Advanced Graduate Courses
offered by the
Mathematics Department
Fall, 2003

MA 542: Theory of Distributions and Applications.
Instructor: Prof. P. Stefanov, office: Math 742, phone: 49–67330, e-mail: stefanov@math.purdue.edu
Time: MWF 11:30
Prerequisite: MA 510 and 525 or equivalent.
Description: Distributions (called sometimes generalized functions) are a natural extension of functions and have the nice property that any distribution can be differentiated infinitely many times even if it is not differentiable in classical sense. The theory of distributions was developed in the second half of the 20-th century and is fundamental in the contemporary theory of Partial Differential Equations (PDE). In this course, we will introduce and study the basic properties of distributions, Fourier transform, convolutions, Schwartz kernels and Sobolev spaces. Particular attention will be paid to applications to PDE. In particular, this course gives the necessary background for those who intend to study Microlocal Analysis in the future.
Text: F. G., Friedlander and M. Joshi *Introduction to the Theory of Distributions*, Second edition, Cambridge University Press, Cambridge, 1999.

MA 557: Abstract Algebra I
Instructor: Prof. B. Ulrich, office: Math 618, phone: 49–41972, e-mail: ulrich@math.purdue.edu
Time: MWF 2:30
Prerequisite: Basic knowledge of algebra (such as the material of MA 503)
Description: The topics of the course will be commutative algebra and introductory homological algebra. We will study basic properties of commutative rings and their modules, with some emphasis on homological methods. The course should be particularly useful to students interested in commutative algebra, algebraic geometry, number theory or algebraic topology. There will be a continuation in the spring.
Text: No particular book is required, but typical texts are: M. Atiyah and I. Macdonald, *Introduction to Commutative Algebra*, Addison-Wesley, and J. Rotman, *An Introduction to Homological Algebra*, Academic Press.

MA 598B: Rational Homotopy Theory
Instructor: Prof. C. Wilkerson, office: Math 450, phone: 49–41955, e-mail: wilker@math.purdue.edu
Time: MWF 10:30
Description: Rational homotopy theory is analogous to the study of linear algebra versus general ring and module theory. On the one hand, it is simpler, but yet has useful applications and predictive power. It isolates in algebraic topology those questions and techniques that deal with non-torsion data.

For example, if M is an orientable manifold, using the differentiable forms, one can calculate the real cohomology of M. Quillen and Sullivan realized 30 years ago that this was just the top level of information available– in fact, information about the entire homotopy type of M is implicit in these differentiable forms. This, along with related methods of localization and completions, marked a change in the approach to the sudy of homotopy types of topological spaces.

The rational homotopy type of a space X provides the backbone, to which more detailed information concerning various primes is attached to build a picture of the homotopy type of X. The techniques involved in studying rational homotopy theory are simpler and more algebraic than those needed in traditional algebraic topology.

This course should be of use to students studying topology, commutative algebra, geometry, and several complex variables. The book by Halperin, et al provides an overall survey of the area.

MA 598E/EAS 591C: Mathematical Models of Earthquakes and Faulting
Instructor: Prof. A. Gabrielov, office: Math 648, phone: 49–47911, e-mail: agabriel@math.purdue.edu
Time: TTh 12:00-1:15
Description: The purpose of this course is to give an introduction into theory and models of rock fracture and earthquake sequences, with a view towards earthquake prediction. The course should be accessible for beginning graduate students in Earth Sciences, Applied Mathematics, and Engineering. No previous knowledge of seismology is required. Mathematics prerequisites: Linear Algebra and Differential Equations (MA 262 or MA 265/266).
The program includes:

- Fracture mechanics: brittle fracture, stress corrosion
- Rock friction: experimental results and theoretical models
- Mechanics of Faulting; incompatibility in fault systems
- Mechanics of earthquakes; self-similarity in earthquake sequences
- Seismotectonic process as a nonlinear dynamical system
- Lattice models of seismicity and self-organized criticality
- Hierarchical models of seismicity and renormalization
- Earthquake prediction: theory and practice

We are going to follow loosely the text: *The Mechanics of Earthquakes and Faulting* by C.H. Scholz (2nd Ed.), with addition of earthquake-related chapters from: *Fractals and Chaos in Geology and Geophysics* by D.L. Turcotte (2nd Ed.).

MA 598G: Advanced Probability and Options, with Numerical Methods
Instructor: Prof. F. Viens, office: Math 504, phone: 49-46035, e-mail: viens@stat.purdue.edu
Time: TTh 10:30-11:45
Prerequisites: Those who have not had MA/STAT 598F as a prerequisite can still hope to enroll in the course by providing evidence that they have equivalent preparation, which includes a graduate background in probability theory, and proficiency in financial mathematics equivalent to the first 10 chapters of the textbook by Bjork.
Description: This is the second course in a two-course sequence on the mathematics of finance, and especially on option pricing. The material will be divided in two parts. First, we will cover theoretical issues regarding: (i) Interest rate term structure models; (ii) American options and stochastic optimal stopping; (iii) finite difference methods. Then we will examine in detail the numerical methods used to solve the partial differential equations and inequalities that determine the prices of options, including the Binomial, Monte-Carlo, and finite difference methods.

MA 598M: Introduction to Representation Theory of Finite Groups
Instructor: Prof. K. Matsuki, office: Math 614, phone: 49–41970, e-mail: kmatsuki@math.purdue.edu
Time: MWF 1:30
Prerequisite: Prerequisites for this course are the basic knowledge of algebra at the level of MA 553 (the notion of groups, etc.) and rudiments of linear algebra.
Description: The basic idea of representation theory is a very simple one: Given a group (or an algebraic object with a certain structure), express it as the set of opeators on a linear space, thus represent its elements as matrices. The depth and importance of representation theory, however, cannot be overemphasized in the realm of modern mathematics.

It is something of an irony, therefore, that the subject of representation theory is not a part of regular requirements at the qualifier level, but everybody assumes you know it (at least the basics) once you pass the qualifier.

In this course, we try to cover the very basics of representation theory of finite groups and, if time permits, Lie groups and Lie algebras. The classical textbook is Serre's "Linear Representation of Finite Groups", where its sharp and concise style might allow one to attempt to finish the material in a two–week-long intensive course. Here we take a more leisurely path paved by the book of Fulton–Harris "Representation Theory: A First Course", where, quoting the words of the authors, "beginners can best learn about a subject by working through examples, with general machinery only introduced slowly and as the need arises."
Text: Fulton–Harris *Represenation Theory: A First Course.*

MA 598W: Mathematical Modeling of Nonlinear Waves
Instructor: Prof. M. Chen, office: Math 818, phone: 49–41964, e-mail: chen@math.purdue.edu
Time: TTh 1:30-2:45
Description: This is an introductory course in the modern theory of nonlinear wave propagation. The course assumes some knowledge of Sobolev spaces and the Fourier transform. Most topics will be developed from scratch, however. It is suitable for graduate students or well–prepared undergraduate students. The course will cover material from the topics listed below.
TOPICS.

1. Derivation of model equations for long waves. One–way models. Two–way models. Weakly three–dimensional models.
2. Initial–value problems.
3. Boundary–value problems.
4. Solitary waves and other travelling–wave phenomena.
5. Numerical simulations. Algorithnms. Analysis.
6. Comparison between various models.
7. Stability and instability – singularity formation.
8. Dissipative effects. Long–time asymptotics of solutions. Comparison with laboratory data.
9. Applications in coastal engineering.

MA 642: Methods of Linear and Nonlinear Partial Differential Equations I
Instructor: Prof. N. Garofalo, office: Math 616, phone: 49–41971, e-mail: garofalo@math.purdue.edu
Time: TTh 10:30-11:45
Prerequisite: MA 523 and 611
Description: Second order elliptic equations including maximum principles, Harnack inequality, Schauder estimates, and Sobolev estimates. Applications of linear theory to nonlinear equation.

MA 663: Algebraic Curves and Functions I
Instructor: Prof. S. Abhyankar, office: Math 600, phone: 49–41933, e-mail: ram@math.purdue.edu
Time: TTh 1:30-2:45
Description: Algebraic geometry, concerned with solutions of systems of polynomial equations, and their graphical representations, has for long been regarded as a very abstract area of mathematics. However, recently, with the advent of high-speed computers, applications have come about in such diverse areas of science and engineering such as theoretical physics, chemical, electrical, industrial and mechanical engineering, computer aided design (CAD), computer aided manufacturing (CAM), optimization, and robotics. These application areas are also increasingly posing fundamental open mathematical questions. This course is intended as an introduction to various relevant topics in algebraic geometry such as:

- Analysis and resolution of singularities
- Rational and polynomial parametrization
- Intersections of curves and surfaces
- Polynomial maps
- Fundamental Groups and Galois groups

The lectures will be expository in nature and so will be accessible to everyone. Thus there are no formal prerequisites and interested students are welcome. In particular the required abstract algebra will be developed simultaneously. The course will continue with its Part II in the Spring.
Text-Books: (1) Shreeram S. Abhyankar, *Algebraic Geometry for Scientists and Engineers* Amer Math Soc.
(2) Shreeram S. Abhyankar, *Resolution of Singularities of Embedded Algebraic Surfaces* Springer Verlag.

MA 665: Algebraic Geometry
Instructor: Prof. D. Arapura, office: Math 642, phone: 49–41983, e-mail: dvb@math.purdue.edu
Time: TTh 12:00-1:15
Prerequisite: some algebra (MA 557) and complex analysis (MA 530?) should be enough
Description: This will be an introductory class in algebraic geometry assuming a basic knowledge of algebra and complex analysis. I taught this course a couple of years ago and the web page for it is still viewable at:
http://www.math.purdue.edu/~dvb/algeom.html This may give you a good indication of what to expect, although this course won't be identical to the previous one. After introducing the basic concepts: curves, affine varieties, projective varieties, regular maps (but not schemes), I was thinking of doing some geometry over finite fields. Perhaps something like the Lang-Weil bound for the number of points. I don't plan to go very far of very fast. People who want an intro to sheaves, cohomology and all that can take a look at my notes from last spring: ~dvb/pub/sheaves.pdf
Text: Harris, J *Algebraic Geomerty, A First Course*, Springer.

MA 690B: Topics in Commutative Algebra
Instructor: Prof. W. Heinzer, office: Math 636, phone: 49-41980, e-mail: heinzer@math.purdue.edu
Time: MWF 12:30
Description: The course will cover material on regular sequences and depth, Cohen-Macaulay rings, the canonical module, Gorenstein rings, Hilbert functions and multiplicities.
Text: W. Bruns and J. Herzog, *Cohen–Macaulay Rings*, Cambridge studies in Advanced MAth, Vol. 39, Revised Edition.

MA 690E: Invitation to the Mathematics of Fermat–Wyles
Instructor: Prof. J. Lipman, office: Math 750, phone: 49–41994, e-mail: lipman@math.purdue.edu
Time: MWF 11:30
Prerequisite: Some knowledge about elliptic curves.
Description: The text offers a panoramic view of the history and methods behind Fermat's Last Theorem, without entering into any advanced details (i.e., the hard stuff that constitutes most of Wiles's proof). It is on reserve in the library, and should be looked at, at least briefly, by anyone thinking about taking the course. The goal will be to instill—in both the instructor and the participants—a "cultural" appreciation of this glorious chapter in Number Theory.

The points of emphasis will be decided on, in part, as we go along. The basic notions are elliptic curves, galois representations, and modular forms. At least the last of these topics will be dealt with in some detail; the first two will be touched on more lightly.
Text: Yves Hellegouarch, *Invitation to the Mathematics of Fermat-Wiles.*

MA 690F: Algebraic and Algebraic Geometric Coding Theory
Instructor: Prof. T. T. Moh, office: Math 638, phone: 49–41930, e-mail: ttm@math.purdue.edu
Time: MWF 9:30
Description: In the past 30 or 40 years, there has been two important developments right before our eyes: super–string theory for physics and algebraic geometry coding theory, both based on algebraic curve theory. The first one is over the complex number field and the second one is over finite fields. Coding theory is the the kernel of the modern information highway which includes telecommunications, CDs, computer technologies, etc.

I plan to cover briefly in the first part basic finite fields, finite dimensional vector spaces over finite fields, polynomial rings of one variable over finite fields. Then it should continuously cover linear and non–linear coding theory. The essence of BCH codes, cyclic codes, polynomial codes and Reed–Soloman codes etc. should be covered in the discussions of linear codes.

In the second part, I will then introduce the genearalized Reed–Soloman codes to present it as a code over the projective line. We should discuss the concepts of "error locator" and "error function." Then there should be a discussion of Algebraic Geometry with an emphasis on the Riemann–Roch theorem for curves. After that one can easily introduce the Goppa code and show its superiority to the classical ones.

MA 691A: Topics in Number Theory and Automorphic Forms
Instructor: Prof. F. Shahidi, office: Math 650, phone: 49–41917, e-mail: shahidi@math.purdue.edu
Time: MWF 10:30
Description: This course will cover topics from analytic theory of automorphic forms that I will choose from Iwaniec's book (see below) as well as a discussion of different aspects of the proof of the new cases of functoriality. The first part will basically concern classical modular forms and Maass forms and some of the related conjectures (Ramanujan, Selberg, ...). The second will discuss some of the steps of the proof of functoriality such as multiplicativity of root numbers which are extremely hard and provincial from the Rankin-Selberg method and quite conceptual from ours although due to its vast generality needs detail which I plan to present in these lectures. I will then connect the two parts at the end.
References: 1. H. Iwaniec *Introduction to the Spectral Theory of Automorphic Forms*
2. F. Shahidi *Park City lecture notes*

MA 692A: Wavelets and Image Processing
Instructor: Prof. B. Lucier, office: Math 634, phone: 49–41979, e-mail: lucier@math.purdue.edu
Time: MWF 9:30
Prerequisite: MA 544 (real analysis through measure theory) and some functional analysis (the first three chapters on Metric Spaces, Banach Spaces, and Hilbert Spaces of "Elements of Applicable Functional Analysis" by Charles W. Groetsch would suffice).
Description: The goal of the course is to describe and analyze nonlinear wavelet algorithms for three fundamental problems in low-level image processing: image compression, Gaussian noise removal, and inverting the Radon transform and other homogeneous linear operators with noisy data (which has application to medical imaging, especially Computed Tomography and Positron Emission Tomography). The main mathematical tool will be Nonlinear Approximation Theory and its relation to the theory of smoothness spaces (i.e., characterizing the smoothness of images in useful ways); we will also use some simple probability theory, variational principles, etc. The main focus of the course will be the rigorous analysis of the algorithms, not the development or theory of wavelet filters per se.
Text: I. Daubechies *Ten Lectures on Wavelets* Although (or perhaps because) the material presented in class will complement the material in this book, every student should have a copy of this book.

MA 693A: K–Theory
Instructor: Prof. M. Dadarlat, office: Math 708, phone: 49–41940, e-mail: mdd@math.purdue.edu
Time: MWF 1:30
Prerequisite: General topology, knowledge of fundamental group is desirable, MA 546 helpful but not really necessary.
Description: The course will offer a gentle introduction to complex K-theory for people with little or no background in algebraic topology. K-theory is a generalization of linear algebra which can be roughly described as the study of abelian invariants of large matrices. Examples of such invariants are the trace and the determinant. We plan to emphasize the connections of K-theory to analysis via Fredholm theory and operator algebras. No textbook is required
References: 1. M. F. Atiyah *K-theory*, 2nd edition
2. M. Rordam, F. Larsen and N. J. Laustsen *An Introduction to K-theory for C^*-algebras*

MA 693B: Riemann Hypothesis
Instructor: Prof. L. de Branges, office: Math 800, phone: 49–46057, e-mail: branges@math.purdue.edu
Time: MWF 10:30
Description: The Euler product and functional identity are obtained for Dirichlet zeta functions. The Hadamard factorization of Dirichlet zeta functions is derived from the factorization of entire functions of Pólya class. An axiomatic treatment is given of Hilbert spaces of entire functions associated with entire functions of Pólya class. A maximal dissipative shift is found in elementary examples of Hilbert spaces of entire functions appearing in Fourier analysis on the complex plane. The Euler product is interpreted as a construction of Hilbert spaces of entire functions associated with Dirichlet zeta functions from the spaces of Fourier analysis. A maximal dissipative transformation is constructed in the Hilbert spaces of entire functions associated with Dirichlet zeta functions. The Riemann hypothesis for Dirichlet zeta functions is obtained from the properties of the maximal dissipative transformation. The course follows Paris lectures in analytic number theory which appear as an appendix in the expository book by Karl Sabbagh on "Dr. Riemann's Zeroes." Students require a facility with Hilbert space techniques in complex analysis, as presented in MA 693B, Spring, 2003.

MA 693C: Commutative and Noncommutative Harmonic Analysis: The Science of Symmetry
Instructor: Prof. L. Lempert, office: MATH 734, e–mail: lempert@math.purdue.edu
Time: MWF 2:30 **NOTE NEW TIME, changed from MWF 10:30 to MWF 2:30.**
Prerequisite: Mathematics as required on the Qualifier Examinations; basic general topology (MA 571 is more than enough); notions of a differentiable manifold, Hilbert space.
Description: The Leitmotive of this course is the view that a large part of mathematics can be understood in terms of symmetries (or: in terms of group representations). This will be illustrated historically starting with XVII-th century number theory and probability and concluding with XX-th century quantum mechanics; symmetries lurk behind all.

Topics touched upon: Early probabilities; representing integers by quadratic forms; Dirichlet's work on: Fourier series, Gauss sums, primes in arithmetic progressions; the birth of noncommutative representation theory; partial differential equations; Weyl's work on: representation theory of compact Lie groups, quantum mechanics; symmetries in quantum mechanics according to Neumann and Wigner.
Recommended Text: G. M. Mackey *The Scope and History of Commutative and Noncommutative Harmonic Analysis,* American Mathematical Society, 1992

MA 694A: Introduction to Backward Stochastic Differential Equations and Their Applications in Finance Theory
Instructor: Prof. J. Ma, office: Math 620, phone: 49–41973, e-mail: majin@math.purdue.edu
Time: TTh 1:30–2:45
Prerequisite: MATH/STAT 538/539, or the consent of the instructor. (Some knowledge on stochastic calculus and partial differential equations will be beneficial.)
Description: The aim of this course is to introduce the theory of backward stochastic differential equations (BSDEs), and forward-backward differential equations (FBSDEs), together with their applications in mathematical finance. Basic concepts and several special methods for solving such equations will be studied in detail. These will include the methods of contraction mappings, of optimal control, and of continuation, etc. Examples of applications of BSDEs and FBSDEs to mathematical finance theory, especially those in option pricing, term structure of interest rates, and utility/risk optimization will be presented. The final phase of the semester will be given to discussions on most recent development of the theory and open problems.
Text: J. Ma and J. Yong, *Forward-Backward Stochastic Differential Equations and Their Applications*, Lecture notes in Mathematics, **1702** (1999), Springer.

MA 696A: Topics in Complex Geometry
Instructor: Prof. S. K. Yeung, office: Math 712, phone: 49–41942, e-mail: yeung@math.purdue.edu
Time: MWF 1:30
Prerequisite: MA 530, 562. Some basic understandings in algebraic geometry and several complex variables will be helpful as well.
Description: In this course, some basic techniques in complex manifolds will be studied. Tentatively, the following topics will be covered.

1. Use of complex analysis in transcendental number theory and diophantine analysis.
2. Harmonic maps and their applications in geometry.
3. L^2-estimates, its applications and generalizations.
4. Kähler-Einstein metrics, existence, uniqueness and applications.

Probably only parts of the materials would be covered, depending on the progress of the course.

Seminars

Algebraic Geometry Seminar, Prof. Abhyankar
Time: Thursday 4:30–6:00

Automorphic Forms and Group Representations Seminar, Prof. Goldberg
Time: Thursdays, 1:30-2:30

Commutative Algebra Seminar, Prof. Ulrich
Time: Wednesdays 4:30-5:20

Computational and Applid Math Seminar, Prof. Shen
Time: Fridays 4:30

Linear and Complex Analysis Seminar, Prof. de Branges
Time: Thursday 10:30-11:20

Operator Algebras Seminar, Prof. Dadarlat
Time: Tuesdays, 2:30-3:20

PDE Seminar, Prof. Bauman
Time: Tuesdays, 9:30-10:20

Probability Seminar, Prof. Banuelos
Time: Mondays 3:30

Scattering Theory and Inverse Problems Seminar, Prof. SaBarreto
Time: Thursdays 10:30-11:20

Toplolgy Seminar, Prof. Mauer–Oates
Time: Thursdays 1:30-2:20

Working Algebraic Geometry Seminar, Prof. Arapura
Time: Wednesday 3:30-4:30