

MA 15200 Lesson 14 Section 1.5 (part 1)

The **general form** of a **quadratic equation** is $ax^2 + bx + c = 0$, where a , b , and c are real numbers and $a \neq 0$. This is a second degree equation.

There are four ways to possibly solve quadratic equations.

1. **Solving by Factoring**
2. **Solving by the Square Root Property**
3. **Solving by Completing the Square**
4. **Solving with the Quadratic Formula**

I Solving by Factoring

Zero-Product Principle: If the product of two algebraic expressions is zero, then at least one of the factors is equal to zero. If $AB = 0$, then $A = 0$ or $B = 0$.

Steps:

1. If necessary, rewrite the equation in general form (zero on one side, polynomial in descending order).
2. Factor the polynomial completely.
3. Apply the zero-product principle, setting each factor containing a variable equal to zero.
4. Solve the resulting equations.

Ex 1: Solve by using factoring.

a) $x^2 = 28 - 3x$

b) $x^2 - 121 = 0$

c) $3x^2 + 30x = 0$

d) $5x + 12x^2 = 2$

Note: If a quadratic equation can be solved by factoring, the solution(s) is(are) rational numbers. If the equation cannot be solved by factoring, the solutions are conjugate irrational numbers or conjugate complex numbers.

II Solving by the Square Root Property

Square Root Property:

If the quadratic equation can be written in the form $u^2 = d$, where u is a variable or an expression with a variable and d is nonzero real number, then the solutions can be found using the equations $u = \sqrt{d}$ and $u = -\sqrt{d}$.

Ex 2: Use the square root property to solve each equation.

a) $(r - 1)^2 = 25$

b) $(2y - 5)^2 - 49 = 0$

c) $9x^2 + 6x + 1 = 100$

d) $8x^2 + 50 = 0$

e) $(n - 2)^2 = -81$

III Solving by Completing the Square

A perfect square trinomial with a leading coefficient of 1 has the form $x^2 + bx + c$. Half of the middle coefficient squared equals the last term. $\left[\frac{1}{2}b\right]^2 = c$. If the leading coefficient is a 1, every binomial of the form $x^2 + bx$ can be made into a perfect square trinomial by adding $\left(\frac{1}{2}b\right)^2$. If this value is added to both sides of an equation of the form $x^2 \pm bx = c$, then the square root property can be used to solve the equation. This process is called the **completing the square** process.

Here is an example:

$$x^2 + 10x + 24 = 0$$

$$x^2 + 10x = -24$$

$$x^2 + 10x + \left(\frac{1}{2} \cdot 10\right)^2 = -24 + \left(\frac{1}{2} \cdot 10\right)^2$$

$$x^2 + 10x + 25 = -24 + 25$$

$$(x + 5)^2 = 1$$

$$x + 5 = \sqrt{1} \quad \text{and} \quad x + 5 = -\sqrt{1}$$

$$x + 5 = 1 \quad \quad \quad x + 5 = -1$$

$$x = -4 \quad \quad \quad x = -6$$

*There is a good visual picture of completing the square on page 141 of the textbook.

Ex 3: What number should be added to each binomial so that it becomes a perfect square trinomial?

a) $x^2 + 14x$

b) $r^2 - 22r$

c) $n^2 + 3n$

Any quadratic equation can be solved using completing the square if each term is divided by the leading coefficient. However, it is recommended you only use this method when the leading coefficient is already a 1.

Here are the steps.

1. Use this procedure only on equations where the leading coefficient is 1.
2. Move the constant to the other side so the equation is in the form $x^2 + bx = c$.
3. Complete the square by taking half of the coefficient of x , squaring it, and adding to both sides of the equation.

4. Factor the perfect square trinomial.
5. Solve by using the square root property.

Ex 4: Solve by completing the square.

a) $x^2 + 8x + 9 = 0$

b) $x^2 - 2 = -6x$

c) $x^2 + x = 6$

d) $x^2 - 6x + 10 = 0$

IV Solving by the Quadratic Formula

The Quadratic Formula is a formula that many of you will recognize. It can be used to solve any quadratic equation. How the formula is derived is found on page 143 of the text. (It is not important that you understand this derivation.)

Quadratic Formula

If a quadratic equation is in general form, $ax^2 + bx + c = 0$, then the solution(s) can be found by the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. It is read, 'x equals the opposite of b plus or minus the square root of $b^2 - 4ac$ all over twice a.'

Note: The quadratic formula was derived by using the completing the square process.

Ex 5: Solve by using the quadratic formula.

a) $2x^2 - x - 15 = 0$

b) $5x + 3x^2 = -1$

c) $7x^2 = 2(x + 1)$

d) $x^2 - 15 = 0$

$$e) \quad x^2 - 2x = -5$$

$$f) \quad x^2 - \frac{2}{3}x + \frac{2}{9} = 0$$