# Advanced Graduate Courses
# offered by the
# Mathematics Department
# Fall, 2002

**MA/STAT 539: Probability Theory II**
**Instructor:** Prof. T. Sellke, office: Math 502, phone: 49–46034, e-mail: tsellke@stat.purdue.edu
**Time:** TTh 1:30-2:45
**Description:** MA/STAT 539 will continue (from MA/STAT 538) to follow the book *Probability and Measure*, by Billingsley, 3rd Edition. We will cover conditional expectation, martingales and Brownian motion.

**MA/STAT 542: Theory of Distributions and Applications**
**Instructor:** Prof. D. Phillips, office: Math 706, phone: 49–41939, e-mail: phillips@math.purdue.edu
**Time:** MWF 1:30
**Prerequisite:** MA 510 and MA 525, or equivalent.
**Description:** Distribution theory allows one to work with differentiation in its most general sense. Because of this, distribution theory is fundamental for work in all areas of analysis, geometry, as well as for linear and nonlinear partial differential equations. In this course the basic properties and definitions for distribution theory are introduced as well as convolution and Fourier transform methods, and applications to partial differential equations.

**MA 546: Introduction to Functional Analysis**
**Instructor:** Prof. M. Dadarlat, office: Math 708, phone: 49–41940, e-mail: mdd@math.purdue.edu
**Time:** MWF 2:30
**Prerequisite:** MA 544
**Description:** Description: Banach spaces and Hilbert spaces; weak topologies; Hahn-Banach theorem; principle of uniform boundedness; open mapping theorem; Krein-Milman theorem and applications (including Stone-Weierstrass theorem). Operators on Hilbert spaces; spectral theorem for hermitian operators; Compact operators; Peter-Weyl theorem. Depending on the interest of the students, I plan to discuss additional topics related to group representations, operator algebras and/or PDEs.

The grade will be based on homework (70%) and a take home final exam (30%).
**References:** Most topics are covered by J. B. Conway's book, *A Course in Functional Analysis*, which is recommended.

**MA 562: Introduction to Differential Geometry and Topology**
**Instructor:** Prof. H. Donnelly, office: Math 716, phone: 49–41944, e-mail: hgd@math.purdue.edu
**Time:** TTh 3:00-4:15
**Prerequisite:** MA 351 and MA 442
**Description:** Topics – Smooth manifolds, tangent vectors, inverse and implicit function theorems, submanifolds, vector fields, integral curves, differential forms, the exterior derivative, partitions of unity, integration on manifolds, fundamentals of Riemannian geometry, Levi–Civita connection, parallel transport, geodesics, curvature tensor.
**Text:** William Boothby, *Introduction to Differentiable Manifolds and Riemannian Geometry*

**MA 571: Elementary Topology**
**Instructor:** Prof. D. Gottlieb, e-mail: gottlieb@math.purdue.edu
**Time:** TTh 10:30-11:45
**Description:** Topics to be covered – sets; mappings; equivalences; a metric on a set; open sets; continuous functions; homeomorphisms and isometries; compactness; connectedness; product spaces; quotient spaces; separation and countability properties; homotopy; fundamental group; Brouwer fixed point theorem; Borsuk–Ulam theorem; Vector fields; Degrees of maps.

**MA 585: Mathematical Logic I**
**Instructor:** Prof. E. Hall, office: Math 632, phone: 49–41982, e-mail: ericeric@math.purdue.edu
**Time:** TTh 12:00–1:15
**Description:** We'll first spend some time developing a good foundation in propositional and predicate logic, including the completeness and compactness theorems. Following that, we will discuss some decidabilty and computabilty and tackle proofs of the Gödel incompleteness theorems. Bonus topics as time and student/professor interest permit and/or demand.

No formal prerequisites. There will be 5 or so homework sets.
**Text:** First four chapters of Peter G. Hinman's *Mathematical Logic and Foundations of Mathematics*, which has not yet been published.

**MA 598B An Introduction to Homological Algebra with examples from group theory**
**Instructor:** Prof. C. Wilkerson, office: Math 450, phone: 49–41955, e-mail: wilker@math.purdue.edu
**Time:** MWF 10:30
**Description:** This course will be based on selected sections from the first six chapters of Weibel. It will be supplemented by examples from group theory and a brief introduction to machine computations using tools such as GAP, CoCoA, and Macaulay 2.
**Text:** C. Weibel, *Introduction to Homological Algebra*, Cambridge Press, Studies in Advanced Mathematics, # 38.

**MA 598C: Numerical Methods for Partial Differential Equations**
**Instructors:** Prof. Z. Cai, office: Math 810, phone: 49–41921, e-mail: zcai@math.purdue.edu
**Time:** TTh 12:00-1:15
**Prerequisite:** MA 523 or consent of instructor
**Description:** This course is designed for two semesters to replace the original one semester course on finite element methods (Math 524). The goal of this course is to teach the basic methodology for developing accurate, robust, and efficient algorithms for the numerical solution of partial differential equations in applied mathematics, science and engineering. The course will provide the mathematical foundation of numerical methods together with important numerical aspects. Applications to some basic problems in mechanics and physics will also be considered.
**Fall Semester 2002.** The course will begin with finite difference and finite element methods for two-point boundary value problems and direct and iterative methods for the resulting algebraic equations. Finite difference and finite element methods will be developed and analyzed for elliptic and parabolic partial differential equations. Iterative solvers including preconditioned conjugate gradient, domain decomposition, and multigrid methods will be introduced for the resulting system of linear and nonlinear equations from the discretizations of elliptic and parabolic problems. Finally, we will discuss numerical methods for hyperbolic partial differential equations. Some implementational aspects will be considered.
**Spring Semester 2003.** The second semester will begin with polynomial approximation theory in Sobolev spaces. We will then develop and analyze mixed finite element methods for both elliptic and parabolic equations. As a special case of the mixed method, we will introduce the finite volume method. Domain decomposition and/or multigrid methods will be further studied. Topics on advanced methods such as methods of characteristics, least squares, and adaptive mesh refinement and on applications such as incompressible Stokes and Navier-Stokes, elasticity, Maxwell, porous media, and pseudo-differential equations are at the discretion of the instructor. These are current topics of very active research in computational mathematics.

**MA 598E/EAS 591B: Instability of Complex Systems and Earthquake Prediction**
**Instructor:** Prof. A. Gabrielov, Office: MATH 648, Phone: 49–47911, e-mail: agabriel@math.purdue.edu
**Time:** TTh 9:00-10:15
**Description:** The course will describe the methodology for understanding and predicting critical transitions in complex nonlinear systems in geosciences and socio-economic sciences, with an emphasis on earthquake prediction.
Topics:

1. Earth crust as a complex nonlinear (chaotic) system: mechanisms of instability in the crust, predictability of geological disasters.
2. Pattern recognition as a powerful tool for understanding complexity. Applications in Earth sciences, geological prospecting, socio-economic and political predictions.
3. Modeling of seismicity: mass-spring models, lattice models and self-organized criticality, hierarchical models and renormalization.
4. Dynamics of fault network. Long-range correlations of seismicity in different fault zones. Symptoms of near-critical state of the fault network.
5. Earthquake prediction: theoretical base, prediction algorithms, success to failure scores.
6. Fundamental implications. What can we learn in earthquake prediction research about Earth and about complex systems in general.
7. Socio-economic predictions: economic recessions, unemployment, elections.

**No preliminary knowledge of seismology is expected. All necessary background on seismicity, nonlinear dynamics, and continuum mechanics will be included in the course.**

**MA 598F: Mathematics of Finance**
**Instructor:** Prof. F. Viens, office: Math 504, phone: 49–46035, e-mail: viens@stat.purdue.edu
**Time:** TTh 9:00-10:15
**Prerequisite:** MA 519 (or equivalent) + MA 261 (or equivalent) + MA 262 or 266 (or equivalent); or consent of the instructor.
**Description:** We will provide an introduction to the mathematical tools and techniques of modern finance theory, in the context of Black-Scholes-style option pricing. The typical (pricing) question is: how much should you charge someone for allowing them the right to purchase a certain stock from you at a given price and given time in the future? The typical (Black-Scholes) assumption is that the differential of the (log of the) stock price is the sum of a constant term (r.dt, constant interest rate) and a random noise term (dW(t), a Brownian increment). Under this assumption, to answer the pricing question, the main mathematical tool is stochastic calculus and its connection to partial differential equations. These mathematics will be the object of a thorough introduction at an elementary level, without measure theory. This toolbag will enable us to derive the main pricing and hedging results in complete and incomplete markets, and to treat many examples of exotic and path-dependent options, as well as an introduction to stochastic optimal control and portfolio optimization.
**Text:** T. Bjork, *Arbitrage Pricing in Continuous Time*, Oxford, 1998.
Suggested additional reading: D. Lambertson and B. Lapeyre, *Stochastic Calculus applied to Finance*, Chapman and Hall/CRC, 1996, reprinted 2000 by CRC Press.; P. Wilmott, S. Howison, J. Dewynne, *The mathematics of financial derivatives. A student introduction.* Cambridge U.P. 1995. Chapters 11 to 16.

**MA 598N: Topics in the Numerical Analysis of Partial Differential Equations**
**Instructor:** Prof. J. Douglas, office: Math 822, phone: 49–41927, e-mail: douglas@math.purdue.edu
**Time:** TTh 1:30-2:45
**Description:** Mixed finite element methods revisited, finite volume methods, numerical methods for transport–dominated diffusion processes, applications to simulation of flow in porous media.

**MA 598T: Stochastic Partial Differential Equations**
**Instructor:** Prof. F. Viens, office: Math 504, phone: 49–46035, e-mail: viens@stat.purdue.edu
**Time:** TTh 10:30-11:45
**Prerequisite:** A non–measure–theoretic working knowledge of probability theory AND stochastic calculus, contained for example in a solid performance in MATH/STAT 598F or in MATH/STAT 532, or in the first few chapters of Oksendal's book on Stochastic Differential Equations.
**Description:** A basic course which introduces the topic of stochastic partial differential equations (SPDEs) via some simple examples that are amenable to detailed calculations, focusing mainly on parabolic equations. We will cover: existence and uniqueness questions, Feynman–Kac–type and particle representation, local and long–term behavior via the theory of Gaussian regularity, and the brand new topic of SPDEs driven by fractional Brownian motion. We will look at specific applications in finance (theory of stochastic interest rates) and filtering (optimal nonlinear stochastic filtering theory).

**MA 598W: Nonlinear Waves**
**Instructor:** Prof. M. Chen, e-mail: chen@math.purdue.edu
**Time:** MWF 3:30
**Description:** This is an advanced course in the modern theory of nonlinear wave propagation. The course assumes some knowledge of real analysis and functional analysis as well as acquaintance with Sobolev spaces and the Fourier transform. Most topics will be developed from scratch, however. It is suitable for second-year graduate students or well-prepared first-year students. The course will cover material taken from the topics listed below.
Topics.

1. Derivation of model equations for long waves; One-way models; Two-way models; Weakly three-dimensional models
2. Initial-value problems
3. Boundary-value problems
4. Solitary waves and other travelling-wave phenomena
5. Numerical simulations; algorithms; analysis
6. Comparison between various models
7. Stability and instability - singularity formation
8. Dissipative effects; Long-time asymptotics of solutions; Comparison with laboratory data
9. Applications in coastal engineering

**MA 642: Methods of Linear and Nonlinear Partial Differential Equations**
**Instructor:** Prof. P. Bauman, Office: MATH 718, Phone: 49–41945, e-mail: bauman@math.purdue.edu
**Time:** MWF 9:30
**Prerequisite:** MA 544 & MA 611 or equivalent
**Description:** This is the first semester of a one-year course in pde theory with applications to nonlinear equations and systems of pde. The first semester will focus on the study of a priori estimates for second order elliptic equations (in divergence and nondivergence form), including maximum principles, Harnack inequalities, classical and Schauder interior and boundary estimates, Sobolev inequalities and imbedding theorems, and applications to nonlinear systems of pde.

The second semester will focus on the theory for time-dependent systems of pde.
**Text:** Gilbarg and Trudinger, *Elliptic Partial Differential Equations of Second Order*, Springer, Second Edition

**MA 690B: Topics in Commutative Algebra**
**Instructor:** Prof. W. Heinzer, office: Math 636, phone: 49–41980, e-mail: heinzer@math.purdue.edu
**Time:** MWF 12:30
**Prerequisite:** MA 557
**Description:** The course will cover selected topics in commutative algebra concerned with the generation of modules and ideals and the number of equations needed to describe an algebraic variety. It will also cover:
(1) properties of extensions rings such as flatness,
(2) valuation rings,
(3) dimension and multiplicity theory.
**Text:** H. Matsumura *Commutative Ring Theory*, Cambridge University Press.

**MA 691A: Topics in Number Theory: A Shortcut to Class Field Theory**
**Instructor:** Prof. F. Shahidi, office: Math 650, phone: 49–41917, e-mail: shahidi@math.purdue.edu
**Time:** MWF 10:30
**Description:** My aim here is to cover the abelian class field theory by minimizing the background in number theory. I will therefore review the necessary number theoretic background, minimizing the necessary preparation. I will assume some knowledge of number fields such as the properties of Dedekind domains. Knowledge of field theory and Galois theory is necessary. But no more than what is in MA553.

**MA 693B: Invariant Subspaces**
**Instructor:** Prof. L. de Branges, office: Math 800, phone: 49–46057, e-mail: branges@math.purdue.edu
**Time:** MWF 10:30
**Description:** A Stone–Weierstass algebra is an algebra of continuous functions on a Hausdorff space $\mathcal{S}$ which contains constants, is closed under conjugation, and contains the inverse of every element of the algebra which is a function without zeros on $\mathcal{S}$. The space $\mathcal{S}$ is the maximal ideal space of the algebra if every homomorphism of the algebra onto the complex numbers, which always has the conjugate value on the conjugate function, is of the form $f$ onto $f(s)$ for a unique element $s$ of $\mathcal{S}$. The duality between the ideals of a Stone–Weierstrass algebra and the topology of its maximal ideal space is an underlying theme of the foundations of analysis which is treated by Leonard Gillman and Meyer Jerison, *Rings of Continuous Functions*, University Series in Higher Mathematics, 1960. A generalization of the Hahn–Banach theorem is introduced in locally convex spaces which are modules over Stone–Weierstrass algebras. An application is an existence theorem for invariant subspaces of continuous linear transformations on certain locally convex spaces, called primitive, for which there are generalizations of the open mapping theorem and the closed graph theorem. A generalization of the trace class applies to continuous linear transformations in primitive locally convex spaces. An algebra of continuous linear transformations on a primitive locally convex space is said to be a dual algebra if it is a Banach algebra whose closed unit disk is compact in the weak topology induced by the trace class. If the identity transformation does not belong to a dual algebra of continuous linear transformations on a primitive locally convex space, then an element $b$ of the space exists which does not belong to the closure of products $Tb$ with $T$ in the algebra. An application is given in the foundations of analysis. A nonnegative measure which is defined on all subsets of a set is said to be trivial if the measure of every set is the least upper bound of the measures of its finite subsets. It is shown that every such measure is trivial. The course is given at the level of texts in the University Series in Higher Mathematics. A level of mathematical maturity is required which is usually obtained in an introductory course in functional analysis (MA 546).

**MA 694A: Variational Inequalities and Free Boundary Problems**
**Instructor:** Prof. D. Danielli, office: Math 802, phone: 49–41920, e–mail: danielli@math.purdue.edu
**Time:** TTh 1:30–2:45
**Description:** The obstacle problem consists of studying the properties of minimizers of the Dirichlet integral

$$J(u) = \int_{\mathcal{D}} |\nabla u(x)|^2 \, dx$$

in a domain $\mathcal{D} \subset \mathbb{R}^n$, among all configurations $u(x)$ with given boundary values $u = f$ on $\partial\mathcal{D}$ and constrained to remain above a prescribed obstacle $\varphi(x)$ in $\mathcal{D}$. Such a problem is motivated by the description of the equilibrium position of a membrane (the graph of $u$) that is "attached" at level $f$ along the boundary of $\mathcal{D}$ and is restricted to remain above $\varphi$. The same mathematical problem appears in many other contexts, such as optimal control and financial math. In the first part of the course we will discuss the classical regularity theory of solutions and free boundaries for this and other variational problems. In the second part we will survey basic techniques developed for studying geometric properties of free boundaries arising in elliptic and parabolic problems. Some knowledge of fundamental properties of harmonic functions might be helpful, but not necessary, since the course will have a completely self-contained character.