<u>Factoring Trinomials Using the *ac* method</u>
<u>or the PRODUCT-SUM METHOD</u>

Some students have difficulty factoring a trinomial of the form $ax^2 + bx + c$ using 'trial-and-error' or 'guessing'. There is a method that works better and will also identify if the trinomial cannot be factored (is prime). This document explain the method, called either the *ac* method or the product-sum method, and gives several examples.

If the trinomial is of the form $x^2 + bc + c$ (leading coefficient is 1)…

1) Find two integers whose product is c and whose sum is b.
2) Let's call the two integers r and s.
3) One factor is $(x + r)$ and the other factor is $(x + s)$.

<u>Ex 1:</u> Factor $x^2 + 16x + 55$

Find two integers whose product is c =55. The pairs are listed in the table at the right.

1	55	-1	-55
5	11	-5	-11

Select the pair that has a sum of b = 16. That pair is 5 and 11. Therefore the factors are…

$$x^2 + 16x + 55 = (x+5)(x+11)$$

<u>Ex 2:</u> Factor $x^2 - 16x + 60$

Find two integers whose product is $c = 60$. The pairs are listed in the table at the right.

1	60	-1	-60
2	30	-2	-30
3	20	-3	-20
4	15	-4	-15
5	12	-5	-12
6	10	-6	-10

Select the pair that has a sum of $b = -16$. That pair is $-6\ and - 10$. Therefore the factors are…

$$x^2 - 16x + 60 = (x - 6)(x - 10)$$

<u>Ex 3:</u> Factor $a^2 + 7a - 18$

Find two integers whose product is $c = -18$. The pairs are listed in the table at the right.

1	-18	-1	18
2	-9	-2	9
3	-6	-3	6

Select the pair that has a sum of $b = 7$. That pair is 9 and -2. Therefore the factors are…

$$a^2 + 7a - 18 = (a+9)(a-2)$$

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Ex 4: Factor $y^2 - 10y - 39$ Find two integers whose product is $c = -39$. The pairs are listed in the table at the right.

1	-39	-1	39
3	-13	-3	13

Select the pair that has a sum of $b = -10$. That pair is $3\ and - 13$. Therefore the factors are…

$y^2 - 10y - 39 = (y-13)(y+3)$

If the trinomial is of the form $ax^2 + bx + c$, there is a little extra effort to find the factors using this method. Here are the steps.

1) Find two integers whose product is ac and whose sum is b.
2) Let's call the two integers r and s.
3) Rewrite the trinomial as a 4 term polynomial as below.

$$ax^2 + rx + sx + c$$

4) Use 'grouping by pairs' to factor. Take the GCF out of the first two terms and out of the second two terms and get a common parentheses. See the steps in the following examples.

Ex 5: Factor $2n^2 + n - 10$ Find two integers whose product is $ac = (2)(-10) = -20$. The pairs are listed in the table at the right.

1	-20	-1	20
2	-10	-2	10
4	-5	-4	5

Select the pair that has a sum of $b = 1$. That pair is $-4\ and\ 5$.
Rewrite the trinomial as follows.

$2n^2 + n - 10$

$= \underline{2n^2 - 4n}\underline{+5n - 10}$ Take out the GCF from each pair.

$= 2n(n-2) + 5(n-2)$

$= (n-2)(2n+5)$

Ex 6: Factor $6x^2 - 17x + 12$ Find two integers whose product is $ac = (6)(12) = 72$. The pairs are listed in the table at the right.

1	72	-1	-72
2	36	-2	-36
3	24	-3	-24
4	18	-4	-18
6	12	-6	-18
8	9	-8	-9

Select the pair that has a sum of $b = -17$. That pair is $-8\ and - 9$.
Rewrite the trinomial as follows.

$6x^2 - 17x + 12$

$= \underline{6x^2 - 8x}\underline{-9x + 12}$

$= 2x(3x-4) - 3(3x-4)$

$= (3x-4)(2x-3)$

Ex 7: Factor $4a^2 + 27a - 40$ Find two integers whose product is $ac = (4)(-40) = -160$. The pairs are listed in the table at the right.

1	-160	-1	160
2	-80	-2	80
4	-40	-4	40
5	-32	-5	32
8	-20	-8	20
10	-16	-10	26

Select the pair that has a sum of $b = 27$. That pair is $-5\ and\ 32$.
Rewrite the trinomial as follows.

$$4a^2 + 27a - 40$$
$$= \underline{4a^2 - 5a}\underline{+32a - 40}$$
$$= a(4a-5) + 8(4a-5)$$
$$= (4a-5)(a+8)$$

Ex 8: Factor $16y^2 + 30y + 9$ Find two integers whose product is $ac = (16)(9) = 144$. The pairs are listed in the table at the right.

1	144	-1	-144
2	72	-2	-72
3	48	-4	-48
4	36	-4	-36
6	24	-6	-24
9	16	-9	-16
12	12	-12	-12

Select the pair that has a sum of $b = 30$. That pair is 6 and 24.
Rewrite the trinomial as follows.

$$16y^2 + 30y + 9$$
$$= \underline{16y^2 + 6y}\underline{+24y + 9}$$
$$= 2y(8y+3) + 3(8y+3)$$
$$= (8y+3)(2y+3)$$

Ex 9: Factor $100c^2 - 140cd + 49d^2$ Find two integers whose product is $ac = (100)(49) = 4900$. The pairs are listed in the table at the right.

Select the pair that has a sum of $b = -140$. The pair is -70 and -70. Rewrite the trinomial as follows.

$$100c^2 - 140cd + 49d^2$$
$$= \underline{100c^2 - 70cd}\ \underline{-70cd + 49d^2}$$
$$= 10c(10c-7d) - 7d(10c-7d)$$
$$= (10c-7d)(10c-7d) \text{ or } (10c-7d)^2$$

1	4900	-1	-4900
2	2450	-2	-2450
4	1225	-4	-1225
5	980	-5	-980
7	700	-7	-700
10	490	-10	-490
14	350	-14	-350
20	245	-20	-245
25	196	-25	-196
28	175	-28	-175
35	140	-35	-140
49	100	-49	-100
50	98	-50	-98
70	70	-70	-70