Slope:

Definition: The slope of a line is the ratio of the 'change' in *y* to the 'change' in *x* (ratio of vertical change to horizontal change) often referred to as rise compared to run. .

Given 2 points of a line, (x_1, y_1) and (x_2, y_2)

$$\text{slope} = m = \frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x} = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

(where $x_1 \neq x_2$)

Caution: *See caution note on page 162 of algebra part of the textbook. See caution note on page 4 of calculus part of the textbook.*

Ex 1: Find the slope of a line containing each pair of points. If the slope is undefined, write 'undefined'.

a) $(-3,-4)$, $(2,-2)$

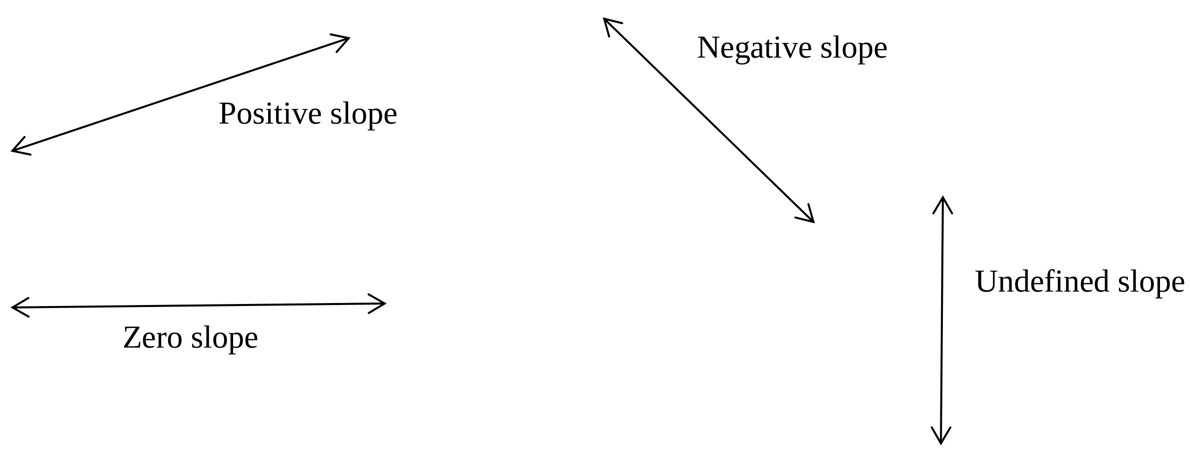
b) $(4,2)$ and $(-7,2)$

c) $(-5,8)$ and $(2,3)$

d) $(3,9)$ and $(3,6)$

As illustrated above and below, there are 4 types of slopes.

1. A line with a **positive slope** rises left to right.
2. A line with a **negative slope** falls left to right.
3. A line with a **zero slope** is horizontal.
4. A line with an **undefined slope** (zero denominator) is vertical.



Linear Equations:

Begin with the slope formula, letting (x_1, y_1) be one specific point of a line and any other point be (x, y). Cross multiply and the result is called the **Point-Slope form** of a line.

$$m = \frac{y - y_1}{x - x_1} \quad \text{Cross multiply...}$$

$$y - y_1 = m(x - x_1) \qquad \textbf{Point - Slope Form of a Linear Equation}$$

Represent the point (x_1, y_1) as the y-intercept $(0, b)$. Solve the equation for y. The result is called the **Slope-Intercept form** of a line.

$$y - b = m(x - 0)$$

$$y - b = mx$$

$$y = mx + b \qquad \textbf{Slope - Intercept Form}$$

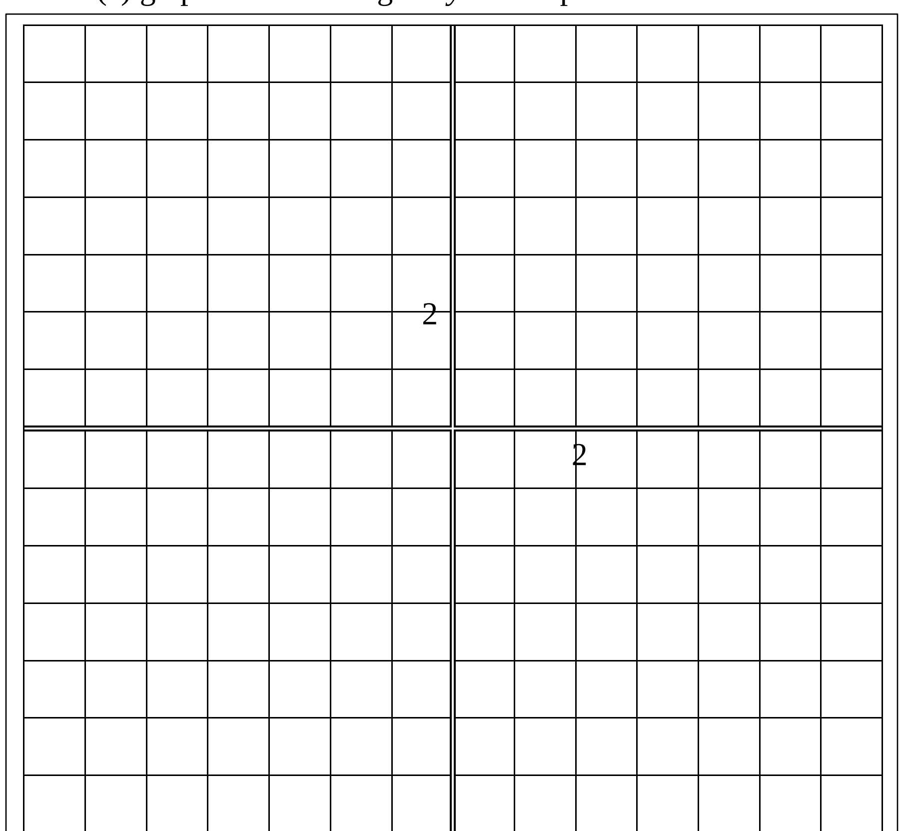
slope is m and the y-intercept is $(0, b)$

Every line can be written in **General Form,** $Ax + By + C = 0$. Some textbooks also define **Standard Form** as $Ax + By = C$. ***A*, *B*, and *C* are integers and *A* is positive.**

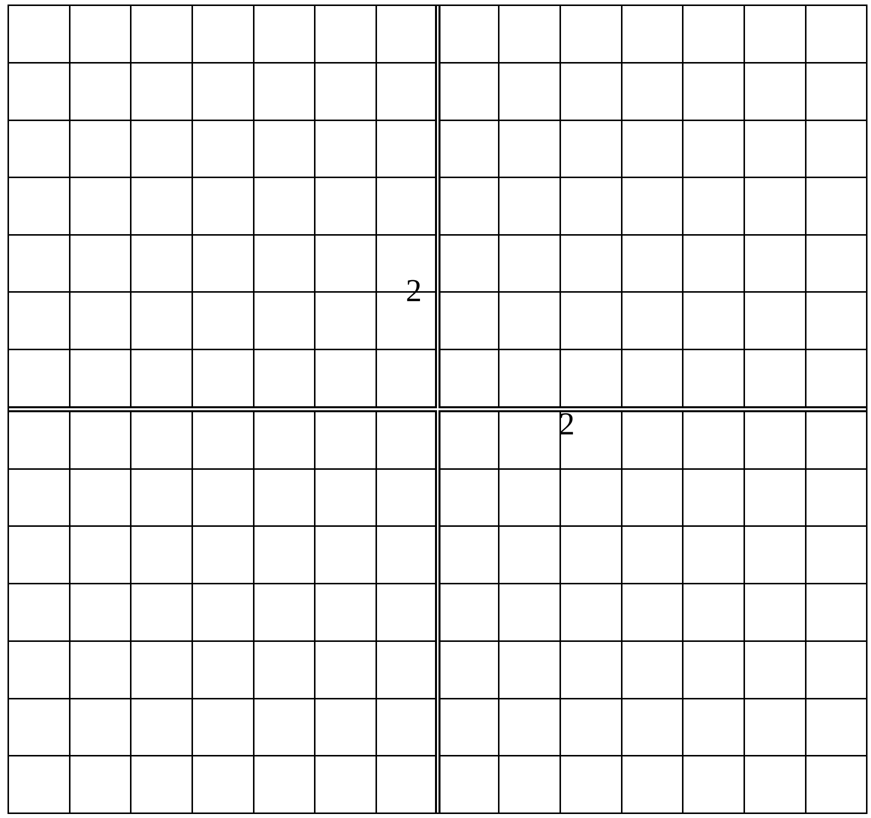
The equation for a **horizontal line** (slope 0) would be $y = b$. A **vertical line** has no slope and cannot be written in slope-intercept form. However, its equation would be $x = a$, where $(a, 0)$ is the x-intercept.

Find equations in (a) slope-intercept form and (b) standard form for the line through the pair of points. Use the slope-intercept form to (c) graph the line using the y-intercept and the rise/run (slope).

<u>Ex 2:</u> $(4, -3)$, $\left(\frac{2}{3}, 2\right)$



Ex 3: (a) Find the general form for the equation of a line with a slope of $-\frac{8}{5}$ containing the point $(5,-3)$. (b) Use the general form to find the *x*-intercept and *y*-intercept and use the intercepts to graph the line.



Ex 4: Find the equation of a horizontal line through the point (-3, 2) and the equation of a vertical line through the point (7, 10).

Ex 5: The average weekly salary for a certain job in 2008 was $248. For the same job, in 2011, the average weekly salary was $285. Assume the salary can be modeled by a linear equation. Write a linear function for the average weekly salary for this job in terms of the number of years since 2008. (t = 0 for 2008, t = 1 for 2009, etc.). Use this function to predict the average weekly salary for this job in 2015, if this trend continues.

Ex 6: A certain car at a rental agency rents for \$75 a week plus \$0.18 per mile driven during the week. (a) Write a linear function that will give the weekly cost C in terms of the number of miles n driven during the week.

b) Find the cost, if 75 miles are driven during a week.

c) How many miles were driven in a week, if the cost of the rental was \$97.50?

Ex 7: A company purchased a carpet shampoo machine for \$375. The machine cost an average of \$15 per day for shampoo supplies and maintenance and the employee who operates the machine is paid \$75 per day.

a) Write a linear function giving the total cost C of operating this carpet shampoo machine for d days after purchase .

b) If revenue from customers who have carpets shampooed averages \$125 per day, write a revenue function R giving the total revenue for d days of use.

c) Write a profit function P that will give the profit from the machine after d days of rental use.

d) Find the approximate number of days the machine must be operated before the company will 'break even'. The company will 'break even' when costs = revenue or profit = 0. (Round to the nearest day.)