

Name: ANSWER KEY

Place your answers in the spaces provided. You must show correct work to receive credit.

(14 pts) 1. Given the functions  $f(x) = 2x^3$  and  $g(x) = 5x - 3$ , find and simplify each of the following:

(4 pts) (a)  $(f - g)(-1)$   
 $= f(-1) - g(-1)$   
 $= -2 - (-8) = -2 + 8 = 6$

6

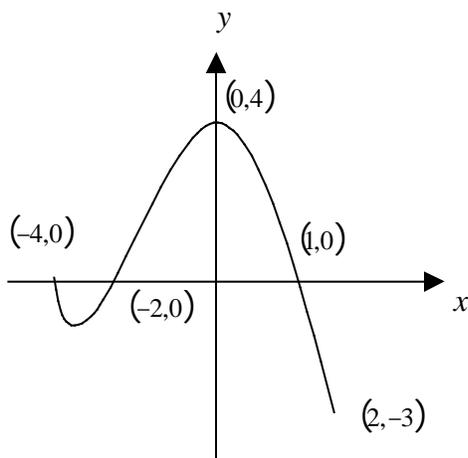
(4 pts) (b)  $f/g(3)$   
 $= \frac{f(3)}{g(3)} = \frac{2(3)^3}{5(3) - 3} = \frac{54}{12} = \frac{9}{2}$

$\frac{9}{2}$  or 4.5

(6 pts) (c)  $(g \circ f)(x)$   
 $= g[f(x)] = g(2x^3)$   
 $= 5(2x^3) - 3 = 10x^3 - 3$

$10x^3 - 3$

(8 pts) 2. Given the graph of a function  $y = f(x)$  below, find each of the following. Express your answers in interval notation.



(2 pts) (a) the domain

[-4, 2]

(2 pts) (b) the range

[-3, 4]

(4 pts) (c) all  $x$  such that  $f(x) < 0$

(-4, -2) (1, 2]

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(12 pts) 3. Solve the following system of equations. Express your answer(s) as an ordered pair(s).

$$2x^2 - 3y = 23$$

$$y - 2x = -5$$

$$y = 2x - 5$$

substitute into the first equation :

$$2x^2 - 3(2x - 5) = 23$$

$$2x^2 - 6x + 15 - 23 = 0$$

$$2x^2 - 6x - 8 = 0$$

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$x = 4, \quad x = -1$$

$$\text{If } x = 4 \quad y = 2(4) - 5 = 3$$

$$\text{If } x = -1 \quad y = 2(-1) - 5 = -7$$

(4,3) and (-1,-7)
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(8 pts) 4. Given the function  $g(x) = \frac{2}{x} - 5$ , find the inverse function  $g^{-1}(x)$ .

$$y = \frac{2}{x} - 5$$

$$x = \frac{2}{y} - 5$$

$$x + 5 = \frac{2}{y}$$

$$y(x + 5) = 2$$

$$y = \frac{2}{x + 5}$$

$g^{-1}(x) = \frac{2}{x + 5}$
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(8 pts) 5. If  $f(x) = 2x^2 + kx - 5k$ , find a number  $k$  such that the graph of  $f$  contains the point  $(-3, 4)$ .

$$4 = 2(-3)^2 + k(-3) - 5k$$

$$4 = 2(9) - 3k - 5k$$

$$-14 = -8k$$

$$k = \frac{14}{8}$$

$k = \frac{14}{8} \text{ or } \frac{7}{4} \text{ or } 1.75$
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(8 pts) 6. Find the minimum value of  $f(x) = 3x^2 + 12x - 1$ .

$$x = \frac{-b}{2a}, \quad x = \frac{-12}{2(3)} = -2$$

Min. value:

$$f(-2) = 3(-2)^2 + 12(-2) - 1 \\ = 12 - 24 - 1 = -13$$

or

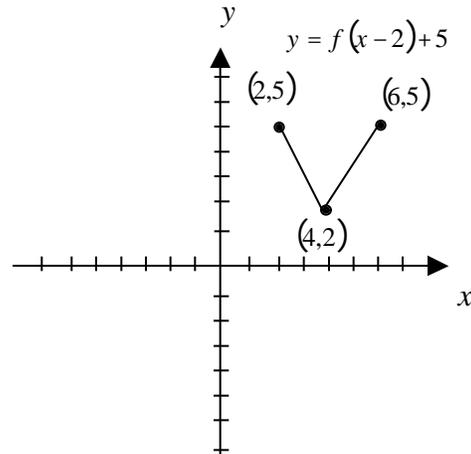
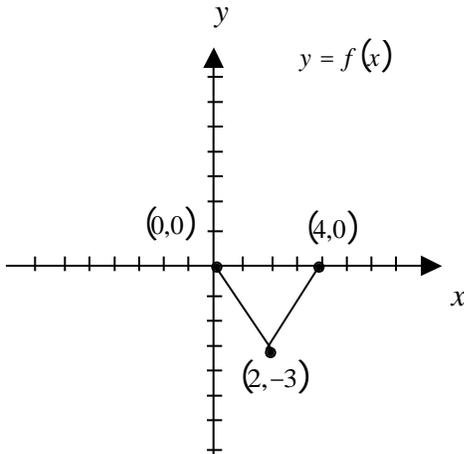
$$f(x) = 3(x^2 + 4x + 4) - 1 + \underline{-12}$$

$$f(x) = 3(x + 2)^2 - 13$$

Vertex:  $(-2, -13)$

-13
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(10 pts) 7. Given the graph of  $y = f(x)$  below with domain  $[0, 4]$ , use the set of axes to sketch the graph of  $y = f(x - 2) + 5$ . Label the three main points on your sketch of the graph.



(10 pts) 8. Given  $f(x) = x^2 - x$ , find and simplify  $\frac{f(a+h) - f(a)}{h}$  ( $h \neq 0$ ).

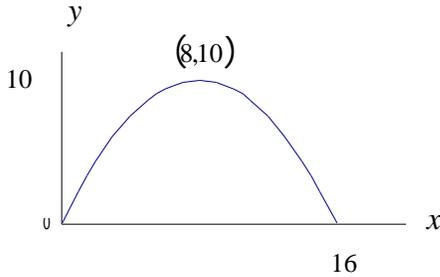
$$\begin{aligned} \frac{f(a+h) - f(a)}{h} &= \frac{(a+h)^2 - (a+h) - (a^2 - a)}{h} \\ &= \frac{a^2 + 2ah + h^2 - a - h - a^2 + a}{h} \\ &= \frac{2ah + h^2 - h}{h} = \frac{h(2a + h - 1)}{h} \end{aligned}$$

$\frac{f(a+h) - f(a)}{h} = 2a + h - 1$
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- (12 pts) 9. A ball is thrown upward from ground level. The path of the ball is in the shape of a parabola. The ball was thrown a distance of 16 yards and the maximum height off the ground was 10 yards (see the figure below). Find the standard equation,  $y = a(x - h)^2 + k$ , for the path of the ball.



$$y = a(x - 8)^2 + 10$$

a point is (0,0)

$$0 = a(0 - 8)^2 + 10$$

$$a = -\frac{10}{64} \text{ or } -\frac{5}{32} \text{ or } -.15625$$

$$y = \boxed{-\frac{10}{64}(x - 8)^2 + 10}$$

- (10 pts) 10. Suppose that the amount of spending money,  $M$ , Purdue students receive from their parents is directly proportional to the square root of the parents' annual income,  $I$ , and inversely proportional to the daily number of hours the student watches television,  $T$ .

(4 pts) (a) Express the statement in terms of  $M, I, T$ , and a constant of proportionality,  $k$ .

$$\boxed{M = \frac{k\sqrt{I}}{T}}$$

- (6 pts) (b) If a student receives \$400, watches 5 daily hours of television, and his parent's annual income is \$62,500, find the value of  $k$  from part (a).

$$400 = \frac{k\sqrt{62500}}{5}$$

$$2000 = k(250)$$

$$k = 8$$

$$k = \boxed{8}$$

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