MA 15910, Lessons 1 (a & b)
Polynomials
Algebra part of Textbook: Section 5.2, Calculus part of Textbook: Section R.1

Definition: A **term** is a number, a variable, the product or quotient of a number and one or more variables (possibly raised to powers). Below are examples of terms.

$$\text{Terms:}\quad -5,\quad x,\quad 3n,\quad a^3,\quad 5x^5 y^2,\quad \frac{3}{4} mn^4 r \ \left(\text{or } \frac{3mn^4 r}{4}\right),\quad \frac{3}{xy^2},\quad \frac{-18z^5}{5x^2 y}$$

If a term has a <u>number factor</u>, that number is called the **coefficient** of the term.
    For the terms listed above, -5 has a coefficient of -5; x has a coefficient of 1, $3n$ has a coefficient of 3, $\dfrac{3mn^4 r}{4}$ has a coefficient of $\dfrac{3}{4}$, and $\dfrac{-18z^5}{5x^2 y}$ has a coefficient of $-\dfrac{18}{5}$.

(Note: The term $\dfrac{3}{xy^2}$ could be written as $3x^{-1} y^{-2}$; the term -5 could be written as $-5x^0$; and the term $\dfrac{-18z^5}{5x^2 y}$ could be written as $-\dfrac{18}{5} x^{-2} y^{-1} z^5$.)

Definition
Any combination of variables or constants (numbers) joined by the operations of addition, subtraction, multiplication, division (except division by 0), powers, or roots is called an **algebraic expression.**

A specific type of algebraic expression is a **polynomial.**

Definition
A polynomial of 1 variable is a term or a finite sum of terms in which all variables have <u>whole number exponents</u> and no variables appear in denominators.
The **degree of the polynomial** is the greatest exponent in the polynomial and the coefficient of the term with the greatest exponent is called the **leading coefficient**. Any number term (no variables) is called the **constant term or simply a constant**.

A polynomial of 1 variable that is written in order of descending powers of the variable is said to be in **standard form**. A polynomial with only one term is a **monomial**. A polynomial with two *unlike* terms is called a **binomial**. One with three *unlike* terms is a **trinomial**.

(See the table on the text page.)

Polynomial	Standard Form	Degree	Leading Term &Coefficient
$4x - 3x^2 + 2 + x^4$	$x^4 - 3x^2 + 4x + 2$	4	$x^4, \quad 1$
$\frac{1}{3}n - 3n^3$	$-3n^3 + \frac{1}{3}n$	3	$-3n^3, \quad -3$
20	20	0	20
$4a - 12a^5 + 2a^3 - 6$	$-12a^5 + 2a^3 + 4a - 6$	5	$-12a^2, \quad -12$

Example A: $\qquad 2x^4 - 3x^5 + 5x - 9 - 5x^3$

a)   Write the polynomial above in descending order.

b)   What is the degree of the polynomial?

c)   Evaluate, if $x = -2$.

> **Evaluate** indicates to substitute the given value for the variable and follow the order of operations to derive a number.

Example B: What type of polynomial is each below (monomial, binomial, trinomial, polynomial)?

> Terms of a polynomial are separated by plus or minus signs.

a) $-3x - x^3$

b) $5n^4 - n^3 + 2n - 7$

c) $3y^2 - 2y + 1$

d) $4a^2bc^8xy^3$

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Example C: Combine terms to simplify.

a) $4a^4 + 2a^2 - 3a^3 + 7a^2 + a^3 - 8a^4$

b) $6 + 3c - (4c + 1) - (2c - 8)$

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(lesson 1b or part 2 of lesson 1)

To add two or more polynomials, remove any grouping and combine 'like' terms. To subtract a polynomial, add the opposite (distribute the negative sign). Polynomials may be added or subtracted in a horizontal format or a vertical format. It is important to remember to distribute the negative (minus) to each term of the polynomial that is subtracted.

Example 1: Add or subtract (combine) where possible in each polynomial expression.

a) $(3x^2 - 2x + 9) - (5x - 2x^2 + 10)$

b) $(4x + 2) - (12x - 9) - 3x + (5 - 7x)$

c) $(2a^2 - 4a + 1) - [(3a^2 - a + 3) - (4a - 9a^2 + 7)]$

d) $[-(b^2 - b + 7) - (2b^2 + 3b - 5)] + (2b^2 - 7b - 5)$

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e) $-5(8a^2 - 2a + 5) - 6(-a^2 + 2a - 11)$

f)   *Add.*
$$-13q^2 - 15q + 3$$
$$\underline{- 5q^2 + 11q - 8}$$

g)   *Add.*
$$6x^3 + 5x^2 \quad + 7x$$
$$- 2x^3 - 3x^2 \quad + x$$
$$\underline{9x^3 - 12x^2 - 2x}$$

h)   *Subtract.*
$$2m^3 - 7m^2 + 4m$$
$$\underline{5m^3 - m^2 \qquad + 9}$$

i)   *Subtract.*
$$-4y^2 - 7y + 2$$
$$\underline{-(-2y^2 + 5y + 3)}$$

Important Note:
The textbook will not have a minus sign before the polynomial to be subtracted, such as *h* at the left (when aligned vertically).

In MyMathLab, there *may* be a minus sign before the polynomial to be subtracted, such as *i* at the left. (As a reminder to distribute the negative to each term).

Example 2:
The four sides of a quadrilateral can be represented by the expressions
$(2a+5)$, $(3a-1)$, $(5a-6)$, and $(4a-3)$. Find a polynomial that represents the <u>perimeter of the quadrilateral</u>. Perimeter is the distance around a figure.

If $a = 12$, what is that perimeter?

Example 3: Add or subtract the following polynomials. (Hint: Find a LCD.)

a) $\left(\dfrac{2}{3}y - \dfrac{3}{2}y^2 + \dfrac{3}{4}\right) + \left(\dfrac{1}{6}y^2 - \dfrac{1}{2}y - \dfrac{2}{3}\right)$

b) $(0.614r^2 - 0.25r + 1.05) - (0.83r^2 - 0.235r + 2.3)$