Seminars and Advanced Graduate Courses offered by the Mathematics Department Fall, 2008

Courses

MA 539: Probability Theory II
Instructor: Prof. Yip, office: Math 432, phone: 49–41941, e-mail: yip@math.purdue.edu
Time: MWF 8:30
Description: This is a continuation of MA/STAT 538. This second course will touch upon the properties of stochastic processes, in particular their limit theorems and long time behaviors. Topics include conditional probability, martingale theory, random walk, Brownian Motions and invariance principle (Chapters 6 and 7 of Billinsgley's book). Depending on time and student interests, additional topics such as Gaussian, stationary, and Levy Processes might be covered.

The prerequisite is MA/STAT 538. But anyone with good background in probability and measure theory (at the level of MA/STAT 519 and MA 544) can also benefit. The main knowledge I will assume is a good understanding of the Weak and Strong Law of Large Numbers and Central Limit Theorem (as covered in Chapters 4, 5 of Billingsley's book).
Text: P. Billingsley, *Probability and Measure*
References: 1. L. Breiman *Probability*
2. R. Durrett *Probability: theory and examples*
3. W. Feller *An Introduction to Probability Theory and Its Applications*, Vol 1 and 2.

MA 542: Theory of Distributions and Applications
Instructor: Prof. Petrosyan, office: Math 610, phone: 49–41932, e-mail: arshak@math.purdue.edu
Time: MWF 12:30
Prerequisite: MA 544 is desirable; minimal requirement is basic measure theory and basic functional analysis.
Description: The theory of distributions is an extension of classical analysis dealing with the most general notion of differentiability. It is of particular importance in partial differential equations (PDEs) and has applications in virtually every field of modern analysis. This is an introductory course where we will study the basic properties of distributions, convolutions and Fourier transforms, Sobolev spaces, as well as applications to PDEs.
Text: F. G. Friedlander and M. Joshi, *Introduction to the Theory of Distributions*, Cambridge University Press, 2nd edition.

MA 546: Introduction to Functional Analysis
Instructor: Prof. L. Brown, office: Math 704, phone: 49–41938, e-mail: lgb@math.purdue.edu
Time: MWF 10:30
Prerequisite: MA 544
Description: The course covers basic functional analysis with emphasis on bounded linear operators on Banach and Hilbert spaces. The topics listed below will be covered, and brief treatments of other topics fitting the interests of the students can be included as time permits.

Banach spaces and Hilbert spaces; Hahn-Banach theorem; closed graph and open mapping theorems; uniform boundedness principle; theory of spectrum for operators on Banach spaces and for compact operators; weak and weak* topologies; reflexivity; Hahn-Banach separation theorem and double polar theorem; operators on Hilbert spaces; spectral theorem for bounded self-adjoint operators on Hilbert spaces.
Text: M. Schechter, *Principles of Functional Analysis*, AMS.

MA 553: Introduction to Abstract Algebra
Instructor: Prof. J-K Yu, office: Math 604, phone: 49–67414, e-mail: jyu@math.purdue.edu
Time: MWF 12:30
Prerequisite: MA 453
Description: Group theory: Sylow theorems, Jordan Hlder theorem, solvable groups. Ring theory: unique factorization in polynomial rings and principal ideal domains. Field theory: ruler and compass constructions, roots of unity, finite fields, Galois theory, solvability of equations by radicals.
Text: P.A. Grillet, *Abstract algebra, 2nd edition, Sprigner-Verlag*
Reference: D.S. Dummit and R.M. Foote, *Abstract algebra, 3rd edition, Wiley*

MA 562: Introduction to Differential Geometry and Topology
Instructor: Prof. Donnelly, office: Math 716, phone: 49–41944, e-mail: hgd@math.purdue.edu
Time: MWF 11:30
Prerequisite: MA 351, 442
Description: Smooth manifolds; tangent vectors; inverse and implicit function theorems; submanifolds; vector fields; integral curves; differential forms; the exterior derivative; DeRham cohomology groups.

MA 584: Algebraic Number Theory
Instructor: Prof. Lipman, office: Math 650, phone: 49–41994, e-mail: lipman@math.purdue.edu
Time: MWF 10:30
Prerequisite: MA 553, MA 554
Description: Dedekind domains, norm, discriminant, different, finiteness of class number, Dirichlet unit theorem, quadratic and cyclotomic extensions, quadratic reciprocity, decomposition and inertia groups, completions and local fields.
References: S. Lang, *Algebraic Number Theory,* Chapters I–VI.
Text: G. Janusz, *Algebraic Number Fields, 2nd edn.*

MA 598E: Self-Similarity and Critical Phenomena in Geosciences
Instructor: Prof. Gabrielov, office: Math 648, phone: 49–47911, e-mail: agabriel@math.purdue.edu
Time: MWF 11:30
Prerequisite: Ordinary differential equations and linear algebra (MA 262 or 265/266)
Description: This course is an introduction to self-similarity, critical phenomena, chaos and fractals.

The topics covered in the course:

Self-similarity, fractals, scaling, renormalization. Pattern formation, self-organization, critical phenomena. Continuous and discrete dynamical systems. Attractors. Regular vs chaotic dynamics. Strange attractors. Instability and unpredictability. Bifurcations and onset of chaos. Self-organized criticality.

Geoscience applications:

Atmospheric circulation, Geomagnetism, Geomorphology, Seismicity, Distribution of ore and sediment deposits, Forest fires, Floods and droughts.

Texts: 1. Manfred Schroeder *Fractals, Chaos, Power Laws: Minutes from an Infinite Paradise.*, W. H. Freemen, either hardcover 1991 or paperback 1992 are ok.
2. Donald L. Turcotte, *Fractals and chaos in geology and geophysics*, 2nd Ed. Cambridge Univ. Press, 1997

MA 598G: Linear Algebraic Groups
Instructor: Prof. Goldberg, office: Math 640, phone: 49–41919, e-mail: goldberg@math.purdue.edu
Time: MWF 10:30
Prerequisite: MA 553, MA 554
Description: This course will cover the classification of linear algebraic groups from first principles. Topics will include some elementary algebraic geometry, definition of algebraic groups, Jordan decompositions, Lie algebras, Chevalley's Theorem, solvable groups, Borel subgroups, parabolic subgroups, linear actions, root spaces and the classification Theorem.

MA 598K: Homological Algebra: Derived Categories
Instructor: Prof. R. Kaufmann, office: Math 710, phone: 49–41205, e-mail: rkaufman@math.purdue.edu
Time: MWF 11:30
Description: Homological algebra is an indispensable tool in many areas of mathematics, such as topology, algebra, algebraic geometry and mathematical physics related to string theory.

We will start with the basic theory of derived functors and derived categories and then go on to discuss triangulated categories and newer concepts like stability. Throughout we will use examples from topology and algebraic geometry.

MA 598T: Bridge to Research Seminar
Instructor: Prof. Milner, office: Math 628, phone: 49–41967, e-mail: milner@math.purdue.edu
Time: Monday 4:30
Description: The seminar has two main goals, both aimed at helping students early in their graduate career find their place in the department. The first is to help students discover what area of mathematics they might be interested in focusing for research, as well as whom they might like to work with. The second is to provide students with an opportunity to interact with faculty in casual setting. This is achieved by having professors from the department give brief talks about their research area at a level that is accessible to those in their first and second year of graduate study.

MA 598W: Introduction to Algebraic Geometry
Instructor: Prof. Wlodarczyk, office: Math 602, phone: 49–62835, e-mail: wlodar@math.purdue.edu
Time: TTh 1:30-2:45
Prerequisite: Basic knowledge in commutative algebra and ring theory.
Description: This is an introductory course in Algebraic Geometry. The course will cover Chapter I and elements of Chapters II form Hartshorne Book "Algebraic Geometry" and related topics in commutative Algebra. The course is aimed at graduate students but with low prerequisites.
Text: R. Hartschorne *Algebraic Geometry* Graduate Texts in Mathematics 52, Springer-Verlag, 1977.
Reference: M.F. Atiyah, I.G. MacDonald, *Introduction to Commutative Algebra* ,1969 Westview Press
I. R. Shafarevich, *Basic Algebraic Geometry*, 1977

MA 642: Methods of Linear and Non-linear Partial Differential Equations I
Instructor: Prof. Bauman, office: Math 718, phone: 49–41945, e-mail:bauman@math.purdue.edu
Time: MWF 9:30
Prerequisite: MA 544 and MA 611 or equivalent.
Description: This is the first semester of a one-year course in pde theory with applications to nonlinear equations. The first semester focuses on second order elliptic equations (in divergence and nondivergence form). Topics to be developed include maximum principles, Harnack inequalities, Schauder theory for classical solutions, and Sobolev estimates for weak solutions.
Text: D. Gilbarg and N. Trudinger *Elliptic Partial Differential Equations of Second order* Second Edition.

MA 650: Commutative Algebra
Instructor: Prof. Ulrich, office: Math 618, phone: 49–41972, e-mail: ulrich@math.purdue.edu
Time: MWF 1:30
Prerequisite: Basic knowledge of commutative algebra (such as the material of MA 557/558)
Description: Description: This is an intermediate course in commutative algebra. The course is a continuation of MA 557/558, but should be accessible to any student with basic knowledge in commutative algebra (localization, Noetherian and Artinian modules, associated primes and primary decomposition, integral extensions, dimension theory, completion, Ext and Tor).

The topics of this semester will include: Graded rings and modules, depth and Cohen-Macaulayness, structure of finite free resolutions, regular rings and normal rings, canonical modules and Gorenstein rings, local cohomology and local duality.

Texts: No specific text will be used, but possible references are:
- H. Matsumura, *Commutative ring theory*, Cambridge University Press
- W. Bruns and J. Herzog, *Cohen-Macaulay rings*, Cambridge University Press
- D. Eisenbud, *Commutative algebra with a view toward algebraic geometry*, Springer.

MA 663: Algebraic Curves and Functions I
Instructor: Prof. Lipman, office: Math 750, phone: 49–41994, e-mail: lipman@math.purdue.edu
Time: MWF 9:30
Prerequisite: MA 553, MA 530
Text: Otto Forster *Lectures on Riemann Surfaces*
Description: The course will cover the basic theory of algebraic curves from the point of view of compact Riemann surfaces. It will serve as an introduction to Algebraic Geometry, Complex Manifolds and Complex Differential Geometry.

<u>Course outline</u>
1. Riemann surfaces and function fields.
2. Survey of elliptic functions.
3. Differential forms.
4. Sheaf cohomology.
5. Riemannn-Roch theorem.
6. Serre Duality.
7. Existence theorems for functions and differentials.
8. Abel's theorm.
9. Jacobi inversion.

MA 690A: Topics in Algebra and Algebraic Geometry
Instructor: Prof. Abhyankar, office: Math 600, phone: 49–41933, e-mail:ram@math.purdue.edu
Time: TTh 3:00-4:15
Description: I shall discuss several topics in algebra and algebraic geometry. There are no prerequisites. All interested students are welcome. I shall use my new book Lectures on Algebra Volume I (published by World Scientific) as a text-book which the students are expected to purchase. Although this is an advanced book, the course will move much slower and can be taken as a basic course, understandable to scientists and engineers. During the course a softer version of the book will be produced. The students may also find it desirable to read my user-friendly book Algebraic Geometry For Scientists And Engineers (published by American Mathematical Society). Here is a list of some of the topics which may be covered:
(1) Expansions of polynomials of any degree in terms of sequences of other polynomials.
(2) Resultants, Discriminants, and solutions of higher degree polynomial equations in several variables.
(3) Newton's Theorem on Fractional Expansions.
(4) Implicit Function Theorem and Inverse Function Theorem.
(5) Intersection Theory and Bezout's Theorem.
(6) Classification and Resolution of Singularities of Curves, Surfaces, and Higher Dimensional Varieties.
(7) Divisors, Differentials, and Genus Formulas.

MA 690B: Topics in Commutative Algebra
Instructor: Prof. Heinzer, office: Math 636, phone: 49–41980, e-mail: heinzer@math.purdue.edu
Time: MWF 3:30
Prerequisite: Basic knowledge about commutative rings.
Description: The course is planned to be a continuation of Math 690B of this spring semester, and will cover additional material from the text by W. Bruns and J. Herzog *Cohen-Macaulay rings*, revised edition. Students enrolled in the course will be encouraged to actively participate by presenting material and exercises from the text.
Text: W. Bruns and J. Herzog, *Cohen-Macaulay rings*, revised edition.

MA 690E: Elliptic Curves
Instructor: Prof. Goins, office: Math 612, phone: 49–41936, e-mail: egoins@math.purdue.edu
Time: MWF 2:30
Prerequisite: MA 553 or equivalent
Description: This course will cover the basic theory of elliptic curves by covering three areas: (1) the Mordell-Weil Theorem, (2) Curves over Finite Fields, and (3) Modular Forms.

The first few weeks will be spent covering the chord-tangent construction. We will present the group law and the Mordell's proof of the Poincaré Conjecture. We will then discuss the relationship between Mordell-Weil groups, Selmer groups, and Shafarevich-Tate groups through Galois cohomology.

The next few weeks will be spent covering properties of elliptic curves over finite fields and local fields. We will discuss properties of Zeta functions, the Riemann Hypothesis, and the Weil Conjectures. We will also discuss L-functions and the conjectures of Birch and Swinnerton-Dyer.

The final few weeks will be spent covering modular forms and applications. We will review classical modular curves, present Serre's mod p modular forms, and introduce Hida's Λ-adic modular forms. The course will conclude with a discussion of the proof of Wiles et al. of Fermat's Conjecture; and the proof of Taylor et al. for the conjectures of Weil, Shimura, and Taniyama.

There will be weekly homework. Students will give in-class presentations in place of a final exam.
References: Dale Husemöller, *Elliptic Curves*; Joseph Silverman, *The Arithmetic of Elliptic Curves*
Text: Dale Husemöller, *Elliptic Curves*

MA 690F: Topics in Automorphic Forms
Instructor: Prof. Shahidi, office: Math 650, phone: 49–41917, e-mail: shahidi@math.purdue.edu
Time: MWF 9:30
Description: This course is aimed at developing different aspects of the theory of automorphic forms. We are planning to complete our study of the theory of automorphic L–functions by means of the theory of Eisenstein series and then attend to the classical theory of P–adic L–functins. We will conclude by giving an overview of recent ideas of Langlands on issues of functionality.
References: 1. F. Shahidi, *Eisenstein Series and Automorphic L–functions*, in preparation
2. S. Lang, *Cyclotomic Fields I4 II*, GTM 121, Sringer-Verlag.
3. R. P. Langlands, *Beyond Endoscopy*, in Shalika volume, Johns Hopkins Press, (H. Hida, R. Ramakrishnan, F. Shahidi, editors)
4. J. Arthur, *Introduction to Trace Formula*, Clay Math. institute, AMS (J. Arthur, R. Kottwitz, D. Ellwood, Editors)

MA 692C: A Posteriori Error Estimation and Adaptive Finite Element Methods
Instructor: Prof. Cai, office: Math 412, phone: 49–41921, e-mail: zcai@math.purdue.edu
Time: TTh 9:00-10:15
Prerequisite: CS/MA 514 and CS/MA 615 or equivalent or consent of instructor.
Description: All computational results for simulating physical phenomena in engineering applications and scientific predictions contain numerical error. Discretization error could be large and unpredictable by classical heuristic means and could invalidate numerical predictions. A posteriori error estimation is a rigorous mathematical theory for estimating and quantifying discretization error in terms of the error's magnitude and distribution. This information provides a basis for error control or verification and for adaptive meshing. The theory of the a posteriori error estimation has become an important area of research and has found application in an increasing number of commercial software products and scientific programs.

This is an introductory course on the a posteriori error estimation and adaptive finite element method. The course will introduce basic principles on how to design viable a posteriori error estimators and basic tools for analyzing estimators

References: [1] M. Ainsworth and J. T. Oden, *A Posteriori Error Estimation in Finite Element Analysis*, John Wiley and Sons, Inc., 2000.
[2] I. Babuska and T. Strouboulis, *The Finite Element Method and Its Reliability*, Oxford Science Publication, New York, 2001.
[3] R. Becker and R. Rannacher, *An optimal control approach to a posteriori error estimation in finite element methods*, Acta Numer., 1-102, 2001.
[4] R. Verfurth, *A Review of A-Posteriori Error Estimation and Adaptive Mesh Refinement Technique*s, John Wiley and Teubner Series. Advances in Numerical Mathematics., 1996.
[5] research articles.

MA 692D: High Performance Computing
Instructor: Prof. Dong, office: Math 436, phone: 49–63875, e-mail: sdong@math.purdue.edu
Time: TTh 3:00-4:45
Prerequisite: Ability to program in any of the high-level languages: FORTRAN, C, or C++.
Description: High performance computing involves the use of the most efficient algorithms on parallel computers capable of the highest performance to solve the most demanding scientific problems. It is an enabling technology for many scientific disciplines such as weather forecasting, genome sequencing, turbulence control, drug design, and cryptography. Without the capabilities enabled by high performance computing, many of today's computational science disciplines will cease to exist.

This course targets graduate students and advanced undergraduate students. It is designed to expose students to the fundamental concepts and principles in high performance computing, and to provide opportunities for students to gain hands-on experience in efficiently programming a spectrum of parallel computers ranging from small workstation clusters with a few CPUs to supercomputers with thousands of CPUs. The course will enable students to efficiently exploit high performance parallel computers in their own research activities.

The course emphasizes practical applications, and will cover several important aspects of high performance computing. Some of the topics include: Supercomputer architecture; parallel programming for distributed- and shared-memory machines; Parallel Input/Output; Parallel numerical algorithms (parallel matrix algorithms; parallel direct/iterative solvers; high performance numerical linear algebra); Performance optimization and analysis; Parallel debugging.

No formal text book. Most materials will be available online.

MA 692E: Inverse Problems
Instructor: Prof. De Hoop, office: Math 422, phone: 49–66439, e-mail: mdehoop@math.purdue.edu
Time: TTh 10:30-11:45
Description: Topics in Imaging.

MA 693A: Bieberbach Conjecture
Instructor: Prof. de Branges, office: Math 800, phone: 49–46057, e-mail: branges@math.purdue.edu
Time: MWF 10:30
Description: A second course in complex analysis is offered to students interested in curent research topics. A fundamental problem is to estimate the coefficients of a power series with constant coefficient zero which represents an injective mapping of the unit disk. A conjecture of Riemann states that every proper region of the plane containing the origin is the image of such a mapping. The proof requires coefficient estimates of which best possible ones were conjectured in 1916 by Bieberbach. The proof of the conjecture in 1984 leaves open the problem of characterizing coefficients. The properties of injective mappings are an introduction to techniques of minimization which originate in the eighteenth century and whose modern formulation applies Hilbert spaces whose elements are analytic functions. The course is an introduction to the Hilbert spaces of functions analytic in the unit disk.

MA 693D: Introduction to Non-commutative Geometry
Instructor: Prof. Dadarlat, office: Math 708, phone: 49–41940, e-mail: mdd@math.purdue.edu
Time: MWF 3:30
Prerequisite: Some prior exposure to algebraic topology and differential geometry would be helpful but not required
Description: We aim for a friendly introduction based on examples to the ideas of noncommutative geometry. Topics will include:

Hochschild cohomology and noncommutative differential forms
Cyclic cohomology
Review of K-theory
Characteristic numbers and the Connes-Chern character
spectral triples and noncommutative Riemannian manifolds

References: 1. A. Connes, *Noncommutative Geometry,* Academic Press, San Diego, CA, 1994, 661 p.
2. A. Connes, *Non commutative differential geometry,* Publ. Math. IHES no. 62 (1985), 41-144
(both available on line: http://www.alainconnes.org/en/bibliography.php)
Text: We will follow no specific text

MA 696Y: Topics in Complex Geometry
Instructor: Prof. Yeung, office: Math 712, phone: 49–41942, e-mail: yeung@math.purdue.edu
Time: MWF 2:30
Prerequisite: MA530, MA562
Description: We will study several topics of current interest in complex geometry.

1. The use of multiplier ideal sheaves in geometric analysis and algebraic geometry. In particular, we will mention results related to Siu's theorem on invariance of plurigenera. In the process we will go through techniques in several complex variables related to L^2-estimates.
2. The relation between the notion of stability and metric with special curvature properties. In particular, we will mention results related to Donaldson's work on metrics with constant scaler curvature. In the process, we will discuss results on Calabi Conjecture as well as extremal metrics.
3. Introduction to geometric flows, including its formulation and its use in harmonic maps, Ricci-flow and Monge-Ampere equation.

Seminars

Algebra and Algebraic Geometry Seminar, Prof. Abhyankar
Time: Thursday 4:30–6:00

Applied Math Lunch Seminar, Prof. Buzzard
Time: Fridays 11:30

Automorphic Forms and Representation Theory Seminar, Prof. Goldberg

Bridge to Research Seminar, Prof. Milner
Time: Mondays 4:30

$\overline{\partial}$**–Neumann Problem Seminar,** Prof. Catlin

Commutative Algebra Seminar, Profs. Heinzer and Ulrich
Time: Wednesdays 4:30

Computational and Applied Math Seminar,
Time: Fridays 3:30

Differential Geometry Seminar, Prof. Donnelly
Time: Thursdays, 3:30

Function Theory Seminar, Prof. Eremenko
Time: Tuesdays, time flexible

Foundations of Analysis Seminar, Prof. de Branges
Time: Thursday 9:30

Geometric Analysis Seminar, Prof. Lempert
Time: Monday 3:30

Number Theory Seminar, Prof. Goins
Time: Thursday 3:30

Operator Algebras Seminar, Prof. Dadarlat
Time: Tuesdays, 2:30

PDE Seminar, Prof. Bauman
Time: Thursday, 3:30

Probability Seminar, Prof. Banuelos
Time: Wednesday, 3:30

Topology Seminar, Prof. Kaufmann
Time: Thursdays 3:30

Working Algebraic Geometry Seminar, Prof. Matsuki
Time: Wednesday 3:30rm