Singularities of Algebraic Varieties

Abstract:

Algebraic geometry is the study of algebraic varieties, or geometric shapes described by polynomial equations. You already know many examples, such as the circle, whose polynomial equation is $x^2 + y^2 = 1$, or a sphere. Algebraic varieties are ubiquitous throughout mathematics and its applications to science and engineering. Not only do they naturally arise in important contexts—the set of all rigid transformations of space, for example, can be given the structure of an algebraic variety—but also all kinds of complicated behavior can be described (or approximated) by polynomials. At the same time, polynomials are relatively easy to manipulate by hand or machine, so algebraic geometry is a tool for scientists, engineers and even artists, as well as a rich source of examples for mathematicians in almost any field. While conic sections and spheres are smooth varieties, in general, a variety can have singular points—places where it is pinched or intersects itself.

In this talk, we will discuss Hironaka's famous theorem on resolution of singularities—a technique to "get rid" of the singular points. We introduce a class of singular varieties called rational singularities that are important because they are well-approximated by their resolutions, and explain how one can use "reduction modulo $p$" to characterize them.

Speaker:
Karen Smith
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Karen E. Smith was born in Red Bank, New Jersey, near the Jersey shore. Although she always loved mathematics and wanted to be a mathematician from a young age, she did not realize that one could have a career as a mathematician until college, when her freshman calculus teacher, Professor Charles Fefferman, suggested it. She graduated from Princeton University in 1987 with a major in mathematics and certification to teach high school mathematics in New Jersey public schools. After teaching high school mathematics for a year, she looked into the possibilities of graduate school and learned that one could actually get full support to work on a Ph.D. At this point, she decided to make a big change, and went off to the midwest for graduate school.

At the University of Michigan, Smith wrote a thesis in commutative algebra under the direction of Professor Melvin Hochester, finishing in 1993. After spending one year working with Craig Huneke at Purdue University on an NSF postdoctoral fellowship, she moved to Massachusetts to be a Moore Instructor at MIT. Although she enjoyed Boston and was promoted to Assistant Professor at MIT, she and her husband moved back to Ann Arbor in 1997, where they had met nine years earlier. Smith is now teaching and doing research in algebraic geometry and commutative algebra at the University of Michigan. She has a daughter, Sanelma, born in 1998 with whom she very much enjoys discussing mathematics, and boy-girl twins, Tapio and Helena, born in 2003.

Karen Smith was awarded the 2001 Ruth Lyttle Satter Prize in recognition of her work in commutative algebra. In 2015, she was named a Fellow of the American Mathematical Society. She was chosen to deliver the AMS-AWM Noether Lecture at the Joint Mathematics Meeting in 2016. In 2019, she was elected to the National Academy of Sciences, and she was inducted into the 2020 class of Fellows of the Association for Women in Mathematics.