Trigonometric Graphs

Find the amplitude, the period, and the phase shift.

$y = 3\cos\left(\frac{1}{2}x - \frac{\pi}{4}\right)$

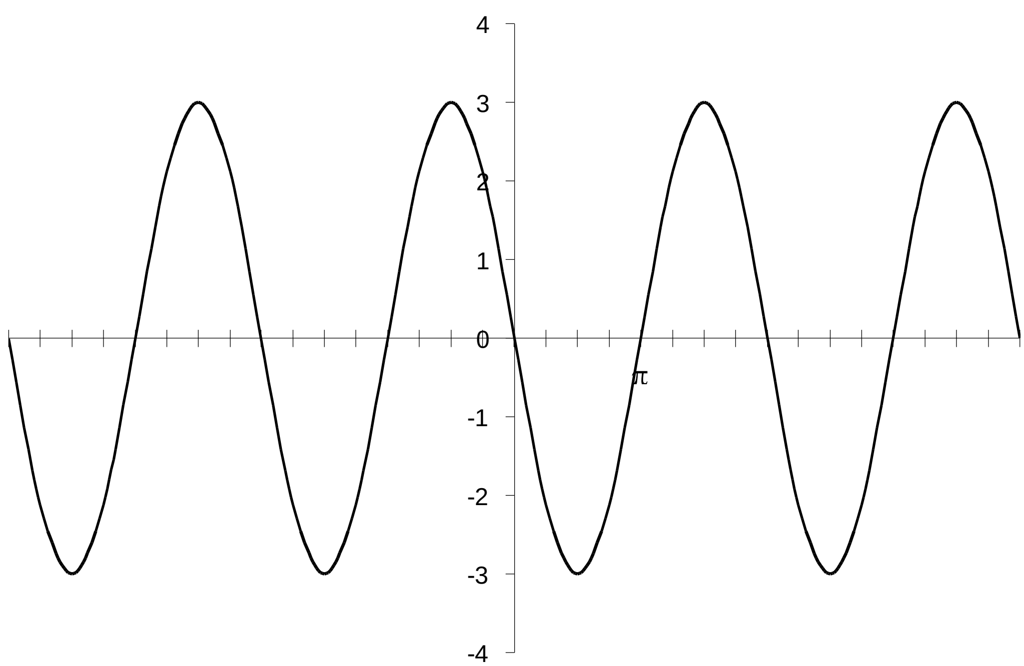
$y = 4\sin(2x + \pi)$

$y = 2\sin\left(\pi x + \frac{\pi}{2}\right)$

$y = -5\cos(\pi x)$

Now we will do the reverse of graphing functions such as those in the previous lesson. Given a graph, we will find the equation. First, shown a curve (such as the one below), we could write either a sine function or a cosine function. We will assume we are writing a sine function. Secondly, we much decide the value of *a*. Could *a* be positive or negative? It could be either depending on where the phase shift is. So we will decide that *a* is positive. We also will assume that *b* is positive and that *c* is the least positive value possible. (There could be several phase shifts. If there is one, we assume it is the one just left of the y-axis. Remember, if both *b* and *c* are positive, the phase shift will be negative.)

1) The graph of the equation is shown in the figure. (a) Find the amplitude, period, and phase shift. (b) Write the equation in the form $y = a \sin(bx + c)$ for $a > 0$, $b > 0$, and the least positive real number *c*.



Note: Only 1 value is marked on the *x*-axis of the graph. You can see that 4 marks equals π.

Amplitude is the distance from the average. In the graph on the previous page it would be 3.
Phase shift is the first **zero** (*x*-intercept) before a maximum left of the *y*-axis. Phase shift of the previous graph would be –π.
To find the period, begin at –π (the average) and determine when one cycle of 'to maximum, back to average, to minimum, back to average' is completed. This is at π. From –π to π gives a period of 2π.

Amplitude: amplitude = $|a| = 3$

$a = 3$

$$y = a\sin(bx + c)$$
$$y = 3\sin(x + \pi)$$

period = $\dfrac{2\pi}{|b|} = 2\pi$

$\dfrac{2\pi}{b} = \dfrac{2\pi}{1}$

$b = 1$

phase shift = $-\dfrac{c}{b} = -\pi$

$-\dfrac{c}{1} = -\pi$

$c = \pi$

2) Find an equation of the form $y = a\sin(bx + c)$, where $a > 0,\ b > 0,$ and c is the least positive value possible for the graph shown at the top of page 3.

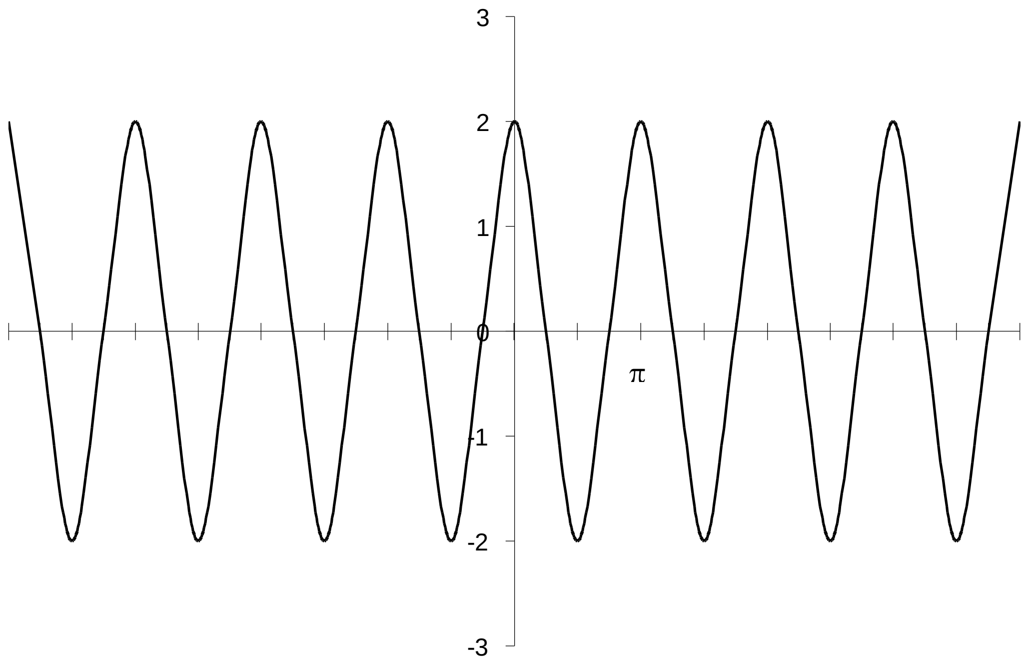
amplitude:

period:

EQUATION:

phase shift:

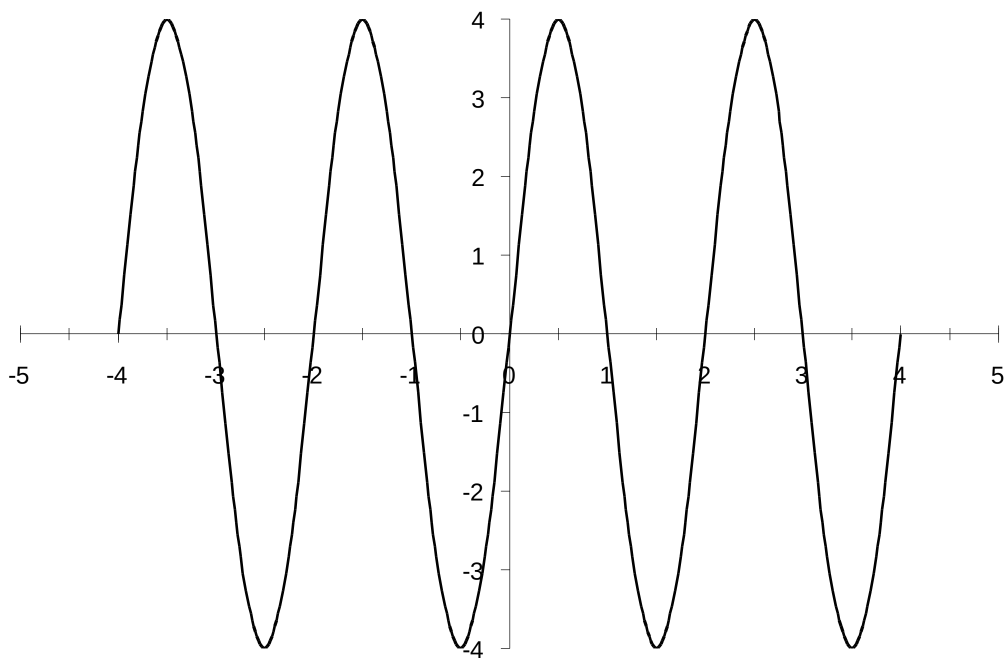
Trigonometric Graphs



3)

amplitude: period:

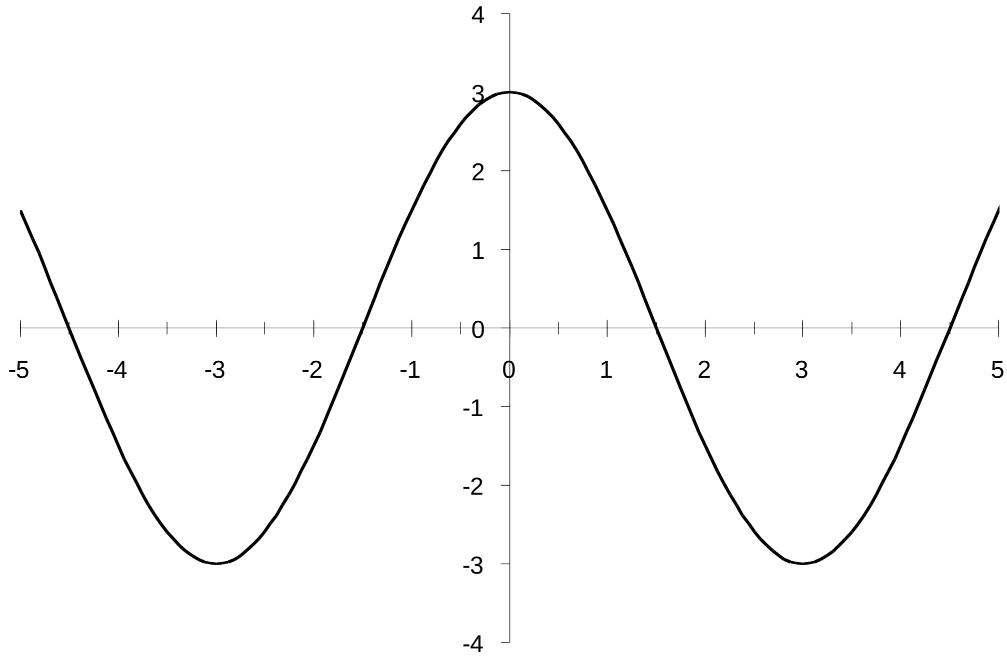
phase shift: EQUATION:



4)

Amplitude: Period:

Phase Shift: EQUATION:



Trigonometric Graphs

Scientists sometimes use the formula

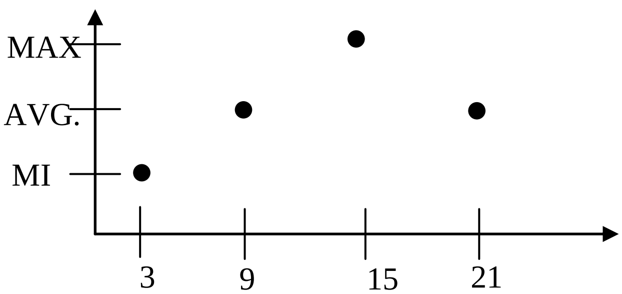
$$f(t) = a\sin(bt + c) + d$$

to simulate temperature variations during the day, with time *t* in hours, temperature $f(t)$ in °C, and *t* = 0 corresponding to midnight, Assume that $f(t)$ is decreasing at midnight.

a) Determine the values of a, b, c and d that fit the information. Note: The graph of this curve will only be to the right of the *y*-axis. Therefore the value of *c* will be negative, since the phase shift will be to the right.

b) Sketch the graph of f for $0 \le t \le 24$

1) The temperature varies between 10°C and 30°C, and the average temperature of 20°C **first occurs at 9 AM**



Period = 24 hours, 'hash marks' would be every 6 hours. Average is at 9 AM, 6 hours earlier (3 AM) would be minimum. Remember, temperature is falling at

Since the average is at 20, the graph has been shifted up 20. *d* = 20

The difference between the maximum of 30 and the average or the minimum of 10 and the average is 10°. amplitude = *a* = 10

Period = $24 = \frac{2\pi}{b} \to 24b = 2\pi \to b = \frac{\pi}{12}$

Phase shift is where the curve is rising at the average:

$$9 = -\frac{c}{\frac{\pi}{12}} \to 9 = -\frac{12c}{\pi} \to 9\pi = -12c \to c = -\frac{3\pi}{4}$$

$$f(t) = 10\sin\left(\frac{\pi}{12}t - \frac{3\pi}{4}\right) + 20$$

2) The high temperature of 28°C occurs at 2 PM and the average temperature of 20°C occurs 6 hours later.