

MA 15910 Lesson 16 Notes  
Section 4.2 (part 2), calculus part of text  
Derivatives of Products and Quotients

**Quotient Rule:**

If  $f(x) = \frac{u(x)}{v(x)}$ , and if the derivatives of  $u$  and  $v$  exist and  $v(x) \neq 0$ ,

then the derivative of  $f$  is...

$$f'(x) = \frac{v(x) \cdot u'(x) - u(x) \cdot v'(x)}{[v(x)]^2}$$

This will help you remember how to use the quotient rule:

**Low Dee High minus High Dee Low, Over the Square of What's Below.**

We will accept this rule as true without a formal proof. Just like the derivative of a product is not the product of the derivative, the derivative of a quotient is NOT the quotient of the derivatives.

**Ex 1:** Find the derivative  $y'$  if  $y = \frac{3x^2 + 5}{2x - 6}$ .

$$y = \frac{3x^2 + 5}{2x - 6}$$

$$y' = \frac{\text{Low(Dee High) - High(Dee Low)}}{\text{Below}^2}$$

$$y' = \frac{(2x - 6)(6x) - (3x^2 + 5)(2)}{(2x - 6)^2}$$

$$= \frac{12x^2 - 36x - 6x^2 - 10}{(2x - 6)^2}$$

$$= \frac{6x^2 - 36x - 10}{[2(x - 3)]^2}$$

$$= \frac{2(3x^2 - 18x - 5)}{4(x - 3)^2}$$

$$= \frac{3x^2 - 18x - 5}{2(x - 3)^2}$$

\*Usually after using the quotient rule, the answers are given with the numerator and denominator factored and simplified.\*

**Ex 2:** Use the quotient rule to find the following derivatives.

a)  $g(x) = \frac{5-2x}{3x+5}$  (Find  $g'(2)$  also)    b)  $D_x \left[ \frac{5x^2 - 12x - 15}{2x+6} \right]$

c)  $y = \frac{2\sqrt{x}}{3x+1}$

d)  $f(u) = \frac{6u+2}{\sqrt{u}}$

**Ex 3:** If  $h(x) = \frac{f(x)}{g(x)}$ , and  $f(4) = 12$ ,  $f'(4) = 9$ ,  $g(4) = 15$ , and  $g'(4) = 2$ , find  $h'(4)$ .

**Ex 4:** Find the derivative of  $k(n)$  two different ways.  $k(n) = \frac{4n^2 + 3n}{n^{1/2}}$

- 1) Use the quotient rule.
- 2) Simplify using division.

**Ex 5:** The total cost (in hundreds of dollars) to produce  $x$  units of a product at a factory is given by the cost function,  $C(x) = \frac{5x+6}{x+2}$ . (a) Write an average cost function. (b) Find the average cost for 8 units and for 20 units. (c) Find the marginal average cost function. (d) Evaluate the marginal average cost function for  $x = 8$  and  $x = 20$ .

**Ex 6:** Find the equation of the tangent line to  $f(x) = \frac{x+5}{x-2}$  at the point (3,8).

**Ex 7:** Find the values of  $x$  where  $f'(x) = 0$ , if  $f(x) = \frac{x-5}{x^2-2}$ , if they exist. Round to the nearest thousandths.