MA 22000, Lessons 1 (a & b)
Polynomials
Algebra part of Textbook: Section 5.2, Calculus part of Textbook: Section R.1

Definition: A **term** is a number, a variable, the product or quotient of a number and one or more variables (possibly raised to powers). Below are examples of terms.

Terms: -5, x, $3n$, a^3, $5x^5y^2$, $\frac{3}{4}mn^4r$ (or $\frac{3mn^4r}{4}$), $\frac{3}{xy^2}$, $\frac{-18z^5}{5x^2y}$

If a term has a number factor, that number is called the **coefficient** of the term. For the terms listed above, -5 has a coefficient of -5; x has a coefficient of 1, $3n$ has a coefficient of 3, $\frac{3mn^4r}{4}$ has a coefficient of $\frac{3}{4}$, and $\frac{-18z^5}{5x^2y}$ has a coefficient of $-\frac{18}{5}$.

Note: The term $\frac{3}{xy^2}$ could be written as $3x^{-1}y^{-2}$; the term -5 could be written as $-5x^0$; tne the term $\frac{-18z^5}{5x^2y}$ could be written as $-\frac{18}{5}x^{-2}y^{-1}z^5$.

Definition
Any combination of variables or constants (numbers) joined by the operations of addition, subtraction, multiplication, division (except division by 0), raising to powers, or taking roots is called an **algebraic expression.**

A specific type of algebraic expression is a **polynomial.**

Definition
A polynomial of 1 variable is a term or a finite sum of terms in which all variables have whole number exponents and no variables appear in denominators.
The **degree of the poynomial** is the greatest exponent in the polynomial and the coefficient of the term with the greatest exponent is called the **leading coefficient**. Any number term (no variables) is called the **constant term or simply a constant**.

A polynomial of 1 variable that is written in order of descending powers of the variable is said to be in **standard form**. A polynomial with only one term is a **monomial**. A polynomial with two *unlike* terms is called a **binomial**. One with three *unlike* terms is a **trinomial**.

(See the table on the text page.)

Polynomial	Standard Form	Degree	Leading Coefficient
$4x-3x^2+2+x^4$	x^4-3x^2+4x+2	4	1
$\frac{1}{3}n-3n^3$	$-3n^3+\frac{1}{3}n$	3	-3
20	20	0	20
$4a-12a^5+2a^3-6$	$-12a^5+2a^3+4a-6$	5	-12

Example A: $2x^4-3x^5+5x-9-5x^3$

a) Write the polynomial above in descending order.

b) What is the degree of the polynomial?

c) Evaluate, if $x=-2$.

Example B: What type of polynomial is each below?

a) $-3x-x^3$

b) $5n^4-n^3+2n-7$

c) $3y^2-2y+1$

Example C: Combine terms to simplify.

a) $4a^4+2a^2-3a^3+7a^2+a^3-8a^4$

b) $6+3c-(4c+1)-(2c-8)$

Like terms are terms with the same variable factors.

'Like' terms may be 'combined' by adding the coefficients; the variable factors stay the same (do not add exponents).

(lesson 1b)

To add two or more polynomials, remove any grouping and combine 'like' terms. To subtract a polynomial, add the opposite (distribute the negative sign). Polynomials may be added or subtracted in a horizontal format or a vertical format. It is important to remember to distribute the negative (minus) to each term of the polynomial that is subtracted.

Example 1: Add or subtract (combine) where possible in each polynomial expression.

a) $(3x^2-2x+9)-(5x-2x^2+10)$

b) $(4x+2)-(12x-9)-3x+(5-7x)$

c) $(2a^2-4a+1)-[(3a^2-a+3)-(4a-9a^2+7)]$

d) $[-(b^2-b+7)-(2b^2+3b-5)]+(2b^2-7b-5)$

e) $-5(8a^2-2a+5)-6(-a^2+2a-11)$

f) *Add.*

$$-13q^2-15q+3$$
$$\underline{-5q^2+11q-8}$$

g) *Add.*

$$6x^3+5x^2+7x$$
$$-2x^3-3x^2+x$$
$$\underline{9x^3-12x^2-2x}$$

h) *Subtract*.

$$2m^3 - 7m^2 + 4m$$
$$\underline{5m^3 - m^2 \qquad + 9}$$

i) *Subtract*.

$$-4y^2 - 7y + 2$$
$$-\underline{(-2y^2 + 5y + 3)}$$

Note:
The textbook will not have a minus sign before the polynomial to be subtracted, such as *h* at the left.

In MyMathLab, there may be a minus sign before the polynomial to be subtracted, such as *i* at the left. (As a reminder to distribute the negative to each term).

Example 2:

The four sides of a quadrilateral can be represented by the expressions $2a+5$, $3a-1$, $5a-6$, and $4a-3$. Find a polynomial that represents the perimeter of the quadrilateral. If $a = 12$, what is that perimeter?

Example 3:

a) $\left(\frac{2}{3}y - \frac{3}{2}y^2 + \frac{3}{4}\right) + \left(\frac{1}{6}y^2 - \frac{1}{2}y - \frac{2}{3}\right)$

b) $(0.614r^2 - 0.25r + 1.05) - (0.83r^2 - 0.235r + 2.3)$