MA 15200 Lesson 1 Section P.1 (part 1)

I Algebraic Expressions

An algebraic expression can be a number, a variable, or a combination of numbers and variables using the operations of addition, subtraction, multiplication, division, powers, absolute values, or roots. **An algebraic equation is not an algebraic expression** (an equation contains algebraic expressions). Most of the problems you will be given in this course will fall either the category of algebraic expression or category of algebraic equation.

Algebraic expression examples: $2x+5$, $3x^2+\sqrt{3x}$, $\dfrac{4x+1}{29x}$

Directions for algebraic expression problems include **simplify, evaluate, add, subtract, multiply, divide, or factor.**

One type of algebraic expression is an **exponential expression**.

$b^n=(b)(b)(b)...(b)$ the factor b used n times

$4^5=4\cdot4\cdot4\cdot4\cdot4$ $(-3)^4=(-3)(-3)(-3)(-3)$

The **base** b is the factor and the **exponent** n describes how many times it is a factor.

II Evaluating Algebraic Expressions

Evaluating an algebraic expression means to find the value of the expression for a given value of the variable(s).

The Order of Operations Agreement

1. **Perform operations within the innermost grouping and work outwards. If the expression involves a fraction, treat the numerator and denominator as if they were each a group. Grouping can include parentheses, brackets, a root, or absolute value bars.**
2. **Evaluate all exponential expressions.**
3. **Perform multiplication and/or division as they occur, working from left to right.**
4. **Perform addition and/or subtraction as they occur, working from left to right.**

Ex 1: Evaluate $3x^3-4(x-2)^2+9$ for $x=4$

Ex 2: Evaluate the following expression, if $x = 3$, $y = -2$, and $z = 4$

$$\frac{4x^2(y-2z)}{xy^2-2z}$$

A formula is an **equation** that uses variables to express a relationship between two or more quantities. The process of determining a formula is called mathematical modeling. For example, in the textbook, the following formula represent the heart rate for a 'couch-potato' (in beats per minute) given the age of the person (in years).

$$H = \frac{1}{5}(220-a)$$

In the above formula or mathematical model, H represent the number of beats of the heart per minute and a is the age of the 'couch-potato'. If a person aged 25 was a 'couch-potato', the following work would find a good approximation for his/her heart rate.

$$H = \frac{1}{5}(220-a)$$

$$H = \frac{1}{5}(220-25)$$ 39 beats per minute

$$H = \frac{1}{5}(195)$$

$$H = 39$$

Ex 3: Suppose $H = \frac{9}{10}(220-a)$ models the heart rate of a person working out. Find the number of beats per minute, if the person is 30 years old.

Ex 4: Use the formula $F = \frac{9}{5}C + 32$ to find the Fahrenheit temperature corresponding to 39°C.

<u>III Sets of Numbers</u>

1. Natural Numbers: *{1, 2, 3, 4, ...}
   *Note: This type of set notation is called **roster set notation**.

2. Whole Numbers: {0, 1, 2, 3, ...}
   Natural numbers + Zero

3. Integers: {..., -3, -2, -1, 0, 1, 2, 3, ...}
   Whole numbers + Opposites of wholes (negatives)

4. Rational Numbers: $*\left\{ \frac{a}{b} \mid a \text{ is an integer and } b \text{ is a nonzero integer} \right\}$

   *Note: This type of set notation is called **set-builder notation**.

   Rational numbers include integers, fractions (proper, improper, or mixed numbers), terminating decimals, and repeating decimals.

   $-8 = -\frac{8}{1}$ $\frac{11}{8} = 1.375$ $\frac{4}{5} = 0.8$

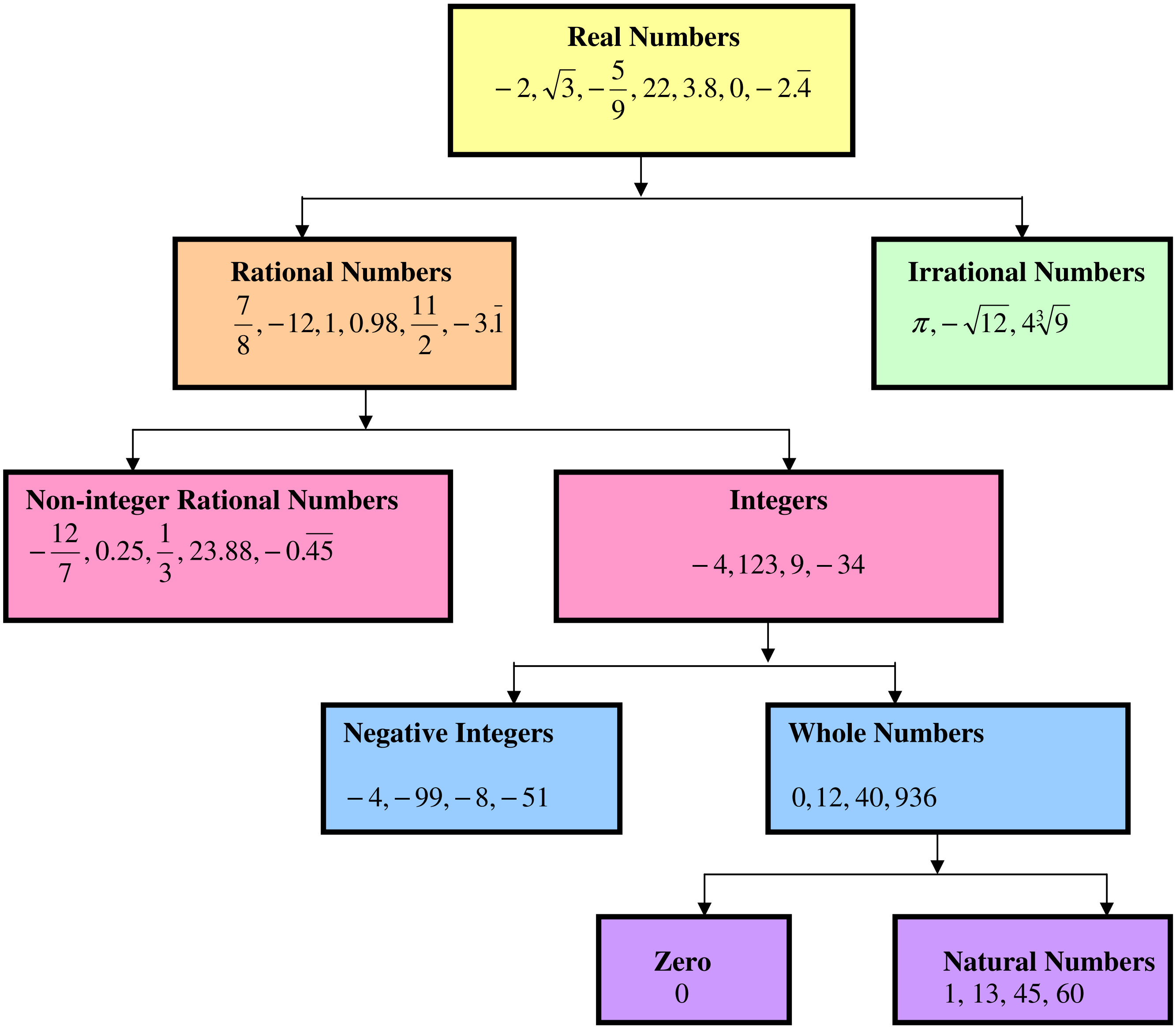
   $-\frac{2}{3} = -0.666\ldots \text{ or } -0.\overline{6}$ $\frac{13}{11} = 1.181818\ldots. \text{ or } 1.\overline{18}$

5. Irrational Numbers: $\{x \mid x$ is a non-terminating or a non-repeating decimal$\}$
   Most irrational numbers are roots. Another well-known irrational number is π.

6. Real Numbers: $\{x \mid x$ is rational or irrational$\}$

Your textbook, on page 7, has one picture of how these sets of numbers are related. Here is another picture that shows how one set of numbers may have other numbers added to it to create a new set of numbers. For example, the set of integers is created when the opposites of each whole number are added to the set of whole numbers.

Relationship between Sets of Numbers and Examples



Ex 5: Describe each statement as true or false.

a) Every natural number is a whole number.

b) Every integer is a whole number.

c) Every irrational number is a real number.

Ex 6: Given the following real numbers,

$$\pi,\ \frac{3}{4},\ -2,\ 0,\ -\sqrt{2},\ \frac{8}{5},\ 3.\overline{4},\ -2.83,\ 2\sqrt{3},\ -6, -\frac{11}{4},\ 20,\ 3$$

a) Which numbers are whole numbers?

b) Which numbers are integers?

c) Which numbers are rational numbers?

d) Which numbers are irrational numbers?

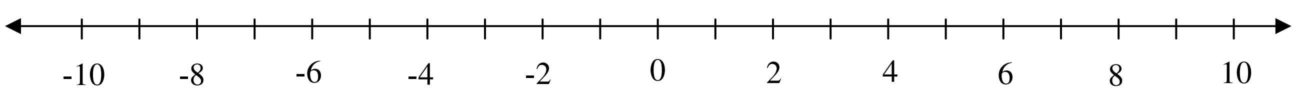
Ex 7: The number -2.3 would be a member of which of the following sets?
natural, whole, integers, rational, irrational, real

Some additional sets of numbers that you should be familiar with are the following.
A **prime number** is a natural number greater than 1 divisible by only 1 and itself.
The first few prime numbers are 2, 3, 5, 7, 11, 13, 17.
A **composite number** is a natural number greater than 1 that is not prime.
The first few composite numbers are 4, 6, 8, 9, 10, 12.
An **even number** is an integer that is divisible by 2. An **odd number** is an integer than is not divisible by 2.
Evens: {..., -4, -2, 0, 2, 4, ...} Odds: {..., -3, -1, 1, 3, ...}

Later in the semester, we will discuss numbers 'beyond' real numbers.

IV Inequalities

An inequality is a statement involving less than or greater than symbols. On a **real number line,** positive numbers are to the right of zero and negatives are to the left of zero. If a number is left of a second number, it is less than the second number. If a number is right of a second number, it is greater than the second number.



A statement such as $a \le b$ is a compound inequality. It says either $a < b$ or $a = b$.

Ex 8: Describe each as true or false.

$a)$ $-5 > -3$

$b)$ $-\frac{3}{5} \le -\frac{3}{5}$

V Absolute Value

The absolute value of a real number *a*, denoted by $|a|$, is the distance from zero to *a* on a real number line. An absolute value is always positive, because distance is always positive.

$$|x| = \begin{cases} x & if \;\; x \geq 0 \\ -x & if \;\; x < 0 \end{cases}$$

Ex 9: Evaluate each.

$a)\;\; \left|\frac{3}{4}\right| =$ $\qquad$ $b)\;\; |-2.8| =$

$c)\;\; |0| =$ $\qquad$ $d)\;\; \frac{|-12|}{3} =$

$e)\;\; |2-\sqrt{7}| =$

$f)\;\; |\pi - 8| =$

Ex 10: Evaluate the following.

$a)\;\; |x-y|$ if $x=-12$ and $y=4$

$b)\;\; |x|-|y|$ if $x=-12$ and $y=4$

VI Distance between two Numbers or two Points on a Real Number Line

The distance between two numbers is represented by an absolute value. Let *a* and *b* represent any two numbers. The distance, or number of units, between the two numbers is given by $|a-b|$ or $|b-a|$.

Ex 11: Find the distance between each pair of numbers.

$a)\;\; -\frac{3}{4}$ and $-\frac{7}{3}$

$b)\;\; -40$ and 62

VII Properties of Real Numbers

The real numbers have the following properties:
If *a, b,* and *c* are real numbers,

1. **Associative Properties for Addition and Multiplication**

   $(a+b)+c = a+(b+c) \qquad (ab)c = a(bc)$

   This property states that changing grouping when adding or multiplying does not affect the sum of product.

2. **Commutative Properties for Addition and Multiplication**
   $a+b=b+a \qquad ab=ba$

This property states that changing the order when adding or multiplying does not affect the sum or product.

3. **Distributive Property of Multiplication over Addition**
   $a(b+c)=ab+ac \qquad a(b-c)=ab-ac$

   Note: This property applies if there are more than 2 terms within parentheses.

This property shows multiplication can be distributed over addition or subtraction and is used most often to either clear parentheses or (in reverse) to factor.

4. **The Double Negative Rule**
   $-(-a)=a$

There are identity and inverse properties for addition and multiplication, but you are not responsible to identify these properties.

Ex 12: Identify which property was used.

$a)\ \ (6+8)+2=2+(6+8)$

$b)\ \ -4+(6+2)+9=(-4+6)+(2+9)$

$c)\ \ 4x+8=4(x+2)$