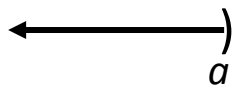
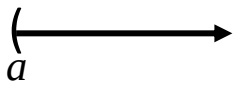
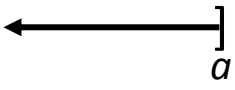
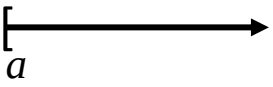
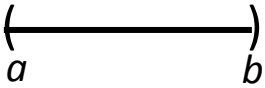
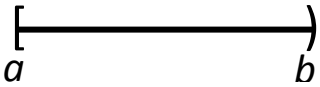
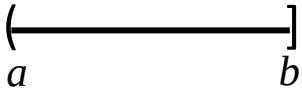
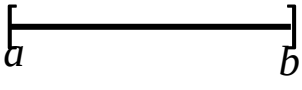
MA 15910, Lessons 2a and 2b Introduction to Functions
Algebra: Sections 3.5 and 7.4
Calculus: Sections 1.2 and 2.1

Representing an Interval Set of Numbers

Inequality Symbol	Number Line Graph	Interval Notation
$x < a$		$(-\infty, a)$
$x > a$		(a, ∞)
$x \leq a$		$(-\infty, a]$
$x \geq a$		$[a, \infty)$
$a < x < b$		(a, b)
$a \leq x < b$		$[a, b)$
$a < x \leq b$		$(a, b]$
$a \leq x \leq b$		$[a, b]$
no solution		$\emptyset$
all real numbers		$(-\infty, \infty)$

You are responsible for knowing all 3 ways above to represent a set of numbers that is an inequality; with the inequality symbol, graphed on a number line, or in interval notation.

You may remember from past experiences of graphing on a number line, that you used a 'closed dot' on the number line to indicate that the number itself was included in the set. You probably used an 'open dot' on the number line to indicate that the number itself was not included in the set. In this class, we will use a parenthesis to replace the 'open dot' and a bracket to replace the 'closed dot'. (The parenthesis or bracket opens in the direction of shading.)Students are expected to know how to represent an interval on a number line as a graph on a number line, using intervals notation, or using the inequality symbols.

Example 0:

(a) Represent the set of numbers $x < 12$, using a number line and interval notation.

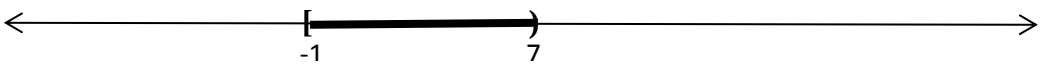
(b) Represent the set of numbers $x \geq 2$, using a number line and interval notation.

(c) Represent the set of numbers $-5 < x \leq 0$, using a number line and interval notation.

(d) Represent the set of numbers $(3, \infty)$, using an inequality symbol and using a number line.

(e) Represent the set of numbers $(-\infty, -100]$, using an inequality symbol and using a number line.

(f) Represent the numbers represented on the number line below in both interval notation and using an inequality symbol.



Definition: A relation is any set of ordered pairs. The set of first components in the ordered pairs is called the **domain** of the relation. The set of second components is called the **range** of the relation. **Relations may be represented as sets, tables, diagrams, graphs, or equations**.

Definition: A **function** is a relation in which no two ordered pairs have the same first components but different second components. Each element or component of the domain (input values) is paired to one and only one element or component of the range (output values).

Relations or functions can be represented by sets of ordered pairs, tables of ordered pairs, mappings of ordered pairs, graphs, or equations (as seen in the next few examples).

Example 1: Which relations below represent functions? State the domains and ranges.

a) $\{(9,81), (4,16), (5,25), (-2,4), (-6,36)\}$ Function?

Domain: Range:

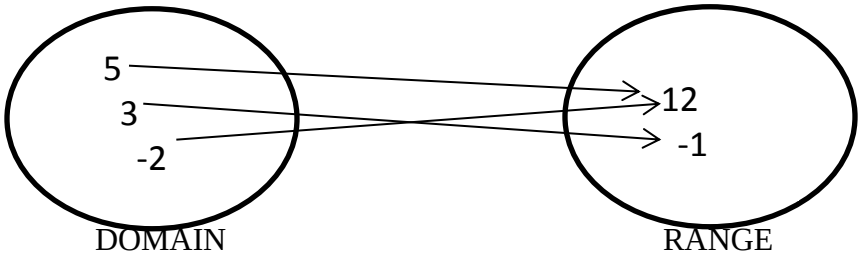
b)

x	-3	4	12	9
y	2	-1	0	2

Function?

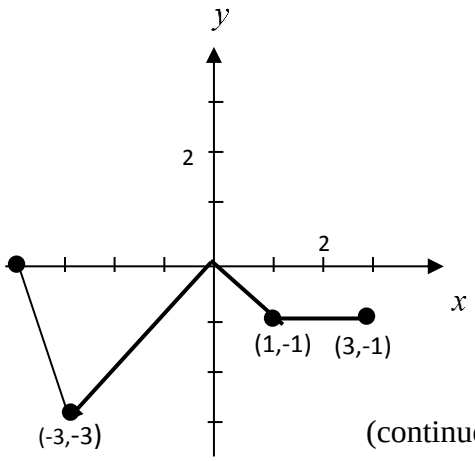
Domain: Range:

c)



Function?

d)



Function?

Domain:

Range:

(continued on the next page)

e) $x = y^2$ Function?
Domain: Range:

$f)$ $y = \dfrac{2x-5}{x+3}$ Function? Domain:

$g)$ $s = \sqrt{3r+2}$ Function? Domain:

Which below represent functions? What is the domain? What is the range?

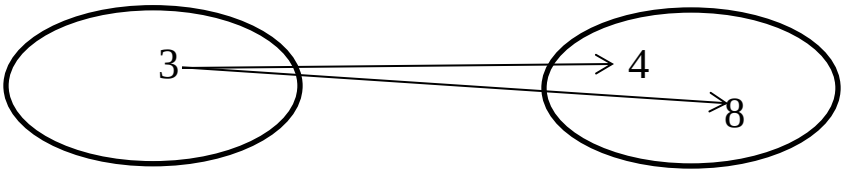
h) $\{(2,6), (7,8), (2,0)\}$

i)

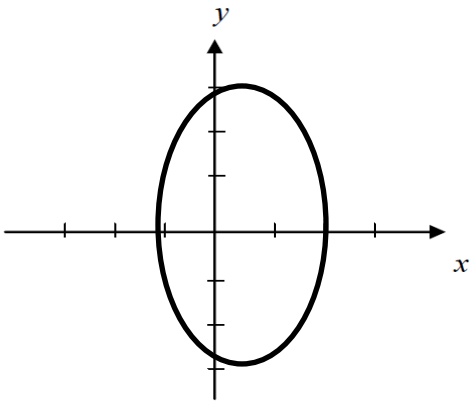
x	$q(x)$
2	5
3	5
9	5
0	5

(continued on the next page)

j)



k)



Function Notation: Functions can be 'named' by using letters. This 'name' can be used to write the function. For example; the function *h* represented by $y = 2x - x^2$ can be written as $h(x) = 2x - x^2$. This type of notation is known as **function notation**. The element of the domain, the input (the *x*) is inside the parentheses and the result, the output, (the *y* or *h*(*x*)) is the matching element of the range.

Example 2: Given the function $g(x) = |3x - 1|$, find the following function values.

a) $g(-2) =$ *b*) $g(0) =$

c) $g(4) =$ *d*) $g(\sqrt{2})$

Example 3: Given the functions $f(x) = x^2 - 2x$ and $F(x) = \dfrac{x}{3x-1}$, find the following.

a) $f(3x) =$

b) $F(a+1) =$

c) $f(r+2) =$

d) $F\left(\dfrac{1}{b}\right) =$

e) $f(x) + f(2) =$

f) $f(x+h) - f(x) =$

g) $F(3x) =$

h) $f(a+1) =$

i) $F(r+2) =$

j) $f\left(\dfrac{1}{b}\right) =$

k) $F(x) + F(2) =$

A function may be defined by more than one equation, such as the **piecewise-defined function** in example 4.

Example 4: Given this piecewise-defined function, find the following.

$$f(x)=\begin{cases} x+2 & \text{if } x<1 \\ x^2 & \text{if } x\geq 1 \end{cases}$$

a) $f(0)=$

b) $f(1)=$

c) $f(x+2)$ if $x\geq 1$

d) $f\left(\frac{x}{2}\right)$ if $x<1$

(lesson 2 part b)
In most functions the implied domain is the set of all real numbers (inputs) that yield real number values for the function values (outputs). In 'real life' problems, the domain and range are only the values that are reasonable.

Example 5: Find the domain of each function.

a) $f(x) = \sqrt{2x-3}$

b) $G(x) = \dfrac{2x}{x^2-3x-4}$

c) $d(t) = 50t,$ where d is the distance in miles traveled by a car in t hours

d) $g(x) = \sqrt{5-10x}$

e) $h(x) = \dfrac{x^2-9}{x^2+5x+6}$

g) volume of a sphere: $V = \frac{4}{3}\pi r^3$

f) A tennis ball is tossed vertically upward from a height of 5 feet according to the height function $h(t) = -16t^2 + 21t + 5$, where *h* us the height of the tennis ball in feet and *t* is time in seconds. When does the ball hit the ground? (Round to the nearest hundredth of a second, if necessary.)

Example 6:

A manufacturer can produce and sell *x* gadgets per week. The total cost of producing these gadgets (in dollars) is given by $C(x) = 11x + 375$. The revenue from the sale of these gadgets (in dollars) is $R(x) = 26.6x$. (a) Find a function to represent the <u>profit</u> from the production and sell of *x* gadgets per week. (b) What is the profit from the sale of 25 gadgets?

Example 7:

Write a <u>cost function</u> (cost as a function of number of songs) for this situation.
An Internet site where a student can download songs charges a $10 registration fee plus $0.75 per song.

Example 8:
The demand function (price/coat) for a type of coat if given by $p = D(n) = 86 - 0.5n$, where p is the price in dollars and n is the quantity demanded (number of coats).

a) Find the price for a demand of 55 coats.

b) Find the quantity demanded for a coat at the price of $32.

Example 9:
Producing x units of hamburger packages costs $C(x) = 12x + 36$. The revenue is $R(x) = 16x$. (Both C and R are in dollars.)

a) Write a profit function $P(x)$. Find the break-even quantity.

b) Find the profit from 250 units of hamburger packages.

c) Find the number of units that must be produced for a profit of $560.

Example 10:
Forensic scientists (CSI) use the lengths of certain bones to calculate the height of a person. The following functions represent a person's height based on the length of the femur (r), the bone from the knee to the hip socket.

For women: $h(r) = 61.41 + 2.3r$

For men: $h(r) = 69.09 + 2.24r$

a) Find the height of a man with a femur measuring 59 cm. 62 cm.

b) Find the height of a woman with a femur measuring 48 cm. 55 cm.

Example 11:
A company that manufactures bicycles has a fixed cost of $100,000 a month. It costs $100 on average to produce each bicycle. The total cost per month for the company is the sum of its fixed cost and the variable costs (costs of production of the bicycles). Write a cost function, C, as a function of the number of bicycles produced, b. Find C (90) and C (200). Interpret each.

Example 12:
A car was purchased for $22,500. The value of the car decreased by $3200 per year for the first six years. Write a function that describes the value of the car, V, after x years, where $0 \le x \le 6$. Find and interpret $V(3)$ and $V(8)$.

Example 13: $p(x) = \begin{cases} 0 & \text{if } x < -4 \\ -x & \text{if } -4 \le x < 0 \\ x^2 & \text{if } x \ge 0 \end{cases}$

a) Find $p(-4)$, $p(0)$, and $p(3)$.

b) Find $p(a+2)$ if $-4 < a + 2 < 0$

c) Find $p(5 - 3a)$ if $5 - 3a \ge 0$