The Addition and Subtraction Formulas

COFUNCTIONS:
We refer to the sine and cosine functions as cofunctions of each other. Similarly, the tangent and cotangent functions are cofunctions, as are the secant and cosecant.

If u is the radian measure of an acute angle, then the angle with radian measure $\left(\frac{\pi}{2}-u\right)$ is complementary to u. If u is the degree measure of an acute angle, then the angle with degree measure $(90°-u)$ is complementary to u.

A trigonometric function value of an angle is equal to the trigonometric cofunction value of the complementary angle. This relationship is stated in the Cofunction Formulas below.

Cofunction Formulas:

If u is a real number or the radian measure of an angle, then

(1) $\cos\left(\frac{\pi}{2}-u\right)=\sin u$ (2) $\sin\left(\frac{\pi}{2}-u\right)=\cos u$

(3) $\tan\left(\frac{\pi}{2}-u\right)=\cot u$ (4) $\cot\left(\frac{\pi}{2}-u\right)=\tan u$

(5) $\sec\left(\frac{\pi}{2}-u\right)=\csc u$ (6) $\csc\left(\frac{\pi}{2}-u\right)=\sec u$

If u is the degree measure of an angle, then

(1) $\cos(90°-u)=\sin u$ (2) $\sin(90°-u)=\cos u$

(3) $\tan(90°-u)=\cot u$ (4) $\cot(90°-u)=\tan u$

(5) $\sec(90°-u)=\csc u$ (6) $\csc(90°-u)=\sec u$

Express as a cofunction of a complementary angle.

tan(23.54°)

sin(1/6)

cos(π/5)

csc(0.74)

The Addition and Subtraction Formulas

We will now talk about the formulas that involve trigonometric functions of $(u + v)$ or $(u - v)$ for any real numbers or angles u and v. These formulas are known as ***addition* and *subtraction formulas*,** respectively, or as ***sum* and *difference identities.*** These formulas are found on the formula sheet shown on the course web page and this formula sheet will be given to you on the remaining exams.

Addition and subtraction formulas for Cosine, Sine and Tangent

$\sin(u+v) = \sin u \cos v + \cos u \sin v$ $\sin(u-v) = \sin u \cos v - \cos u \sin v$

$\cos(u+v) = \cos u \cos v - \sin u \sin v$ $\cos(u-v) = \cos u \cos v + \sin u \sin v$

$\tan(u+v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v}$ $\tan(u-v) = \dfrac{\tan u - \tan v}{1 + \tan u \tan v}$

CAUTION: Never use the distributive property as below.

$\sin(u+v) \neq \sin u + \sin v$

$\tan(u-v) \neq \tan u - \tan v$

Find the exact values. Notice: parts (a) of each problem have nothing to do with the formulas above.

1a) $\sin(2\pi/3) + \sin(\pi/4)$

b) $\sin(11\pi/12)$
(Hint: Use $11\pi/12 = 2\pi/3 + \pi/4$)

2a) $\tan(30°) + \tan(225°)$

b) $\tan(255°)$
(Hint: Use $225° = 30° + 225°$)

The Addition and Subtraction Formulas

3a) $\cos(\pi/3) - \cos(\pi/4)$

b) $\cos(\pi/12)$
(Hint: Use $\pi/12 = \pi/3 - \pi/4$)

Express as a trigonometric function of one angle. (Match with sum or difference formulas.)

$\cos 17° \cos 25° - \sin 17° \sin 25°$

$\sin 25° \cos 17° - \cos 25° \sin 17°$

<u>These formulas can be used to find the quadrant where the terminal side of the sum of two angles is found.</u>

If α and β are acute angles such that $\tan\alpha = \frac{15}{8}$ and $\sin\beta = \frac{12}{13}$, find

a) $\sin(\alpha + \beta)$

b) $\cos(\alpha + \beta)$

c) The quadrant containing $\alpha + \beta$

The Addition and Subtraction Formulas

If α and β are acute angles such that $\csc\alpha = \frac{5}{3}$ and $\cot\beta = \frac{24}{7}$, find

a) $\sin(\alpha - \beta)$

b) $\tan(\alpha - \beta)$

c) The quadrant containing $\alpha - \beta$

The Addition and Subtraction Formulas

Using addition formulas to find the quadrant containing an angle.

If $\tan\alpha = \frac{15}{8}$ and $\sec\beta = -\frac{13}{12}$ for <u>a third-quadrant angle α</u> and <u>second-quadrant angle β,</u> find

a) $\sin(\alpha+\beta)$

b) $\cos(\alpha+\beta)$

c) The quadrant containing $\alpha + \beta$

The Addition and Subtraction Formulas

If α and β are <u>fourth-quadrant angles</u> such that $\sec\alpha = \frac{5}{4}$ and $\cot\beta = -\frac{24}{7}$, find

a) $\cos(\alpha - \beta)$

b) $\tan(\alpha - \beta)$

c) The quadrant containing α - β

The Addition and Subtraction Formulas

Verify each identity:

$$\cos\left(\theta+\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}(\cos\theta-\sin\theta)$$

$$\tan\left(x-\frac{\pi}{4}\right)=\frac{\tan x-1}{\tan x+1}$$

$$\cos(h+t)+\cos(h-t)=2\cos(h)\cos(t)$$