Seminars and Advanced Graduate Courses offered by the Mathematics Department Fall, 2006

Courses

MA 531: Functions of Complex Variables II
Instructor: Prof. A. Eremenko, office: Math 450, phone: 49–41975, e-mail: eremenko@math.purdue.edu
Time: TTh 3:00-4:15
Prerequisite: MA 530
Description: We will cover the chapters of Ahlfors that were not covered in Math 530.

Normal families, Potential theory, Uniformization theorem, Elliptic functions, analytic continuation, Riemann surfaces and linear differential equations. Compact Riemann surfaces (=complex algebraic curves) will be considered in greater depth then Ahlfors' book does, including the theory of Abelian integrals and the Riemann-Roch theorem.

If time permits, additional topics will be considered, like quasiconformal mappings, Nevanlinna theory and/or additional chapters of Potential theory.
Text: Ahlfors *Complex Variables*

MA 539: Probability Theory II
Instructor: Prof. A. Yip, office: Math 710, phone: 49–41941, e-mail: yip@math.purdue.edu
Time: MWF 11:30
Description: This is a continuation of MA/STAT 538. This second course will touch upon the properties of stochastic processes, in particular their limit theorems and long time behaviors. Topics include martingale theory, random walk, Brownian Motions, invariance principle, stationary and Gaussian processes and ergodic theorems. Depending on time and interests, additional topics such as Markov and diffusion processes, and large deviation theory will be covered.

The prerequisite is MA/STAT 538. But anyone with good background in probability and measure theory (at the level of MA/STAT 519 and MA 544) can also benefit. The main knowledge I will assume is a good understanding of the Weak and Strong Law of Large Numbers, Central Limit Theorem, and the concept of Conditional Probability (as covered in Chapters 4, 5, and 6 of Billingsley's book).
References: W. Feller *An Introduction to Probability Theory and Its Applications*, Vol 1 and 2.
Texts: 1. P. Billingsley *Probability and Measure*
2. L. Breiman, *Probability*

MA 542: Theory of Distributions and Applications
Instructor: Prof. A. Petrosyan, office: Math 610, phone: 49–41932, e-mail: arshak@math.purdue.edu
Time: TTh 10:30-11:45
Prerequisite: MA 544 and a knowledge of basic linear algebra
Description: The theory of distributions is an extension of classical analysis dealing with the most general notion of differentiability. It is of particular importance in partial differential equations (PDEs) and has applications in virtually every field of modern analysis. This is an introductory course where we will study the basic properties of distributions, convolutions and Fourier transforms, Sobolev spaces, as well as applications to PDEs.
Text: F. G. Friedlander and M. Joshi, *Introduction to the Theory of Distributions*, Cambridge University Press, 2nd edition.

MA 557: Abstract Algebra I
Instructor: Prof. B. Ulrich, office: Math 618, phone: 49–41972, e-mail: ulrich@math.purdue.edu
Time: MWF 1:30
Prerequisite: Basic knowledge of algebra (such as the material of MA 503).
Description: The topics of the course will be commutative algebra and introductory homological algebra. We will study basic properties of commutative rings and their modules, with some emphasis on homological methods. The course should be particularly useful to students interested in commutative algebra, algebraic geometry, number theory or algebraic topology. There will be a continuation in the spring.
Text: No particular book is required, but typical texts are:
1. M. Atiyah and I. Macdonald, *Introduction to commutative algebra, Addison-Wesley.*
2. J. Rotman, *An introduction to homological algebra*, Academic Press.

MA 562: Introduction to Differential Geometry and Topology
Instructor: Prof. Y-J Lee, office: Math 734, phone: 49–47919, e-mail: yjlee@math.purdue.edu
Time: TTh 12:00-1:15
Prerequisite: MA 351 and MA 442
Description: Topics – Smooth manifolds, tangent vectors, inverse and implicit function theorems, submanifolds, vector fields, integral curves, differential forms, the exterior derivative, partitions of unity, integration on manifolds, fundamentals of Riemannian geometry, Levi–Civita connection, parallel transport, geodesics, curvature tensor.
Text: William Boothby, *Introduction to Differentiable Manifolds and Riemannian Geometry*

MA 584: Algebraic Number Theory
Instructor: Prof. J. Lipman, office: Math 750, phone: 49–41994, e-mail: lipman@math.purdue.edu
Time: MWF 9:30
Prerequisite: MA 553, 554, or consent of Instructor.
Description: An introduction to classical foundations for the study of algebraic curves and algebraic number fields.

Dedekind domains, norm, discriminant, different, finiteness of class number, Dirichlet unit theorem, quadratic and cyclotomic extensions, quadratic reciprocity, decomposition and inertia groups, completions and local fields.
Text: G. Janusz *Algebraic Number Fields*, 2nd edition

MA 598B: Riemann Mapping
Instructor: Prof. L. de Branges, office: Math 800, phone: 49–46057, e-mail: branges@math.purdue.edu
Time: MWF 9:30
Description: A Riemann mapping is an injective analytic mapping of the unit disk onto a plane region. The existence of Riemann mappings for arbitrary regions requires an estimation theory. An estimation theory is offered which is capable of generalization to three dimensions and which is related to the proof of the Riemann hypothesis. Nested families of regions are parameterized for the construction of mappings. A plane region is seen as created by the paths of particles issuing from the origin and ending on the boundary. A generalization of Newton's laws determines motion. The estimation theory of mappings is based on an energy conservation principle. Hilbert spaces are constructed whose elements are functions analytic in the unit disk. The estimation problem reduces to one for star-like regions about the origin where a generalization of the Radon transformation is applied. Students need a knowledge of complex analysis required for doctoral qualifying examinations.

MA 598C Numerical Methods of Continuum Mechanics
Instructor: Prof. Z. Cai, office: Math 810, phone: 49–41921, e-mail: zcai@math.purdue.edu
Time: TTh 9:00-10:15
Prerequisite: MA/CS 514 or equivalent or consent of instructor.
Description: This course is an introduction to numerical methods for problems arising from solid and fluid mechanics. We shall derive basic mathematical equations for Darcy's flow in porous media, elasticity, and incompressible Newtonian fluid flow based on physical laws. We will then concentrate on basic numerical methods such as finite element, mixed finite element, and stabilized finite element for these partial differential equations. A review of fast iterative solvers such as multigrid and domain decomposition for algebraic systems resulting from discretization will also be presented. This course will focus on fundamental issues of numerical methods rather than implementation.

MA 598D/EAS 591C: Mathematics of Upscaling
Instructor: Prof. J. Cushman, office: Math 816, phone: 49–48040, e-mail: jcushman@math.purdue.edu
Time: TTh 12:00-1:15
Prerequisite: Basic pde's and some probability/stochastic process theory
Description: All fields of science and engineering are concerned with upscaling problems, i.e. answering the question: How do the many degrees of freedom on smaller scales get rationally filtered into a far smaller number of degrees of freedom on larger scales? The goal, of course, is to not lose the interesting physics, chemistry and biology during the upscaling process. A closely related question is: How do the tools employed in the filtering process change when considering discrete as opposed to continuous hierarchies? Borrowing examples from turbulence, porous media physics, and statistical and continuum thermodynamics, a large number of methodologies will be illustrated. Amongst these are matched asymptotics, central limit theorems, renormalization groups, mixture and polar field approaches, projection operators, stochastic perturbation, moments, generalized hydrodynamics and Monte Carlo.

MA 598E/EAS 591G: Introduction to Continuum Mechanics
Instructor: Prof. A. Gabrielov, office: Math 648, phone: 49–47911, e-mail: agabriel@math.purdue.edu
Time: TTh 10:30-11:45
Description: This course is an introduction to the fundamental ideas and methods of continuum mechanics, with a view towards geophysical applications. The goal is to provide beginning graduate students with the physics background and mathematical tools necessary for more advanced, special topics courses in continuum mechanics and its applications in the geosciences. Prerequisites include standard undergraduate calculus sequence: linear algebra, differential equations, and multivariate calculus.

The topics to be covered include:

- Essential Mathematics
- Stress Principles
- Kinematics of Deformation and Motion
- Fundamental Laws and Equations
- Thermodynamics
- Linear Elasticity
- Linear Viscoelasticity
- Friction and Fracture
- Classical Fluids
- Geophysical Fluid Dynamics

The course will be based on the book *Continuum Mechanics for Engineers* by G. T. Mase and G. E. Mase, with additional material on Friction and Fracture and Geophysical Fluid Dynamics from other sources.

MA 598G: Basic Algebra I
Instructor: Prof. S. Abhyankar, office: Math 600, phone: 49–41933, e-mail: ram@math.purdue.edu
Time: TTh 1:30-2:45
Description: This will be a two-semester introduction to algebra (with Part II to be given in Spring 2007). There are no prerequisites and all interested students are welcome. The idea is to start at the bottom and deal with most of the material usually covered in 553-554 and 557-558 in an integrated manner.
Text: Shreeram S. Abhyankar *Lectures on Algebra I*, being Published by World Scientific (Copies will be made available)

MA 598K: Algebraic Coding Theory
Instructor: Prof. T. T. Moh, office: Math 638, phone: 49–41930, e-mail: ttm@math.purdue.edu
Time: MWF 9:30
Prerequisite: Grade of at least B in a Linear Algebra course, and a mature attitude.
Description: This course, suitable for graduates students, will introduce some basic ideas about "self-correcting codes," a very useful subject in the real world. For instance, in our information era, signals might be received incorrectly, in e-mails, telephone calls, CD, or remote sensing (pictures from Mars). What is needed are "self-correcting codes" to eliminate the errors. The treatment in this course is purely mathematical. We will start with Linear Algebra and then do some Algebraic Geometry over a Finite Field, especially the Riemann-Roch theorem. Because of possible unfamiliarity with the tools of Algebraic Geometry involved, plenty examples will be given to help students grasp the concepts. We will show that there is no distinction here between pure and applied mathematics.
Text: Instructor's notes

MA 598Z: Introduction to Numerical Methods for Partial Differential Equations.
Instructor: Prof. B. Lucier, office: Math 634, e-mail: lucier@math.purdue.edu
Time: MWF 11:30
Prerequisite: The prerequisite is linear algebra and some concepts of partial differential equations (at the level of MA 511, MA 523).
Description: This course will introduce basic finite difference and finite element techniques for numerically approximating the solutions of partial differential equations. We shall study the stability and convergence of these methods applied to elliptic, parabolic, and hyperbolic equations. Other topics depending on time and interests of the students.

MA 620: Mathematical Theory Of Optimal Control
Instructor: Prof. D. Danielli, office: Math 802, phone: 49–41920, e-mail: danielli@math.purdue.edu
Time: TTh 1:30-2:45
Prerequisite: MA 544 or instructor consent.
Description: This course is an introduction to the mathematical theory of optimal control of processes governed by ordinary differential equations. In recent years, control problems have arisen in very diverse areas, such as production planning, chemical and electrical engineering, and flight mechanics. The course will focus on the mathematical formulation of such problems and the existence of optimal controls both with and without convexity assumptions. One of the crucial tools for the characterization of optimal controls, namely the maximum principle, will be illustrated and (if time permits) proved. Finally, the relationship with the Calculus of Variations and applications will be discussed.
Text: notes will be distributed

MA/STAT 638: Stochastic Process II
Instructor: Prof. M. Roeckner, office: Math 432, phone: 49–41963, e-mail: roeckner@math.purdue.edu
Time: TTh 9:00-10:15
Description: The course will consist of the following topics:

I. Introduction to the pathwise Ito-calculus I
II. Semimartingales and stochastic integration
III. Markov processes
IV. Girsanov transformation
V. Weak solutions of stochastic differential equations and martingale problems
VI. Potential theory of Brownian motion

MA 642: Methods Of Linear And Nonlinear Partial Differential Equations I
Instructor: Prof. P. Bauman, office: Math 718, phone: 49–41945, e-mail: bauman@math.purdue.edu
Time: MWF 8:30
Prerequisite: Ma 544 & Ma 611 or equivalent
Description: This is the first semester of a one-year course in pde theory with applications to nonlinear equations and systems of pde. The first semester will focus on the study of second order elliptic equations (in divergence and nondivergence form), including maximum principles, Harnack inequalities, classical and Schauder interior and boundary estimates, Sobolev inequalities and imbedding theorems, and applications to nonlinear systems of pde.

The second semester will focus on the theory of time-dependent equations and systems of pde.
Text: Gilbarg & Trudinger, *Elliptic Partial Differential Equations of Second Order*, Springer, Latest Edition

MA 646: Banach Algebras and C*-algebras
Instructor: Prof. L. Brown, office: Math 704, phone: 49–41938, e-mail:lgb@math.purdue.edu
Time: MWF 10:30
Prerequisite: MA 546
Description: Banach algebras, Gelfand theory, the commutative Gelfand-Naimark theorem and applications to normal operators. C*-algebras and representations, the noncommutative Gelfand-Naimark theorem, von Neumann algebras, and Murray-von Neumann equivalence. Some operator algebras theory or other topics may be included as time permits.

MA 650: Commutative Algebra
Instructor: Prof. B. Ulrich, office: Math 618, phone: 49–41972, e-mail: ulrich@math.purdue.edu
Time: MWF 2:30
Prerequisite: Basic knowledge of commutative algebra (such as the material of MA 557/558).
Description: This is an intermediate course in commutative algebra. The course is a continuation of MA 557/558, but should be accessible to any student with basic knowledge in commutative algebra (localization, Noetherian and Artinian modules, associated primes and primary decomposition, integral extensions, dimension theory, completion, Ext and Tor).

The topics of this semester will include: Graded rings and modules, depth and Cohen-Macaulayness, structure of finite free resolutions, regular rings and normal rings, canonical modules and Gorenstein rings, local cohomology and local duality.
Text: No specific text will be used, but possible references are:
1. H. Matsumura, *Commutative ring theory*, Cambridge University Press
2. W. Bruns and J. Herzog, *Cohen-Macaulay rings*, Cambridge University Press
3. D. Eisenbud, *Commutative algebra with a view toward algebraic geometry*, Springer.

MA 690B: Topics in Commutative Algebra
Instructor: Prof. W. Heinzer, office: Math 636, phone: 49–41980, e-mail: heinzer@math.purdue.edu
Time: MWF 3:30
Description: The course will cover material from the first four chapters of the text by W. Bruns and J. Herzog titled *Cohen-Macaulay Rings*, revised edition. These chapters are Regular sequences and depth, Cohen-Macaulay rings, The canonical module. Gorenstein rings, Hilbert functions and multiplicities. Students enrolled in the course will be encouraged to actively participate by presenting material and exercises from the text.

MA 690F: Modular Forms
Instructor: Prof. J. Lipman, office: Math 750, phone: 49–41994, e-mail: lipman@math.purdue.edu
Time: MWF 10:30
Prerequisite: MA 553, 554, 530
Description: The theory of modular forms, initiated before 1850 by Jacobi and Eisenstein, and constantly developed since, provides an entry point into much of modern number theory. It is a rich subject, synthesizing ideas from complex analysis, algebraic geometry and representation theory as well as number theory. The text — available in the library, on reserve, for perusal — revolves around the Modularity Theorem (explained there in multiple ways, though not proved), which gives a correspondence between modular forms and elliptic curves, and has, e.g., Fermat's Last Theorem as a consequence.

The course will be run as a seminar — no homework, no exams, but with students expected to take responsibility for lucid classroom presentation of the material. It is hoped to cover the first half of the book. If there is sufficient interest, a second semester may cover the rest.
Text: F. Diamond, J. Shurman, *A First Course in Modular Forms*

MA 690M: Schemes, Sheaves and Cohomology in Algebraic Geometry
Instructor: Prof. K. Matsuki, office: Math 614, phone: 49–41970, e-mail: kmatsuki@math.purdue.edu
Time: MWF 11:30
Description: This is an introductory course in algebraic geometry. We will go over the established but "hard to swallow for a beginner" - notions, such as schemes, sheaves, and cohomology, etc. at a slow pace. We will also be ambitious to see how these notions have developed into stacks, derived categories, etc. more recently. No prior knowledge of algebraic geometry is assumed, but a good background in basic algebra is expected.
Texts: 1. Hartshorn, *Algebraic Geometry*
2. Gelfand – Manin, *Methods of Homological Algebra*

MA 690R: Representations of p-adic Groups
Instructor: Prof. S. Spallone, office: Math 850, phone: 49–67968, e-mail: sspallon@math.purdue.edu
Time: MWF 12:30
Description: This class will serve as an introduction to the theory of admissible representations of p-adic reductive groups. Familiarity with classical representation theory of Lie groups will be assumed. The primary reference will be Casselman's notes.

MA 690S: Topics in Automorphic Forms
Instructor: Prof. F. Shahidi, office: Math 650, phone: 49–41917, e-mail: shahidi@math.purdue.edu
Time: MWF 9:30
Prerequisite: MA 544 and MA 684
Description: The course will be based partly on a new book published by the Clay Math Institute based on the CMI 2003 Summer School at the Fields Institute in Toronto, and partly on my own notes (to be published eventually as a book). The main topics in the book of interest to us will be Arthur's on trace formula and Kotwitz's on the harmonic analysis.
Text: 1. (Recommended) *Proceedings of the 2003 CMI Summer School at the Fields Institute.*, CMI Publications.
2. My notes, which may be made available for the course.

MA 692C: Fourier Integral Operators II
Instructor: Prof. A. SaBarreto, office: Math 604, phone: 49–41965, e-mail:sabarre@math.purdue.edu
Time: MWF 12:30
Prerequisite: MA 692B (taught in the spring of 2006), or permission from the instructor.
Description: This is the continuation of MA 692B which was taught during the spring 2006. The course will consist mostly of applications of the calculus of Fourier integral operators studied during the spring semester, and some of its refinements. The topics covered will include spectral theory of differential operators and certain generalizations of Radon transforms.

MA 692D: Applied Inverse Problems
Instructor: Prof. M. de Hoop, office: Math 822, phone: 49–66439, e-mail: mdehoop@math.purdue.edu
Time: TTh 10:30-11:45
Description: Theme: global seismology and inverse problems

In this course we develop the mathematical framework describing the normal modes in the Earth and associated inverse problems. Topics include a hierarchy of simplified models, the Watson transform connecting normal mode summation to the asymptotic description of seismic waves, and spectral perturbation theory. Imaging is formulated both from a tomographic point of view as well as from an inverse scattering point of view. We also discuss the inverse source problem.

MA 692F: Special Topics in Mathematical Biology
Instructor: Prof. Z. Feng, office: Math 814, phone: 49–41915, e-mail: zfeng@math.purdue.edu
Time: MWF 12:30
Description: This course is an introduction to the application of mathematical methods and concepts to the description and analysis of biological processes. The mathematical contents consist of difference and differential equations and elements of stochastic processes. The topics to be covered include dynamical systems theory motivated in terms of its relationship to biological theory, deterministic models of population processes, epidemiology, coevolutionary systems, structured population models, nonlinear dynamics, and stochastic simulations. Bio-mathematical research projects (in small group) may be carried out.
Texts: 1. Class notes and Handouts and Articles
2. Brauer and Castillo-Chavez, *Mathematical Models in Population Biology and Epidemiology* (optional).
3. Kot, *Elements of Mathematical Ecology* (optional)
4. Thieme, *Mathematics in Population Biology* (optional)

MA 692S: Special Topics in Multi-scale Modelling and Computation
Instructor: Prof. D. Sheen, office: MATH 845, e-mail: sheen@math.purdue.edu
Time: TTh 9:00-10:15
Prerequisite: Knowledges in standard theories of both elliptic partial differential equations and finite element methods are certainly quite sufficient, but students who have any experience in either elliptic PDEs or FEMs are welcome.
Description: Multiscale problems arise in many important scientific and engineering research areas, whose examples include composite materials with fine microstructures, highly heterogeneous porous media, turbulent transport in high Reynolds number flows, the transport of lipids on the cell membranes, chemical reactions, and so on. Depending on a specific problem, the range of space scale varies from 10^3 to 10^6, or 10^9 from the finest (microscopic) level to the coarsest (macroscopic) level. Similar phenomena occur for the time scale, too. Most multiscale problems require suitable modeling techniques, which can provide reasonable computational model, without which the current computational resources are not sufficient to treat any single of such problems.
In this course, we give a series of survey lectures on the following topics.

- Examples of multiscale problems
- Introduction to the theory of homogenization and upscaling ideas
- Introduction to multiscale finite element methods
- Analysis of multiscale finite element methods
- Introduction to heterogeneous multiscale methods
- Analysis of heterogeneous multiscale methods
- Introduction to variational multiscale methods
- Analysis of variational multiscale methods

References:
[1] S. C. Brenner and L. R. Scott. *The Mathematical Theory of Finite Element Methods.* Texts in Applied Mathematics. Springer, New York, second edition, 2002.
[2] D. Gilbarg and N. S. Trudinger, *Elliptic Partial Differential Equations of Second Order*, Springer–Verlag, Berlin, Heidelberg, second edition, 1983.
[3] V. V. Jikov, S. M. Kozlov, and O. A. Oleinik. *Homogenization of differential operators and integral functions.* Springer–Verlag, Berlin, Heidelberg, New York, 1994.
[4] E. Sanchez–Palencia. *Non–homogeneous Media and Vibration Theory*, volume 127, Springer–Verlag, Berlin and New York, 1980.

MA 694A: Introduction to Nonlinear Elliptic Systems in Applications
Instructor: Prof. D. Phillips, office: Math 706, phone: 49–41939, e-mail: phillips@math.purdue.edu
Time: MWF 1:30
Prerequisite: MA 642
Description: This course covers topics on the existence, regularity, and qualitative structure of solutions to systems arising as models for liquid crystals.
Course content:

i) At least one half of the time will be spent introducing the basics from Giaquinta's monograph, [1].
ii) Introduction to harmonic maps and the Frank-Oseen energy.
iii) Stability of liquid crystal configurations.
iv) Models coupling the Ginzburg-Landau energy with the Frank-Oseen energy.

Students should have covered the first eight chapters of Gilbarg and Trudinger, *Elliptic PDEs of 2nd Order.* No background with systems is needed.
References:
[1] *Multiple Integrals in the Calculus of Variations and Nonlinear Elliptic Systems*, M. Giaquinta.
[2] *Variational Theories for Liquid Crystals*, E.G. Virga.

MA 694R: Malliavin Calculus
Instructor: Prof. M. Roeckner, office: Math 432, phone: 49–41963, e-mail: roeckner@math.purdue.edu
Time: TTh 10:30-11:45
Description: This course will give a detailed introduction to the Malliavin calculus. It will be based on D. Nualart's book (see references below). Particular, emphasis at the beginning will be given to the relation of the various approaches to the Weiner chaos expansions, i.e., via multiple stochastic integrals, via Gaussian renormalization on Fack space representations. Subsequently, the general theory will be developed and finally applications, to prove smoothness of densities of the marginals of solutions to stochastic differential equations, will be presented.
References: D. Nualart, *The Malliavin Calculus and Applications*, Springer, 1995.

Seminars

Algebra and Algebraic Geometry Seminar, Prof. Abhyankar
Time: Thursday 4:30–6:00

Automorphic Forms and Representation Theory Seminar, Prof. Yu
Time: Thursdays, 1:30-2:30

Commutative Algebra Seminar, Profs. Heinzer and Ulrich
Time: Wednesdays 4:30

Computational and Applied Math Seminar, Prof. Shen
Time: Fridays 3:30

Function Theory Seminar, Prof. Eremenko
Time: Tuesdays, time flexible

Foundations of Analysis Seminar, Prof. de Branges
Time: Thursday 9:30

Geometric Analysis Seminar
Time: Monday 3:30

Number Theory Seminar, Prof. Goins
Time: Thursday 3:30

Joint Geometry/Topology Seminar, Prof. Y. J. Lee
Time: TBA

Operator Algebras Seminar, Prof. Dadarlat
Time: Tuesdays, 2:30

PDE Seminar, Prof. Phillips
Time: Thursday, 3:30

Probability Seminar, Prof. Banuelos
Time: Monday, 3:30

Spectral and Scattering Theory Seminar, Prof. SaBarreto
Time: Wednesday 4:30

Stochastic Models and PDEs Seminar, Profs. Roeckner and Yip
Time: To Be Arranged

Student Tea Time Seminar on Applied Analysis, Profs. N. Nguyen, A. Petrosyan, M. Torres, A. Yip
The purpose of this seminar is to introduce to the students various topics and techniques in analysis and differential equations which are useful in applications. It will be broadly based. The talks will be initiated by various faculties. Student presentations are certainly welcome. The topics will be determined by the participants. The main prerequisite is the basic knowledge of PDEs (MA 523). Cookies will be served for the participants!
Time: Tuesday 3:00

Topology Seminar, Prof. McClure
Time: Tuesday 1:30-2:20

Working Algebraic Geometry Seminar, Prof. Arapura
Time: Wednesday 3:30-4:30