materials defects appear. Their modeling is of great interest in materials science since material properties, including rigidity and conductivity, can be strongly influenced by the presence of defects such as dislocations.

In this talk we will use methods from the calculus of variations and partial differential equations to model and mathematically analyze the onset of quantum dots, the regularity and evolution of their shapes, and the nucleation and motion of dislocations.

Geometric structures and Invariants of 3-manifolds

Plenary Speaker: Efstratia Kalfagianni

Abstract

After introducing some background, I will survey open conjectures and recent results on relations between Geometric structures of 3-manifolds (in the sense of Thurston) and Quantum invariants of 3-manifolds.

Symplectic fillings, caps, and the Weinstein conjecture

Bahar Acu

Abstract

Contact and symplectic geometry, motivated by mathematical formalism of classical mechanics, is the study of certain geometric structures on odd and even, respectively, dimensional smooth topological manifolds. Under certain conditions, contact geometric data can be studied on the boundary of symplectic manifolds. We then call those symplectic manifolds "symplectic fillings" of the bounding contact manifolds. Several aspects of "planar" contact 3-manifolds have been intensively studied. In this talk, we define "iterated planar contact manifolds", a higher-dimensional analog of planar contact manifolds, by using topological tools such as "open book decompositions" and "Lefschetz fibrations". We provide some history on existing low-dimensional results regarding symplectic fillings and caps of contact manifolds and explain generalization of those results to higher dimensions via iterated planar structure.

An optimization-based coupling strategy for classical and nonlocal elasticity

Marta D'Elia

Abstract

The use of nonlocal models in science and engineering applications has been steadily increasing over the past decade. The ability of nonlocal theories to accurately capture effects that are difficult or impossible to represent by Partial Differential Equations (PDEs) motivates and drives the interest in this type of simulations. However, the improved accuracy of nonlocal models comes at the price of a significant increase in computational costs. As a result, it is important to develop local-to-nonlocal coupling strategies, which aim to combine the accuracy of nonlocal models with the computational efficiency of PDEs. We develop an optimization-based method for the coupling of nonlocal and local problems in the context of nonlocal elasticity. The approach formulates the coupling as a control problem where the states are the solutions of the nonlocal and local equations, the objective is to minimize their mismatch on the overlap of the nonlocal and local domains, and the controls are virtual volume constraints and boundary conditions. Numerical results for nonlocal diffusion and nonlocal elasticity in three-dimensions illustrate key properties of the optimization-based coupling method; these numerical tests provide the groundwork for the development of efficient and effective engineering analysis tools.

Efficient Numerical Treatment of High-Contrast Composite Materials

Yuliya Gorb

Abstract

This talk concerns a robust numerical treatment of an elliptic PDE with high contrast coefficients. A finite-element discretization of such an equation yields a linear system whose conditioning worsens as the variations in the values of PDE coefficients become large. We introduce a procedure by which a discrete system obtained from a linear finite element discretization of the given continuum problem is converted into an equivalent linear system of the saddle point type. A robust preconditioner for the Lancsoz iterative method to solve this saddle point problem is proposed. We demonstrate that the number of iterations is independent of the contrast and the discretization scale.

Connected Heegaard Floer homology and homology cobordism

Kristen Hendricks

Abstract

We study applications of the 3-manifold invariant Heegaard Floer homology to integer homology cobordism. In particular, to a homology sphere Y, we define a module $HF_{conn(Y)}$, called the connected Heegaard Floer homology of Y, and show that this module is invariant under homology cobordism and isomorphic to a summand of $HF_{red(Y)}$. We use this to define a new filtration on the homology cobordism group, and to give a reproof of Furuta's theorem. This is joint work with Jen Hom and Tye Lidman .

A fast spectral method for the Boltzmann collision operator with general collision kernels

Jingwei Hu

Abstract

We propose a simple fast spectral method for the Boltzmann collision operator with general collision kernels. In contrast to the direct spectral method (Pareschi and Russo 00, Gamba and Tharkabhushanam 09), which requires $O(N^6)$ memory to store precomputed weights and has $O(N^6)$ numerical complexity, the new method has complexity $O(MN^4 \log N)$, where N is the number of discretization points in each of the three velocity dimensions and M is the total number of discretization points on the sphere and $M \ll N^2$. Furthermore, it requires no pre-computation for the variable hard sphere model and only $O(MN^4)$ memory to store precomputed functions for more general collision kernels. Although a faster spectral method is available (Mouhot and Pareschi 06) (with complexity $O(MN^3 \log N)$), it works only for hard sphere molecules, thus limiting its use for practical problems. Our new method, on the other hand, can apply to arbitrary collision kernels. A series of numerical tests is performed to illustrate the efficiency and accuracy of the proposed method. Joint work with I. Gamba (UT-Austin), J. Haack (LANL) and C. Hauck (ORNL).

Lagrangians and constructible sheaves

Xin Jin

Abstract

I will give an overview of sheaf theoretic method of studying Lagrangian submanifolds, which is a topological and combinatorial alternative of Floer theory. If time permits, I will present recent results on quantizing exact Lagrangian submanifolds in cotangent bundles (joint with D. Treumann) and some interesting examples beyond cotangent bundles.

Biology and Computation

Kay Kirkpatrick

Abstract

We will discuss newly defined machines that out-perform Turing machines. In his unpublished 1948 paper, Intelligent Machinery, Alan Turing identified several types of machines, with one dichotomy that is false, between active and controlling machines. I'll introduce a new kind of machine and define a subtype, an automatic biochemical machine, that is equivalent to a deterministic Turing machine with two oracle machines as adjuncts.

Quantitative Stochastic Homogenization for Elliptic Equations

Jessica Lin

Abstract

Stochastic homogenization is concerned with identifying the asymptotic behavior of solutions to PDEs with random coefficients. Specifically, we are interested in the following: if the coefficients are randomly varying on a microscopic lengthscale, then on average, do the random solutions exhibit the same deterministic behavior? When this is indeed the case, we say that the random equation "homogenizes." Furthermore, from both the theoretical and applied perspective, an important issue is to understand the quantitative aspects of this homogenization process. In this talk, I will present an overview of the subject of stochastic homogenization for nondivergence form elliptic equations. I will discuss the interplay between PDE and probabilistic techniques used to study these types of problems. In addition, I will present a recent quantitative result which yields the optimal error estimates on the size of the fluctuations of the random solutions. This talk is based on joint work with Scott Armstrong.

On minimizers and critical points for anisotropic isoperimetric problems

Robin Neumayer

Abstract

Anisotropic surface energies are a natural generalization of the perimeter that arise in models for equilibrium shapes of crystals. We discuss some recent results for anisotropic isoperimetric problems concerning the strong quantitative stability of minimizers, bubbling phenomena for critical points, and a weak Alexandrov theorem for non-smooth anisotropies. Part of this talk is based on joint work with Delgadino, Maggi, and Mihaila.

Regularity of interfaces for a Pucci-type segregation problem
Stefania Patrizi

Abstract

Motivated by a model studied by V. Quitalo and describing population segregation, we consider a free boundary problem involving Pucci-type operators. We show the existence of a Lipschitz viscosity solution and prove that the set of regular points of the free boundary, i.e. the boundary of the positivity set of the solution, is relatively open and locally of class $C^{1,\alpha}$ (joint work with L. Caffarelli, V. Quitalo and M. Torres).

A localized view of nonlocal models
Petronela Radu

Abstract

The emergence of nonlocal theories as promising models in different areas of science (continuum mechanics, biology, image processing) has led the mathematical community to conduct varied investigations of systems of integro-differential equations. In this talk I will present some recent results on existence and regularity of solutions to integral equations with weakly singular kernels. This work is part of a developing theory that is the nonlocal counterpart of elliptic regularity; indeed, many nonlocal results mimic well-known properties and theorems that hold for elliptic systems. The implications could have far-reaching applications in mathematics, at both theoretical and applied levels.

The signature modulo 8 of surface bundles
Carmen Rovi

Abstract

In general the signature of a fiber bundle is not multiplicative. In 1973, Werner Meyer used group cohomology to show that a surface bundle has signature divisible by 4 always. Divisibility by 8 is does not hold in general. In this talk, I will discuss the connection between the signature and the Arf and Brown-Kervaire invariants which sheds light onto the problem of multiplicativity of the signature modulo 8. I will also explain the connection to current work with David Benson, Caterina Campagnolo and Andrew Ranicki where we are using group cohomology and representation theory of finite groups to detect non-trivial signatures modulo 8 of surface bundles.

Dispersive Lamb Systems

Natalie Sheils

Abstract

Under periodic boundary conditions, a one-dimensional dispersive medium driven by a Lamb oscillator exhibits a smooth response when the dispersion relation is asymptotically linear or superlinear at large wave numbers, but unusual fractal solution profile emerge when the dispersion relation is asymptotically sublinear. Strikingly, this is exactly the opposite of the superlinear asymptotic regime required for fractalization and dispersive quantization, also known as the Talbot effect, of the unforced medium induced by discontinuous initial conditions.

Nonuniform dependence on initial data for compressible gas dynamics: The periodic Cauchy problem

Feride Tiglay

Abstract

We start with the classic result that the Cauchy problem for ideal compressible gas dynamics is locally well posed in time in the sense of Hadamard; there is a unique solution that depends continuously on initial data in Sobolev space H^s for $s > d/2 + 1$ where d is the space dimension. We prove that the data to solution map for periodic data in two dimensions, although continuous, is not uniformly continuous (joint work with B. Keyfitz).

Asymptotic structure of self-shrinkers of mean curvature flow

Lu Wang

Abstract

Self-shrinkers are a special class of solutions of mean curvature flow, in which a later time slice is a scale-down copy of earlier ones. In this talk, we will show that each end of any noncompact self-shrinker in 3-Euclidean space of finite topology is smoothly asymptotic to a cone with isolated singularities or a round cylinder, thus confirming an assertion of Ilmanen.