June 26, 2001 MA 266 Name:_____

McQuinn ID#:_____

Exam I

Instructions:

- Write your name on each page.
- You have 60 minutes to complete all problems.
- Write your final answers in the boxes provided, where appropriate.
- Do all work to be graded on the FRONT of the exam sheets. Use the back of the exam sheets for scratch paper, and do not use any other scratch paper. No work on the back of the exam sheets will be graded.
- **No credit will be given for any problem without work being shown.**
- No calculators, no notes, no books, no formula sheets may be used.
- It is advisable to check your work carefully before turning in the exam.
- This exam has 5 pages plus this cover sheet; please ensure that you have all pages.

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Page 4		15
Page 5		22
Total		100

Write your answers in the boxes provided. Be sure to show all work.

(12) 1. For the differential equations below, circle each property that applies (if any). You do not need to justify your answers.

$(\ln x)y' + y = xy$ linear homogeneous exact separable

$(3x^2y + y^2) + (2yx + x^3)\dfrac{dy}{dx} = 0$ linear homogeneous exact separable

$(x^2 + y^2)\,dx - 2xy\,dy = 0$ linear homogeneous exact separable

(10) 2. Solve the initial value problem for y (explicitly)

$$y' + \frac{1}{x}y = \frac{4}{x}e^{2x} \quad \text{and} \quad y(2) = 2e^4$$

$y =$

(10) 3. Find all solutions (explicitly).

$$\frac{dy}{dx} = \frac{xy - y^2}{x^2}$$

(9) 4. Determine all equilibrium solutions, and classify them as stable, unstable, or semi-stable.

$$y' = y(1-y)(y+2)$$

(10) 5. A tank holds 50 gallons of fresh water. Brine containing 2 pounds per gallon of salt is pumped in at the rate of 3 gallons per minute, and the mixture is allowed to flow out at the same rate. Let t be the time in minutes, and $Q(t)$ the amount of salt at time t (in pounds). Set up an *initial value problem* relating t and $Q(t)$. *Do not solve the initial value problem.*

(12) 6. Find all solutions (implicitly if necessary).

$$y'' + 2y(y')^3 = 0$$

(15) 7. Without solving, determine whether the initial value problem is *guaranteed* to have a unique solution. Justify your answer.

(a) $(\sin \pi t)\, y' + e^{t^2} y = 3 \quad \text{and} \quad y(2) = 2$

(b) $y' = \ln x + 1/y \quad \text{and} \quad y(1) = -1$

(c) $y' = x\sqrt{y} \quad \text{and} \quad y(1) = 0$

8. A rocket takes off from an initial height of R miles above the surface of the Earth, where R is the Earth's radius. Let m be the mass of the rocket (which is constant), x the height in miles above the ground and v the velocity at time t. The only force acting is the force due to gravity, with magnitude $\frac{gmR^2}{(R+x)^2}$. The initial velocity is $\sqrt{\frac{gR}{2}}$.

(14) (a) Set up *and solve explicitly* an equation that relates v to x at time t. Leave your answer in terms of the constants R and g. Remember that the rocket may be *rising or falling* for different values of t.

$v =$

(8) (b) What is the rocket's maximum height (in miles) above the Earth's surface (in terms of R)?

BONUS (The Simpsons):

What is the name of Homer Simpson's brother?

What was Bart's first word?