Advanced Graduate Courses offered by the Mathematics Department Fall, 2000

Courses

MA 546: Introduction to Functional Analysis
Instructor: Prof. L. Brown, office: Math 704, phone: 49–41938, e-mail: lgb@math.purdue.edu
Time: MWF 11:30
Prerequisite: MA 544
Description: This is a course in Banach and Hilbert spaces and bounded linear operators on such spaces, culminating in the spectral theorem for bounded self–adjoint linear operators on Hilbert space. Other topics include dual spaces and the Hahn-Banach theorem, compact operators and their spectral theory, spectrum and holomorphic functional calculus for general bounded operators, Fredholm operators and index, reflexivity, and separation of convex sets and the double polar theorem.
Text: Schechter, *Principles of Functional Analysis*, Academic Press

MA 557: Abstract Algebra I
Instructor: Prof. L. Avramov, office: Math 640, phone: 49–41978, e-mail: avramov@math.purdue.edu
Time: TTh 1:30-2:45
Description: The course will cover basic techniques and results of the theory of commutative rings and their modules, in the spirit of the classical text *Commutative Algebra* by M. F. Atiyah and I. G. Macdonald. The prerequisites are the material from the standard algebra courses (MA 553 and MA 554), some preparation for the fact that two pages of the textbook may require three hours of study, and willingness to work on numerous exercises that look like theorems.

MA 585: Mathematical Logic I
Instructor: Prof. L. Lipshitz, office: Math 428, phone: 49–41907, e-mail: lipshitz@math.purdue.edu
Time: MWF 9:30 (**Note new time. The time of MA 585 has been changed from the time that was originally announced.**)

MA 587: General Set Theory
Instructor: Prof. J. Rubin, office: Math 400, phone: 49–41986, e-mail: jer@math.purdue.edu
Time: MWF 10:30
Prerequisite: mathematical maturity, familiarity with mathematical proofs.
Description: This is a course in axiomatic set theory. Topics include operations on sets, cardinal and ordinal numbers, the axiom of choice, the well ordering theorem, Zorn's lemma, consistency and independence results.
Text: K. Devlin *Fundamentals of Contemporary Set Theory*, Springer-Verlag.

MA 598A Intermediate Abstract Algebra
Instructor: Prof. J. Lipman, office: Math 750, phone: 49–41994, e-mail: lipman@math.purdue.edu
Time: MWF 9:30
Description: This course will cover fundamentals on groups, rings, and fields (excluding galois theory). It is intended to prepare students who don't have enough background in algebra for the faster-paced Ph.D. qualifier courses MA 553 and 554, as well as to introduce those whose goals may not include the algebra qualifiers to some of the basic concepts.
Text: Dummit and Foote *Abstract Algebra*, Prentice Hall, 2nd edition.

MA 598B Computational Commutative Algebra
Instructor: Prof. J. Lipman, office: Math 750, phone: 49–41994, e-mail: lipman@math.purdue.edu
Time: MWF 11:30
Prerequisite: MA 553. Acquaintance with CoCoA will be useful, but not strictly essential.
Description: The course will be concerned with basic operations in commutative algebra which lend themselves to explicit computer calculations. While the theoretical underpinnings - Gröbner bases - will be explained, the emphasis will be on working out specific problems and applications using a computer algebra system. The system used by the textbook is CoCoA (short for the title of the text and of the course), which will be available on the Math. Dept. computers, and also downloadable (free, with online manual) for PC's and Mac's. <http://cocoa.dima.unige.it>
Sample Problems:

- Division of a multivariable polynomial by an ideal, with remainder.
- Perform elementary operations ($I \cap J, I : J, \cdots$) on pairs (I, J) of polynomial ideals.
- Eliminate variables from systems of equations, and study the solution set of such systems (i.e., affine algebraic varieties).
- Compute kernels and images of ring– and module–maps.

Sample applications:

- Nullstellensatz.
- Primary ideal decomposition.
- Factoring polynomials over finite fields.
- Integer programming.

Text: Kreuzer and Robbiano *Computational Commutative Algebra*. (This exists presently only in draft form, which will be distributed to students if the published version hasn't appeared.)

MA 598C: Numerical Methods for Partial Differential Equations
Instructors: Prof. J. Douglas, office: Math 822, phone: 49–41927, e-mail: douglas@math.purdue.edu
and Prof. Z. Cai, office: Math 810, phone: 49–41921, e-mail: zcai@math.purdue.edu
Time: TTh 1:30-2:45
Prerequisite: MA 523 or consent of instructor
Description: This course is designed for two semesters to replace the original one semester course on finite element methods (Math 524). The goal of this course is to teach the basic methodology for developing accurate, robust, and efficient algorithms for the numerical solution of partial differential equations in applied mathematics, science and engineering. The course will provide the mathematical foundation of numerical methods together with important numerical aspects. Applications to some basic problems in mechanics and physics will also be considered.
Fall Semester 2000. The course will begin with finite difference and finite element methods for two-point boundary value problems and direct and iterative methods for the resulting algebraic equations. Finite difference and finite element methods will be developed and analyzed for elliptic and parabolic partial differential equations. Iterative solvers including preconditioned conjugate gradient, domain decomposition, and multigrid methods will be introduced for the resulting system of linear and nonlinear equations from the discretizations of elliptic and parabolic problems. Finally, we will discuss numerical methods for hyperbolic partial differential equations. Some implementational aspects will be considered.
Spring Semester 2001. The second semester will begin with polynomial approximation theory in Sobolev spaces. We will then develop and analyze mixed finite element methods for both elliptic and parabolic equations. As a special case of the mixed method, we will introduce the finite volume method. Domain decomposition and/or multigrid methods will be further studied. Topics on advanced methods such as methods of characteristics, least squares, and adaptive mesh refinement and on applications such as incompressible Stokes and Navier-Stokes, elasticity, Maxwell, porous media, and pseudo-differential equations are at the discretion of the instructor. These are current topics of very active research in computational mathematics.

MA 598F: Mathematics of Finance
Instructor: Prof. V. Weston, office: Math 746, phone: 49–41959, e-mail: weston@math.purdue.edu
Time: MWF 8:30
Prerequisite: Knowledge of Ordinary Differential Equations (Ma 262, 361 or equivalent). Multivariate Calculus (curves, line integrals, double integrals, chain rule for partial derivatives), is required, and some knowledge of elementary Probability Theory would be useful (only needed for very small part of course).
Course Objective: For students in mathematics and related disciplines, to introduce them to the mathematical models of Financial Derivatives, and the theory and technique for solving the associated partial differential equations. Course is self contained in that it will include an introduction to the terminology of mathematical finance and supplementary background material.
Description: Course Outline:

1. Mathematical Models: basic definitions, European and American options, Mathematical model of assets (brief introduction to random walk), Stochastic differential equation for options, Ito's Lemma, development of Black-Scholes equation, terminal and boundary conditions.
2. Diffusion Equation: well-posedness, initial and terminal boundary conditions, explicit solutions to the Black–Scholes equation.
3. Brief introduction to distribution theory and weak solutions, delta and Heaviside function, fundamental solution, and solution (Greens function representation, optional) of the initial and terminal boundary-value problem.
4. American Options: Free boundary problems, formulation in terms of Greens functions or otherwise, local analysis using self-similar solutions
5. Diffusion Equation with Time Dependent Parameters: formulation and solutions of Black- Scholes equation with time dependent parameters, formulation when dividends are paid out in discrete time intervals.
6. (if time permits)Variational Formulation of the American Option: variational formulation of free -boundary problems.
7. Exotic Options: models of look-back, barrier, Asiatic options, etc., solutions of the corresponding equations.
8. Bond models: Vasicek and other equations for modelling bond pricing.
9. Binomial Approach to option pricing.

References: J.C. Hull *Options ,Futures, and Other Derivative Securities*
Text: The course will be based upon Chapters 1-6, 8-12, and 14 (and if time, chapter 7 on variational techniques) of the book *Option Pricing, Mathematical Models and Computation* by P.Wilmott, J.Dewynne,S.Howison , Oxford Financial Press ,1997 . Since this book is too expensive, students will not be required to buy it. Instead the following simple condensed afordable version will be required: P. Wilmott, J. Dewynne and S. Howison, *Mathematics of Financial Derivatives, a Student Edition*, Oxford Financial Press, 1999.

MA/STAT 638: Stochastic Processes I
Instructor: Prof. B. Davis, office: Math 538, phone: 49–46048, e-mail: bdavis@stat.purdue.edu
Time: MWF 8:30
Prerequisite: Math-Stat 539 is perfect preparation for the course. Minimal preparation is probability at a measure theoretic level, including martingales.
Description: We will certainly cover the first five chapters of the text, which give the core material on stochastic integrals and stochastic differential equations. Applications to boundary value problems of partial differential equations will also be treated. Other possible topics include optimal stopping, filtering, and stochastic control.
Text: B. Oksendal *Stochastic Differential Equations*

MA 642: Methods of Linear and Nonlinear Partial Differential Equations
Instructor: Prof. D. Phillips, office: Math 706, phone: 49–41939, e-mail: phillips@math.purdue.edu
Time: MWF 2:30
Prerequisite: MA 523 & MA 611
Description: Second order elliptic equations including maximum principles, Harnack inequality, Schauder estimates, and Sobolev estimates. Applications of linear theory to nonlinear problems.
Text: Gilbarg and Trudinger, *Elliptic Partial Differential Equations of Second Order*, Springer, 2nd Ed.

MA 663: Algebraic Curves and Functions I
Instructor: Prof. S. Abhyankar, office: Math 432, phone: 49–41933, e-mail: ram@math.purdue.edu
Time: TTh 3:00-4:15
Description: Algebraic geometry, concerned with solutions of systems of polynomial equations, and their graphical representations, has for long been regarded an abstract area of mathematics. However, recently, with the advent of high-speed computers, applications have come about in such diverse areas of science and engineering such as theoretical physics, chemical, electrical, industrial and mechanical engineering, computer aided design (CAD), computer aided manufacturing (CAM), optimization, and robotics. These application areas are also increasingly posing fundamental open mathematical questions. This course is intended as an introduction to various relevant topics in algebraic geometry such as:

- Analysis and resolution of singularities
- Rational and polynomial parametrization
- Intersections of curves and surfaces
- Polynomial maps
- Fundamental Groups and Galois groups
- Solutions of non–linear algebraic systems with uniqueness and multiplicity criterion.

The lectures will be expository in nature and so will be accessible to everyone.
Text: Shreeram S. Abhyankar, *Algebraic Geometry for Scientists and Engineers*, Amer Math Soc
References: Shreeram S. Abhyankar, *Resolution of Singularities of Embedded Algebraic Surfaces*, Springer Verlag

MA 665: Algebraic Geometry
Instructor: Prof. S. Archava, office: Math 748, phone: 49–43173, e-mail: archava@math.purdue.edu
Time: TTh 10:30-11:45
Prerequisite: MA 530
Description: This will be a first course on algebraic curves and Riemann surfaces for students who had taken some complex analysis, for example MA 530. Starting from the multivalued analytical functions and analytical continuation I will introduce Riemann surfaces, study their basic properties and consider many concrete examples: Riemann surfaces of algebraic functions, meromorphic functions and differentials, residue theorem, genus, Riemann–Hurwitz formula, how to cut and spread a Riemann surface flat. Along the way I will introduce some more advanced techniques such as sheaves, vector bundles, cohomology, to prove Riemann–Roch formula, Serre duality, Hodge–deRham theorem and show that every compact Riemann surface corresponds to an algebraic curve. Besides being a natural continuation of a basic complex analysis course this class can serve as an introduction to algebraic geometry.
Texts: 1. O. Forster *Riemann Surfaces* (main text)
2. R. Narasimhan *Compact Riemann Surfaces*

MA 690A: Homological Algebras
Instructor: Prof. L. Avramov, office: Math 640, phone: 49–41978, e-mail: avramov@math.purdue.edu
Time: TTh 12:00-1:15
Description: Homological Algebra is a language in which many parts of modern mathematics are written. The course will concentrate on the grammar. The aim will be to present the basic aspects of the subject at three levels, each one subsumed by the next: modules, complexes, differential graded modules. Guidelines may be found in any textbook named *Homological Algebra*, but preference will be given to the one by J. Rotman for the first level, and the one by C. Weibel for the second level.

MA 690B: Topics in Commutative Algebra
Instructor: Prof. W. Heinzer, office: Math 636, phone: 49–41980, e-mail: heinzer@math.purdue.edu
Time: MWF 12:30
Prerequisite: MA 650 or 558
Description: The course will cover selected topics in commutative algebra concerned with the generation of modules and ideals and the number of equations needed to describe an algebraic variety.
Text: Ernst Kunz *Introduction to Commutative Algebra and Algebraic Geometry*

MA 692A: Special Topics in Numerical Analysis
Instructor: Prof. J. Douglas, office: Math 822, phone: 49–41927, e-mail: douglas@math.purdue.edu
Time: TTh 3:00-4:15
Description: Numerical solution of differential equations, modeling of flows and waves in porous media and their numerical approximation, inverse and not–well–posed problems.

MA 692B: Wavelets and Image Processing
Instructor: Prof. B. Lucier, office: Math 634, phone: 49–41979, e-mail: lucier@math.purdue.edu
Time: MWF 9:30
Prerequisite: MA 544 (real analysis through measure theory) and some functional analysis (the first three chapters on Metric Spaces, Banach Spaces, and Hilbert Spaces of *Elements of Applicable Functional Analysis* by Charles W. Groetsch would suffice).
Description: The goal of the course is to describe and analyze nonlinear wavelet algorithms for three fundamental problems in low-level image processing: image compression, Gaussian noise removal, and inverting the Radon transform and other homogeneous linear operators with noisy data (which has application to medical imaging—Magnetic Resonance Imaging, Computed Tomography, and especially Positron Emission Tomography). The main mathematical tool will be Nonlinear Approximation Theory and its relation to the theory of smoothness spaces (i.e., characterizing the smoothness of images in useful ways); we will also use some simple probability theory, variational principles, etc. The main focus of the course will be the rigorous analysis of the algorithms, not the development or theory of wavelet filters per se.
Text: I. Daubechies, *Ten Lectures on Wavelets.* Although (or perhaps because) the material presented in class will complement the material in this book, every student should have a copy of this book.

MA 693B: Riemann Theta Functions
Instructor: Prof. L. de Branges, office: Math 800, phone: 49–46057, e-mail: branges@math.purdue.edu
Time: MWF 10:30
Description: A Riemann theta function is a function analytic in the upper half-plane whose Mellin transform is a Riemann zeta function. These zeta functions, which satisfy the expectations of the Riemann hypothesis, are characterized by an Euler product and a functional identity. A Riemann theta function is accordingly characterized by its invariance under the weighted action of a Hecke subgroup of the modular group and the finiteness of the Petersen norm. It is an eigenfunction of Hecke operators which are self–adjoint for the resulting scalar product. An adelic treatment is given of the estimate of eigenvalues conjectured by Ramanujan. The course is intended as preparation for a thesis in the structure theory of Riemann theta functions.