NAME ____________________

STUDENT ID ____________________

REC. INSTR. ____________________ REC. TIME ____________________

INSTRUCTIONS:

1. Verify that you have all the pages (there are 10 pages).
2. Fill in your name, your student ID number, and your recitation instructor's name and recitation time above. Write your name, your student ID number and division and section number of your recitation section on your answer sheet, and fill in the corresponding circles.
3. Mark the letter of your response for each question on the mark–sense answer sheet.
4. There are 25 problems. Each problem is worth 8 points.
5. No books or notes or calculators may be used.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!},\ |x| < \infty$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1},\ |x| < \infty$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n},\ |x| < \infty$$

$$(1+x)^s = \sum_{n=0}^{\infty} \binom{s}{n} x^n,\ |x| < 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n},\ |x| < 1$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n,\ |x| < 1$$

$$\sin^2 x = \frac{1-\cos(2x)}{2}$$

$$\cos^2 x = \frac{1+\cos(2x)}{2}$$

$$\sin(2x) = 2\sin x\cos x$$

$$1+\tan^2 x = \sec^2 x$$

The angle of rotation θ, $0 < \theta < \pi/2$, that eliminates the xy term from the second degree equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ satisfies the equation $\tan 2\theta = \dfrac{B}{A-C}$, provided $A \neq C$. If $A = C$, then $\theta = \pi/4$.

$x = (\cos\theta)X - (\sin\theta)Y$ and $y = (\sin\theta)X + (\cos\theta)Y$, where the XY coordinate system is obtained by rotation the x and y axes through the angle θ about the origin.

1. Find the angle between the vectors $\mathbf{u} = \mathbf{j} - \mathbf{k}$ and $\mathbf{v} = -2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $\frac{3\pi}{4}$

E. π

2. A parallelepiped has sides $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{c} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$. Its volume is

A. 0

B. 1

C. 2

D. 3

E. 4

3. $\lim_{x \to 0} \frac{1 - \cos x}{\sin^2(\pi x)} =$

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{2\pi}$

D. $\frac{1}{\pi}$

E. $\frac{1}{2\pi^2}$

4. $\int_1^2 x\ln(x^2)dx =$

A. $4\ln 2 - \frac{3}{2}$

B. $4\ln 2 - 2$

C. $3\ln 2 - 1$

D. $2\ln 2 + \frac{1}{2}$

E. $2\ln 2 - \frac{1}{2}$

5. $\int_0^2 \sqrt{4-x^2}\,dx =$

A. 2π

B. $2\pi - 1$

C. π

D. $\pi - 1$

E. $\pi - 2$

6. $\int_0^1 \frac{2}{(x+1)(x+3)}\,dx =$

A. $\ln 2 - \ln 3$

B. $\ln 3 - \ln 2$

C. $\frac{1}{2}(\ln 3 - \ln 2)$

D. $\frac{1}{4}$

E. $\frac{1}{2}$

7. $\displaystyle\int_2^{\infty} \frac{1}{x(\ln x)^2}\,dx =$

A. $\frac{1}{2}$

B. $\frac{1}{\ln 2}$

C. $\frac{1}{(\ln 2)^2}$

D. the integral is divergent

E. 1

8. Find the volume of solid generated by revolving about the x axis the region between the graphs of $y = x^2$ and $y = x^3$.

A. $\frac{\pi}{6}$

B. $\frac{1}{6}$

C. $\frac{\pi}{10}$

D. $\frac{2\pi}{35}$

E. $\frac{\pi}{20}$

9. It takes 4 ft-lbs of work to stretch a spring to 2 ft. beyond neutral position. How much force is required to hold the spring 3 ft. from the neutral position?

A. 2 lbs

B. 3 lbs

C. 4 lbs

D. 5 lbs

E. 6 lbs

10. The force of gravity on a rocket of mass m is given by $F(x) = \dfrac{-GMm}{x^2}$, where M is the mass of the earth, G is a universal constant, and x is distance (in miles) between rocket and the center of the earth. Assuming the radius of earth is 4000 miles and the rocket is launched from the surface of earth, find the limit of the amount of work required to send the rocket arbitrarily far away.

A. $-\dfrac{GMm}{(4000)^2}$

B. $-\dfrac{GMm}{4000}$

C. infinite

D. $\dfrac{GMm}{4000}$

E. $\dfrac{GMm}{(4000)^2}$

11. The third degree Taylor polynomial about $a = 0$ for $f(x) = \sin\left(\frac{\pi}{4} + 2x\right)$ is

A. $\left(\frac{\pi}{4} + 2x\right) - \frac{1}{6}\left(\frac{\pi}{2} + 2x\right)^3$

B. $\frac{1}{\sqrt{2}} + \sqrt{2}x - \sqrt{2}x^2 - \frac{2\sqrt{2}}{3}x^3$

C. $1 + x - \frac{1}{2}x^2 - \frac{1}{6}x^3$

D. $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}x - \frac{1}{2\sqrt{2}}x^2 - \frac{1}{6\sqrt{2}}x^3$

E. $1 + x + x^2 + x^3$

12. $\displaystyle\sum_{n=3}^{\infty} \left(-\frac{2}{3}\right)^n =$

A. $-\dfrac{8}{45}$

B. $\dfrac{3}{5}$

C. $-\dfrac{3}{2}$

D. $\dfrac{5}{7}$

E. ∞

13. The radius of convergence of $\sum_{n=0}^{\infty} \frac{2^n}{n!} x^n$ is

A. 0

B. $\frac{1}{2}$

C. 1

D. 2

E. ∞

14. Which of the following series converge?

I. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{n^2+1}}$

II. $\sum_{n=1}^{\infty} \frac{(-1)^n + n}{2^n}$

III. $\sum_{n=2}^{\infty} \frac{1}{n^2 \ln(n)}$

A. I, II, III

B. I, II

C. I, III

D. II, III

E. III

15. Which of the following series converge absolutely?

I. $\sum_{n=3}^{\infty} \frac{(-1)^n}{n \ln(n)}$

II. $\sum_{n=3}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$

III. $\sum_{n=3}^{\infty} \frac{(-1)^n}{\ln n}$

A. I, II, III

B. I, II

C. I, III

D. II, III

E. II

16. The interval of convergence of $\sum_{n=1}^{\infty} nx^n$ is

A. $(-\infty, \infty)$

B. $(-1, 1)$

C. $[-1, 1)$

D. $(-2, 2)$

E. $[0, 0]$

17. The Taylor series of $\dfrac{x^2}{1+x^2}$ about $x = 0$ is

A. $1 - x + x^2 - \dots$

B. $1 - x^2 + x^4 - \dots$

C. $x^2 + x^4 + x^6 + \dots$

D. $x^2 - x^3 + x^4 - \dots$

E. $x^2 - x^4 + x^6 - \dots$

18. $(1 + 2x)^{\frac{1}{5}} =$

A. $1 + \dfrac{1}{5}x - \dfrac{1}{50}x^2 + \dots$

B. $1 - 2x - 4x^2 + \dots$

C. $1 + \dfrac{2}{5}x - \dfrac{8}{25}x^2 + \dots$

D. $1 + \dfrac{2}{5}x - \dfrac{4}{25}x^2 + \dots$

E. $1 - x + x^2 + \dots$

19. $\int_0^1 e^{-x^4}\,dx =$

A. $1 - \frac{1}{120} + \frac{1}{1780} - \ldots$

B. $1 - \frac{1}{2} + \frac{1}{12} - \ldots$

C. $1 + \frac{1}{2} + \frac{1}{12} + \ldots$

D. $1 + \frac{1}{5} + \frac{1}{18} + \ldots$

E. $1 - \frac{1}{5} + \frac{1}{18} - \ldots$

20. The curve $r = -\frac{3}{2}\cos\theta$ is a circle with radius

A. 3

B. $\frac{4}{3}$

C. $\frac{3}{2}$

D. $\frac{3}{4}$

E. $\frac{2}{3}$

21. The length of the curve $x = \cos t + t\sin t$, $y = \sin t - t\cos t$, $0 \le t \le \frac{\pi}{2}$, is

A. $\frac{\pi}{2}$

B. $\frac{\pi^2}{8}$

C. $\frac{1}{2}$

D. $\frac{1}{8}$

E. 1

22. The area of the region inside both polar curves $r = \sin\theta$ and $r = \sqrt{3}\cos\theta$ is

A. $\frac{1}{2}\int_0^{\frac{\pi}{4}} \sin^2\theta d\theta + \frac{3}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2\theta d\theta$

B. $\frac{3}{2}\int_0^{\frac{\pi}{3}} \cos^2\theta d\theta + \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin^2\theta d\theta$

C. $\frac{1}{2}\int_0^{\frac{\pi}{3}} \sin^2\theta d\theta + \frac{3}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos^2\theta d\theta$

D. $\frac{3}{2}\int_0^{\frac{\pi}{4}} \cos^2\theta d\theta + \frac{1}{2}\int_0^{\frac{\pi}{4}} \sin^2\theta d\theta$

E. $\frac{1}{2}\int_0^{\frac{\pi}{6}} \sin^2\theta d\theta + \frac{3}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta d\theta$

23. The ellipse $5(x-1)^2 + 9y^2 = 45$ has foci

A. $(-1, 0), (3, 0)$

B. $(1-\sqrt{5}, 0), (1+\sqrt{5}, 0)$

C. $(-2, 0), (4, 0)$

D. $(1, -2), (1, 2)$

E. $(1, -\sqrt{5}), (1, \sqrt{5})$

24. The hyperbola $4x^2 - 9y^2 - 8x - 36y = -4$ has asymptotes given by

A. $2y - 3x = -7$
$2y + 3y = -1$

B. $3y + 2x = -4$
$3y - 2x = -8$

C. $3y - 2x = 0$
$3y + 2x = 0$

D. $3y - 2x = 7$
$3y + 2x = -1$

E. $2y + 3x = -8$
$2y - 3x = 4$

25. The graph of the equation $x^2 - 2xy + y^2 + 2y = 1$ is

A. an ellipse

B. a parabola

C. a hyperbola

D. a pair of lines

E. a circle