Seminars and Advanced Graduate Courses
offered by the
Mathematics Department
Fall, 2009

Courses

MA 53900 CRN:23406 Probability Theory II
Instructor: Prof. Yip, office: Math 432, phone: 49–41941, e-mail: yip@math.purdue.edu
Time: MWF 12:30 **NOTE NEW TIME**
Description: This is a continuation of MA/STAT 53800. This second course will touch upon the properties of stochastic processes, in particular their limit theorems and long time behaviors. Topics include martingale theory, random walk, Brownian Motions, invariance principle, stationary and Gaussian processes and ergodic theorems. Depending on time and interests, additional topics such as Markov and diffusion processes, and large deviation theory will be covered.

The prerequisite is MA/STAT 53800. But anyone with good background in probability and measure theory (at the level of MA/STAT 51900 and MA 54400) can also benefit. The main knowledge I will assume is a good understanding of the Weak and Strong Law of Large Numbers, Central Limit Theorem, and the concept of Conditional Probability (as covered in Chapters 4, 5, and 6 of Billingsley's book).
Text: P. Billingsley, *Probability and Measure*

MA 54600: CRN:23409 TITLE
Instructor: Prof. L. Brown, office: Math 704, phone: 49–41938, e-mail: lgb@math.purdue.edu
Time: MWF 10:30
Prerequisite: MA 544
Description: The course covers basic functional analysis with emphasis on bounded linear operators on Banach and Hilbert spaces. The topics listed below will be covered, and brief treatments of other topics fitting the interests of the students can be included as time permits.

Banach spaces and Hilbert spaces; Hahn-Banach theorem; closed graph and open mapping theorems; uniform boundedness principle; theory of spectrum for operators on Banach spaces and for compact operators; weak and weak* topologies; reflexivity; Hahn-Banach separation theorem and double polar theorem; operators on Hilbert spaces; spectral theorem for bounded self-adjoint operators on Hilbert spaces.
Text: M. Schechter, *Principles of Functional Analysis*, AMS.

MA 55700: CRN:23413 Abstract Algebra I
Instructor: Prof. Abhyankar, office: Math 600, phone: 49–41933, e-mail: ram@math.purdue.edu
Time: TTh 3:00-4:15
Description: This course and its Spring sequel MA55800 will cover the basic algebra needed for studying advanced algebra, algebraic geometry, number theory, complex analysis, and applications to computer aided design and geometric modelling. I shall use my book *Lectures on Algebra* Volume I (published by World Scientific) as a text-book which the students are expected to buy. The students may also find it desirable to read my user-friendly book *Algebraic Geometry For Scientists And Engineers* (published by American Mathematical Society). Here is a list of possible topics to be dealt with:

(1) Group Theory including Sylow Theorem and Burnside's Theorem.
(2) Rings and Modules incluing, Euclidean Domains, Principal Ideal Domains, and Unique factorization Domains.
(3) Fundamental Theorems of Galois Theory.
(4) Polynomials and Power series incluing Hensel's Lemma and Newton's Theorem as well as the Preparaton Theorem of Weierstrass.
(5) Valuation Theory and Integral Dependence.
(6) Resultants and Discriminants leading to solutions of higher degree polynomial equations in several variables.
(7) Primary decomposition in noetherian rings and noetherian modules.
(8) Artinian rings and lengths of modules.
(9) Local Rings and Graded Rings.
(10) Algebraic Varieties including their spectral and modelic versions.
(11) Hilbert Nullstellensatz and Noether Normaliztion.
(12) Cohen-Macaulay Rings and Gorenstein Rings.
(13) Hibert Syzygies and Unique Factorization in Regular Local Rings.
(14) Resolution of Singularities by means of Quadratic and Monoidal Transformations.

MA 56200: CRN:23414 Introduction to Differential Geometry and Topology
Instructor: Prof. Albers, e-mail: palbers@math.purdue.edu
Time: TTh 10:30-11:45
Prerequisite: MA 35100 and MA 44200
Description: This course introduces and studies differential calculus on curved spaces (= differentiable manifolds). This is a core topic with applications throughout all of mathematics.

In the first part we will introduce the concept of differentiable manifolds, tangent spaces, and study the implicit function theorem. We will discuss the theorem of Sard and Brouwer's fixed point theorem. After developing some degree theory we study vector fields and the famous Poincare-Hopf index theorem.

In the second part we revisit the theory manifolds from a more abstract point of view. We return to vector fields and their flows on manifolds and introduce the Lie bracket and derivative. Then we define differential forms and the exterior derivative. This leads itself to the definition of deRham cohomology. Finally we study some global properties of manifolds e.g. the Gauss-Bonnet theorem.

Text:
Milnor – *Topology from the Differentiable Viewpoint*, Princeton University Press
Gallot, Hulin, Lafontaine – *Riemannian Geometry*, Springer
References:
Guillemin, Pollack – *Differential Topology*, Prentice-Hall
Hirsch – *Differential Topology*, Springer

MA 59800: CRN:23432 Selmer Groups and Galois Representation
Instructor: Prof. Goins, office: Math 612, phone: 49–41936, e-mail: egoins@math.purdue.edu
Time: MWF 1:30 **NOTE NEW TIME**
Prerequisite: MA 55300 or equivalent. MA 58400 is preferred but not necessary.
Description: The celebrated Chinese Remainder Theorem asserts that the integral solutions to linear equations can be found by considering solutions to congruences modulo prime powers. In 1921, Helmut Hasse, expanding upon ideas of Hermann Minkowski, generalized this to quadratic equations. This lead to the so-called "local-global principle". In 1951, Ernst Selmer found a class of cubic curves for which this principle fails. Nowadays, in order to find rational points on abelian varieties, one constructs the Selmer group of the curve by searching for "local" points, and uses a type of descent procedure to determine which actually come from those which are "global".

This course seeks to motivate the study of Selmer groups by discussing Galois cohomology. We will review the Galois theory of number fields, introduce the p-adic numbers and other completions, and study decomposition groups. We will cover Hasse's results on quadratic forms as well as Selmer's counterexamples for cubic forms. Along the way, we will discuss algorithms for finding solutions to Pell's equations and rational points on elliptic curves. The course will conclude with applications to Galois representations. We will study the Tate module of an abelian variety, with special emphasis on elliptic curves. We will discuss the Shimura-Taniyama-Weil conjecture (now proved by Wiles, et al.) by considering the deformation ideas of Mazur via the study of Selmer groups.

MA 59800: CRN:23433 Introduction to Homological Algebra **COURSE CANCELLED**
Instructor: Prof. Wilkerson, office: Math 700, phone: 49–41955, e-mail: wilker@math.purdue.edu
Time: TTh 12:00-1:15
Description: This course is intended for students in algebraic topology, algebraic geometry, and commutative algebra. Beginning with chain complexes, projective and injective modules, resolutions and the Ext and Tor functors will be defined. Then applications to ideas of homological dimension will be given. Next spectral sequences as a means of computation will be discussed. (This block of material is in the first five chapters of Weibel's book.)

The remaining third or so of the course will treat topics from group cohomology, Andre-Quillen homology and Hochschild homology.

Students may be asked to present some material in class and and packages such Macaulay 2, Singular, or CoCoa will be used for examples and projects.
Text: Charles A. Weibel *An introduction to homological algebra*, Cambridge Studies in in advanced mathematics 38.

MA 59800: CRN:23437 Theory of Analytic Surfaces
Instructor: Prof. Matsuki, office: Math 614, phone: 49–41970, e-mail: kmatsuki@math.purdue.edu
Time: MWF 10:30
Description: We will study the theory of algebraic surfaces, i.e., algebraic varieties of dimension 2. After reading Hartshorne as the first introductory textbook into algebraic geometry, one may still feel lost in the deluge of abstract notions such as schemes, sheaves, and cohomology, etc. The theory of algebraic surfaces provides a very good exercising ground to see how these abstract notions can be effectively used to study the concrete geometry. It can be seen as a natural extension of the theory of algebraic curves, and also as a bridge to the theory of higher dimensional algebraic varieties, a hot subject in the recent research arena. The course is aimed at not only the students who took the basic courses with Profs. Wlodarczyk and Arapura in 2008 and intend to continue their study, but also the students who want to learn the subject as a general background toward their own special interests (number theory, complex analysis of several variables etc.).
We will use the following three books as the textbooks/reference books for the course:
1. *Complex Algebraic Surfaces* by A.Beauville
2. *Introduction to the Mori Program* by K.Matsuki
3. *Compact Complex Surfaces* by W.Barth, C.Peters, and A.Van de Ven

MA 59800: CRN:23431 Topological Galois Theory
Instructor: Prof. Gabrielov, office: Math 648, phone: 49–47911, e-mail: agabriel@math.purdue.edu
Time: MWF 2:30
Description: Problems of solvability and non-solvability of algebraic, transcendental and differential equations in explicit form, including Liouville theory, classical and differential Galois theory, and Picard-Vessiot theory, will be addressed from the geometry and analysis point of view. The course will be based on the book *Topological Galois Theory* by A. G. Khovanskii (2008, in Russian). Most of its contents can be found in Khovanskii's review paper "On solvability and unsolvability of equations in explicit form," Russian Math. Surveys, v. 59, pp. 661-736.

MA 63100: CRN:37874 Several Complex Variables
Instructor: Prof. Lempert, office: Math 728, phone: 49–41952, e-mail: lempert@math.purdue.edu
Time: TTh 12:00-1:15
Prerequisite: MA 53000
Description: Power series, holomorphic functions, representation by integrals, extension of functions, holomorphically convex domains. Local theory of analytic sets (Weierstrass preparation theorem and consequences). Functions and sets in the projective space Pn (theorems of Weierstrass and Chow and their extensions).

MA 64200: CRN:23438 Methods of Linear and Nonlinear Partial Differential Equations I
Instructor: Prof. Danielli, office: Math 620, phone: 49–41920, e-mail: danielli@math.purdue.edu
Time: MWF 9:30
Prerequisite: MA 54400 and MA 61100, or instructor's approval
Description: This is the first semester of a one-year course in the theory of PDEs with applications to nonlinear equations. This semester focuses on second order elliptic equations, in both divergence and nondivergence form. The topics covered include Laplace's and Poisson's equations, the classical maximum principle and the Harnack inequality, Schauder's estimates, Sobolev spaces, and the regularity of weak solutions.
Text: D. Gilbarg and N. S. Trudinger *Elliptic Partial Differential Equations of Second Order*, Second Edition.

MA 68400: CRN:37875 Class Field Theory
Instructor: Prof. Shahidi, office: Math 650, phone: 49–41917, e-mail: shahidi@math.purdue.edu
Time: MWF 9:30
Prerequisite: MA 58400 of the instructors approval.
Description: Class field theory is that of understanding abelian extensions of local and global fields. It is a crowning achievement of number theory in the 20th century and the main motivating objects for the Langlands program. We will treat the subject by mainly concentrating on number fields.
Syllabus: Ideles, adeles, L-functions, first and second inequalities, Artin symbol, reciprocity, local and global class fields, Kronecker-Weber theorem.
Recommended books:
1. S. Lang, *Algebraic Number Theory*, GTM, Springer
2. J. Cassels and A. Frohlich, *Algebraic Number Theory*, Thompson book company, 1967

MA 69000: CRN:23442 Topics in Commutative Algebra
Instructor: Prof. Heinzer, office: Math 636, phone: 49–41980, e-mail: heinzer@math.purdue.edu
Time: MWF 3:30
Prerequisite: Math 55800 or its equivalent
Description: The course will cover selected topics in commutative algebra mainly take from the book by Irena Swanson and Craig Huneke titled *Integral Closure of Ideals, Rings and Modules* London Mathematical Society Lecture Notes Series 336. There is a corrected online version of the book that may be freely downloaded. Students in the class will be encouraged to actively participate by presenting exercises and/or text material.

MA 69000: CRN:23444 Gröbner Bases in Commutative Algebra
Instructor: Prof. Caviglia, office: Math 608, phone: 49–41973, e-mail: gcavigli@math.purdue.edu
Time: MWF 12:30
Prerequisite: This course is intended to graduate students with a certain familiarity with the basic concepts of Commutative Algebra. A previous knowledge of Commutative Algebra, for instance the material covered in 65000, would be useful although not required.
Description: The course will focus on the theory of Gröbner bases and its applications in Commutative Algebra. I will discuss both theoretical and computational aspects. Three central topics will be covered:

- Graded free resolutions and their homological invariants.
- Generic initial ideals.
- Determinantal ideals and secant varieties.

References for the material regarding the first two topics can be found in:
1. *Algebraic Combinatoric* (B. STURMFELS, E. MILLER)
2. *Gröbner bases and convex politopes* (B. STURMFELS)
3. *Commutative Algebra* (D. EISENBUD)
4. *Generic Initial Ideals* (M. GREEN)

The reference for the last topic (determinantal ideals) will be a recent article of B. Sturmfels and S. Sullivan.

MA 69200: CRN:23449 Topics in Imaging
Instructor: Prof. De Hoop, office: Math 422, phone: 49–66439, e-mail: mdehoop@math.purdue.edu
Time: TTh 10:30-11:45

MA 69300: CRN:23453 Riemann Proof
Instructor: Prof. de Branges, office: Math 800, phone: 49–46057, e-mail: branges@math.purdue.edu
Time: MWF 10:30
Description: A proof of the Riemann hypothesis is offered as a second course in complex analysis assuming information from the text by Ahlfors and acquaintance with integration on the real line. The proof begins with the Stieltjes representation of positive linear functionals on polynomials as integrals on the real line and with the Hermite generalization of polynomials as entire functions admitting a factorization in terms of zeros. Hilbert spaces of entire functions are introduced which axiomatize the Stieltjes representation and generalize it to the entire functions discovered by Hermite. The proof applies a generalization of the Riemann hypothesis for Hilbert spaces of entire functions as published in the Bulletin of the American Mathematical Society 1986 following the proof of the Bieberbach conjecture, and appears as an electron preprint on Riemann spaces of entire functions.

Seminars

Algebra and Algebraic Geometry Seminar, Prof. Abhyankar
Time: Thursday 4:30–6:00

Applied Math Lunch Seminar, Prof. Buzzard
Time: Fridays 11:30

Automorphic Forms and Representation Theory Seminar, Prof. Goldberg
Time: Thursdays 1:30

Bridge to Research Seminar, Prof. Bell
Time: Mondays 4:30

Commutative Algebra Seminar, Profs. Heinzer and Ulrich
Time: Wednesdays 4:30

Commutative Algebra Reading Seminar, Prof. Kummini
Time: Tuesday 3:30

Computational and Applied Math Seminar, Prof. Shen
Time: Fridays 3:30

Function Theory Seminar, Prof. Eremenko
Time: Wednesdays, 1:30

Foundations of Analysis Seminar, Prof. de Branges
Time: Thursday 9:30

Geometric Analysis Seminar, Prof. Lempert
Time: Monday 3:30

Number Theory Seminar, Prof. Goins
Time: Thursday 3:30

Operator Algebras Seminar, Prof. Dadarlat
Time: Tuesdays, 2:30

PDE Seminar, Prof. Bauman
Time: Thursday, 3:30

Probability Seminar (STAT 69100 CRN 29295), Prof. Sellke
Time: Mondays, 3:30

Spectral and Scattering Theory Seminar, Prof. SaBarreto
Time: Thursday 4:30

Topology Seminar, Profs. Kaufmann and McClure
Time: Thursdays 3:30

Working Algebraic Geometry Seminar, Profs. Arapura and Matsuki
Time: Wednesday 3:30