Seminars and Advanced Graduate Courses
offered by the
Mathematics Department
Fall, 2010

Courses

MA 54200: CRN:45582 Theory of Distributions and Applications
Instructor: Prof. Donnelly, office: Math 716, phone: 49–41944, e-mail: hgd@math.purdue.edu
Time: MWF 12:30
Description: Definition and basic properties of distributions; convolution and Fourier transforms; applications to partial differential equations; Sobolev spaces.

MA 55700: CRN:23413 Abstract Algebra I
Instructor: Prof. Ulrich, office: Math 618, phone: 49–41972, e-mail: ulrich@math.purdue.edu
Time: TTh 3:00-4:15
Prerequisite: Basic knowledge of algebra (such as the material of MA 50300).
Description: The topics of the course will be commutative algebra and introductory homological algebra. We will study basic properties of commutative rings and their modules, with some emphasis on homological methods. The course should be particularly useful to students interested in commutative algebra, algebraic geometry, number theory or algebraic topology. There will be a continuation in the spring.
Texts: No particular book is required, but typical texts are:
M. Atiyah and I. Macdonald, *Introduction to commutative algebra*, Addison-Wesley.
- J. Rotman, *An introduction to homological algebra*, Academic Press.

MA 56200: CRN:23414 Introduction to Differential Geometry and Topology
Instructor: Prof. Donnelly, office: Math 716, phone: 49–41944, e-mail: hgd@math.purdue.edu
Time: MWF 2:30
Description: Smooth manifolds; tangent vectors; inverse and implicit function theorems; submanifolds; vector fields; integral curves; differential forms; the exterior derivative; DeRham cohomology groups; surfaces in E3., Gaussian curvature; two dimensional Riemannian geometry; Gauss-Bonnet and Poincare theorems on vector fields.

MA 59800: CRN:23437 Introduction to Algebraic Geometry
Instructor: Prof. Matsuki, office: Math 614, phone: 49–41970, e-mail: kmatsuki@math.purdue.edu
Time: MWF 10:30
Prerequisite:
- basic knowledge of algebra (at the level of MA 55300 and MA 55400)
- basic knowledge of commutative algebra at the level of Atiyah-McDonald
- some exposure to the theory of (complex or real) manifolds
- some exposure to basic theory of homological algebra

Description: The purpose of this course is to give a concise introduction to algebraic geometry, discussing the materials covered in Chapters I, II, III, IV of Hartshorne's textbook. We will cover the basics of schemes, sheaves, cohomology, culminating in the theory of algebraic curves.

The course is aimed not only at the students who want to specialize in algebraic geometry proper, but also at those who have other major disciplines in mind, such as number theory and commutative algebra, where the language of scheme is becoming more and more ubiquitous.

I would like to give lectures at a leisurely pace with emphasis on the ideas, while boosting the confidence level of the students by going through the exercise problems on Hartshorne with rigor.

MA 59800: CRN:23432 Using Algebraic Geometry
Instructor: Prof. Gabrielov, office: Math 648, phone: 49–47911, e-mail: agabriel@math.purdue.edu
Time: MWF 12:30
Prerequisite: Abstract and Linear Algbra.
Description: This course, oriented towards beginning graduate and advanced undergraduate students, will introduce some basic ideas from Algebraic Geometry, emphasizing computational aspects and interactions with linear algebra and combinatorics.

We will cover roughly chapters 1-3 and 7 of the text: *Using Algebraic Geometry* by D. Cox, J. Little and D. O'Shea, Second Edition, with some additional material from *Solving Systems of Polynomial Equations* by B. Sturmfels.

Our principal subject will be solving systems of polynomial equations, both algebraically and geometrically. Two principal computational approaches are based on Groebner bases and resultants. For sparse systems solving, connection with polytopes and toric varieties will be discussed.

Some homework will use Maple, but no prior experience with Maple is expected.

MA 59800: CRN:45583 Pseudodifferential Operators
Instructor: Prof. Sa Barreto, office: Math 410, phone: 49–41965, e-mail: sabarre@math.purdue.edu
Time: NOTE NEW TIME IS MWF 10:30
Prerequisite: MA 54400 or permission of the instructor.
Description: We will cover the following topics:

1) Quick review of the theory of distributions.
2) The calculus of pseudodifferential operators in Hormander's class, including things like the sharp Garding inequality.
3) Applications to hyperbolic equations, spectral theory and Hormander's theorem on propagation of singularities.
4) Extra topics: If time allows, we will do the Weyl calculus and semiclassical pseudodifferential operators. **Text:** We will have no textbook, but I will post my lecture notes on my home page. There are many very good references, including the following:
    1) *Pseudodifferential operators and spectral theory.* M. A. Shubin
    2) *Microlocal Analysis for differential operators, an introduction.* A. Grigis and J. Sjostrand
    3) *An introduction to semiclassical and microlocal analysis.* A. Martinez
    4) *Semiclassical Analysis.* M. Zworski and C. Evans (can be downloaded from http://math.berkeley.edu/ zworski)

MA 59800: CRN:23431 Modeling and Computation in Optics and Electromagnetics
Instructor: Prof. Peijun Li, office: Math 440, phone: 49–40846, e-mail: lipeijun@math.purdue.edu
Time: TTh 1:30-2:45
Description: This course addresses some recent developments on the mathematical modeling and the numerical computation of problems in optics and electromagnetics. The fundamental importance of the fields is clear, since they are related to technology with signicant industrial and military applications. The recent explosion of applications from optical and electromagnetic scattering technology has driven the need for modeling the relevant physical phenomena and developments of fast, efficient numerical algorithms. The course will provide introductory material to the areas in optics and electromagnetics that offer rich and challenging mathematical problems. It is also intended to convey some up-to-date results to students in applied and computational mathematics, and engineering disciplines as well.

Particular emphasis of this course is on the formulation of the mathematical models and the design and analysis of computational approaches. Topics are organized to present model problems, physical principles, mathematical and computational approaches, and engineering applications corresponding to each of these problems. The following is a brief outline of the topics.

Topic 1 concerns modeling and computation of diffractive optics. Variational formulations and well- posedness results will be addressed for model problems involving the two-dimensional Helmholtz and three-dimensional Maxwell equations. Computationally, finite element methods will be discussed. Recent results on inverse and optimal design problems will be highlighted.

Topic 2 deals with photonic crystals, also known as photonic band gap structures. A photonic crystal is a periodic dielectric low-loss material for which there exist intervals of frequency for which electro- magnetic waves cannot propagate in this medium. The mathematical formulation, analysis, and bandgap calculations based on Dirichlet-to-Neumann maps will be covered.

Topic 3 describes the modeling of electromagnetic cavities. Coupling of finite element and boundary integral equation methods will be introduced to deal with the radiation and scattering of complex cavity- backed conformal antennas.

Topic 4 is devoted to important mathematical and computational issues for scattering problems of acoustic and electromagnetic wave propagation. The two basic problems in classical scattering theory are the scattering of time-harmonic acoustic or electromagnetic waves by a penetrable inhomogeneous medium of compact support and by a bounded impenetrable obstacle. Boundary integral methods based on potential theory will be introduced for solving the obstacle scattering problem. The medium scattering problem will be reduced to a bounded domain by introducing artificial boundary conditions, e.g., nonlocal transparent boundary conditions, local absorbing boundary condition, perfectly matched layer techniques. Existence and uniqueness of the weak solutions will be examined by variational approaches.

Topic 5 investigates the analysis of boundary integral formulation and variational formulation for scattering by unbounded rough surfaces. By rough surface, it refers to a usually nonlocal perturbation of an infinite plane surface such that the whole surface lies within a finite distance of the original plane. Such problems arise frequently in applications from modeling acoustic or electromagnetic wave propagation over outdoor ground and sea surfaces.
Text: No textbook is required. Lecture notes will be made available to students.
Grades: No exams. Students are required to present course-related material in class.
References:
1. G. Bao, L. Cowsar, and W. Master, *Mathematical Modeling in Optical Science*
2. D. Colton and R. Kress, *Inverse Acoustic and Electromagnetic Scattering Theory*
3. J. Jin, *The Finite Element Method in Electromagnetics*
4. P. Monk, *Finite Element Methods for Maxwell's Equations*
5. J.-C. Nedelec, *Acoustic and Electromagnetic Equations: Integral Representations for Harmonic Problems*

MA 64200: CRN:38721 Methods of Linear and Nonlinear Partial Differential Equations I
Instructor: Prof. Phillips, office: Math 706, phone: 49–41939, e-mail: phillips@math.purdue.edu
Time: MWF 9:30
Prerequisite: MA 54400 and MA 61100
Description: This is the first semester in a one-year course on the theory of PDE. The Fall semester focuses on linear second order elliptic equations. Topics to be covered include Laplace's equation, the maximum principle, Poisson's equation and the Newtonian potential, Schauder's estimates for classical solutions, Sobolev Spaces, weak solutions and their regularity.
References: D.Gilbarg and N.S.Trudinger *Elliptic Partial Differential Equations of Second Order*

MA 64400: CRN:45584 Calculus of Variations
Instructor: Prof. Danielli, office: Math 620, phone: 49–41937, e-mail: danielli@math.purdue.edu
Time: MWF 11:30
Prerequisite: The course will be essentially self-contained. The only prerequisites are familiarity with the Lebesgue integral and the first properties of L^p spaces, as well as a basic knowledge of Sobolev spaces.
Description: The fundamental problem of the calculus of variations is the minimization of the integral functional

$$F(u;\Omega)=\int_\Omega F(x,u,Du)dx$$

among all the functions u taking prescribed boundary values on the boundary of Ω. In this course we will focus on its solution via the so-called direct methods, which consist in proving the existence of the minimum of the integral functional F directly, rather than resorting to its Euler equation. The central idea is to consider F as a real-valued mapping on the manifold of functions taking on $\partial\Omega$ the given boundary values and applying to it a generalization of Weierstrass' theorem on the existence of the minimum of a continuous function. There are two main issues in this approach. The first one is that semicontinuity, rather than continuity, is the key assumption to apply Weierstrass theorem to the functional F, whereas the second one is to identify the Sobolev spaces as the proper function spaces for compactness results to hold. On the other hand, the solution of the existence problem in the Sobolev class opens up a series of questions about the regularity of the minimizers. This was a long-standing open problem, until the way to its solution (in a non-direct fashion, since it involves the Euler equation) was opened by the celebrated De Giorgi-Nash-Moser result concerning the Hölder continuity of solutions to uniformly elliptic PDEs in divergence form with bounded and measurable coefficients. A first step towards the use of direct methods in the regularity issue came from a 1982 paper by Giaquinta and Giusti, who proved the Hölder continuity of quasi-minima, that is functions u for which

$$F(u;K)\leq QF(u+\varphi;K)$$

for every φ with compact support $K\subset\Omega$. The notion of quasi-minima reduces of course to the one of minimum when $Q=1$, but it is substantially more general, since it includes solutions of linear and nonlinear elliptic equations and systems. The course will follow the path outlined above, and it will combine the classical approach to the subject with its latest developments. In particular, emphasis will be placed on presenting an unified treatment of the regularity of the minima of functionals in the calculus of variations, and of the solutions to elliptic equations and systems in divergence form. We will also explore the connections with the study of minimal surfaces and applications to optimal control theory.
Texts: E. Giusti, *Direct Methods in the Calculus of Variations*, World Scientific, 2002.
References: B. Dacorogna, *Introduction to the Calculus of Variations*, Imperial College Press, 2008
I. Fonseca and G. Leoni, *Modern Methods in the Calculus of Variations: L^p Spaces*, Springer, 2007.

MA 66300: CRN:45585 Algebraic Curves and Functions I
Instructor: Prof. Abhyankar, office: Math 600, phone: 49–41933, e-mail: ram@math.purdue.edu
Time: TTh 3:00-4:15
Description: Algebraic geometry, concerned with solutions of systems of polynomial equations, and their graphical representations, has for long been regarded as a very abstract area of mathematics. However, recently, with the advent of high-speed computers, applications have come about in such diverse areas of science and engineering such as theoretical physics, chemical, electrical, industrial and mechanical engineering, computer aided design (CAD), computer aided manufacturing (CAM), optimization, and robotics. These application areas are also increasingly posing fundamental open mathematical questions. This course is intended as an introduction to various relevant topics in algebraic geometry such as:

- Analysis and resolution of singularities
- Rational and polynomial parametrization
- Intersections of curves and surfaces
- Polynomial maps
- Expansions of polynomials of any degree in terms of sequences of other polynomials.
- Resultants and solutions of higher degree polynomial equations in several variables.
- Divisors, Differentials, and Genus Formulas.

The lectures will be expository in nature and so will be accessible to everyone. Thus there are no formal prerequisites and all interested students are welcome. In particular the required abstract algebra will be developed simultaneously. The course will continue with its Part II in the Spring.
Texts:(1) Shreeram Abhyankar, *Algebraic Geometry for Scientists and Engineers*, Amer Math Soc
(2) Shreeram Abhyankar, *Resolution of Singularities of Embedded Algebraic Surfaces*, Springer Verlag

MA 69000: CRN:45591 L-Functions and Automorphic Forms
Instructor: Prof. Shahidi, office: Math 650, phone: 49–41917, e-mail: shahidi@math.purdue.edu
Time: MWF 9:30
Prerequisite: MA58400 (Algebraic Number Theory) and MA68400 (Class Field Theory).
Description: I hope to cover the theory of complex and p-adic L-functions attached to automorphic forms which are central in Langlands program. While the first subject is rather well-studied in great generality, p-adic L-functions are understood only in a few cases. They are both very central in the developement of modern number theory. I hope to address both in some detail.
References: No books are required, but here are some recommended ones:
1.) N. Koblitz, *P-adic Numbers, P-adic Analysis and zeta functions*, GTM 58
2.) L. Washington, *Intro. to Cyclotomic Fields*, GTM 83
3.) S. Lang, *Cyclotomic Fields I & II*, GTM 121
4.) D. Bump, *Automorphic Forms*, Cambridge U. Press
5.) Subject to availability my book: *Eisenstein Series and Automorphic L-functions*, AMS Colloquium Series

MA 69000: CRN:23442 Topics in Commutative Algebra
Instructor: Prof. Heinzer, office: Math 636, phone: 49–41980, e-mail: heinzer@math.purdue.edu
Time: MWF 3:30
Description: I hope to cover various topics in Commutative Algebra. Relevant references for these topics are the following books: *Local Rings* by Masayoshi Nagata, *Commutative Algebra* by Hideyuki Matsumura, *Commutative ring theory* by Hideyuki Matumura, *Cohen-Macaulay rings* by Winfried Bruns and Jürgen Herzog and *Integral Closure of Ideals, Rings and Modules* by Irena Swanson and Craig Huneke. There will be no specific textbook for the course.

MA 69000: CRN:23444 Introduction to Contact 3-manifolds
Instructor: Prof. Yi-Jen Lee, e-mail: yjlee@math.purdue.edu
Time: MWF 4:30
Prerequisite: Introductory Differential Geometry (MA 56200)
Description: Description: A contact manifold is an odd dimensional manifold with a "maximally non-integrable" hyperplane field. It occurs naturally as the boundary of a symplectic manifold (e.g. a Kahler manifold). Due to availability of tools, contact 3-manifolds are much bettter understood than higher dimensional ones. Having its origin from classical mechanics, contact geometry today stands in the crossroad of low dimensional topology, dynamical systems, and complex geometry. I will introduce some basic notion and tools, describe some recent advances, with emphasis on its interplay with low dimensional topology.
References: H. Geiges *Introduction to contact topology*, Cambridge University Press, 2008.

MA 69200: CRN:45593 Selected Topics on Spectral Methods for Partial Differential Equations
Instructor: Prof. Shen, office: Math 406, phone: 49–41923, e-mail: shen@math.purdue.edu
Time: MWF 8:30
Prerequisite: Special topic course: Introduction to Spectral Methods for Scientific Computing
Description: This is an advanced course on the numerical analysis of spectral methods for PDEs. The topics will include: polynomial and rational polynomial approximation theory in bounded and unbounded domains; construction of efficient spectral algorithms for PDEs in high-dimensional spaces; error analysis of spectral methods for elliptic equations, Helmholtz/Maxwell equations; miscellaneous applications.

No textbook is required. Lecture notes will be provided.

MA 69200: CRN:45590 Special Topics in Mathematical Biology
Instructor: Prof. Feng, office: Math 414, phone: 49–41915, e-mail: zfeng@math.purdue.edu
Time: MWF 9:30
Description: This course is an introduction to the application of mathematical methods and concepts to the description and analysis of biological processes. The mathematical contents consist of difference and differential equations, basic probability theory and elements of stochastic processes. The topics to be covered include dynamical systems theory motivated in terms of its relationship to biological theory, deterministic models of population processes, life history evolution, epidemiology,ecology, coevolutionary systems, structured population models, and stochastic processes. Bio-mathematical research projects (in small group) may be carried out.
References: various handouts.

MA 69200: CRN:23449 Adaptive Finite Element Method and A Posteriori Error Estimation
Instructor: Prof. Cai, office: Math 412, phone: 49–41921, e-mail: zcai@math.purdue.edu
Time: TTh 9:00-10:15
Prerequisite: CS/MA 51400 and CS/MA 61500 or equivalent or consent of instructor.
Description: All computational results for simulating physical phenomena in engineering applications and scientific predictions contain numerical error. Discretization error could be large and unpredictable by classical heuristic means and could invalidate numerical predictions. A posteriori error estimation is a rigorous mathematical theory for estimating and quantifying discretization error in terms of the error's magnitude and distribution. This information provides a basis for error control or verification and for adaptive meshing. The theory of the a posteriori error estimation has become an important area of research and has found application in an increasing number of commercial software products and scientific programs.

This is an introductory course on the a posteriori error estimation and adaptive finite element method. The course will introduce basic principles on how to design viable a posteriori error estimators and basic tools for analyzing estimators.

References:
[1] M. Ainsworth and J. T. Oden, *A Posteriori Error Estimation in Finite Element Analysis*, John Wiley & Sons, Inc., 2000.
[2] I. Babuska and T. Strouboulis, *The Finite Element Method and Its Reliability*, Oxford Science Publication, New York, 2001.
[3] R. Becker and R. Rannacher, *An optimal control approach to a posteriori error estimation in finite element methods*, Acta Numer., 1-102, 2001.
[4] R. Verfurth, *A Review of A-Posteriori Error Estimation and Adaptive Mesh Refinement Techniques*, John Wiley and Teubner Series. Advances in Numerical Mathematics., 1996.
[5] research articles.

MA 69300: CRN:45587 Riemann Mapping
Instructor: Prof. de Branges, office: Math 800, phone: 49–46057, e-mail: branges@math.purdue.edu
Time: MWF 9:30
Description: Riemann mapping is a persitent source of research in complex analysis which is now generalized to functions of a quaternion variable. The quaternion skew-plane is a skew-field rich in subfields isomorphic to the complex plane. An analytic function of a variable in the skew-plane is treated by analytic functions in subplanes. Classical theorems of complex analysis are generalized to a new context in which the classical proofs apply. The Riemann mapping theorem applies in the skew-plane as conjectured by Poincare. The structure of mapping functions and the estimates of coefficients apply the 1984 proof of the Bieberbach conjecture. Riemann mapping for a skew-plane is preparation for Fourier analysis appearing in the proof of the Riemann hypothesis. No previous experience of complex analysis is required, but doctoral students are advised to complete qualifying examinations before taking the course.

MA 69400: CRN:45588 Free Boundary Problems of Obstacle Type
Instructor: Prof. Petrosyan, office: Math 610, phone: 49–41932, e-mail: arshak@math.purdue.edu
Time: TTh 10:30-11:45
Prerequisite: MA64200, MA64300 desirable; MA52300, MA54400 absolute minimum.
Description: Free boundaries are apriori unknown sets, coming up in solutions of partial differential equations and variational problems. Typical examples are the interfaces and moving boundaries in problems on phase transitions and fluid mechanics. Main questions of interest are the regularity (smoothness) of free boundaries and their structure.

A well-known (and well-studied) example is the obstacle problem of minimizing the energy of the membrane subject to remaining above a given obstacle: the free boundary is the boundary of the contact set. The objective in this course is to give an introduction to the theory of the regularity of the free boundaries in problems of the obstacle type, pioneered in the works of Luis Caffarelli, et al.

We are going to discuss classical and more recent methods in such problems, including the optimal regularity of solutions, monotonicity formulas, classification of global solutions, geometric and energy criterions for the regularity of the free boundary, singular points.
Text: A. Petrosyan, H. Shahgholian, N. Ural'tseva *Regularity of free boundaries in obstacle-type problems*, lecture notes, unpublished

Seminars

Algebra and Algebraic Geometry Seminar, Prof. Abhyankar, **Thursday 4:30–5:45**

Applied Math Lunch Seminar, Prof. Buzzard, **Fridays 11:30**

Automorphic Forms and Representation Theory Seminar, Prof. Goldberg, **Thursdays 1:30**

Bridge to Research Seminar, Prof. Bell, **Mondays 4:30**

Commutative Algebra Seminar, Profs. Heinzer and Ulrich, **Wednesdays 4:30**

Commutative Algebra Reading Seminar, Prof. Kummini, **Tuesday 1:30**

Computational and Applied Math Seminar, Prof. Shen, **Fridays 3:30**

Function Theory Seminar, **Time To Be Determined**

Geometric Analysis Seminar, Prof. Yeung, **Monday 3:30**

Number Theory Seminar, Prof. Goins, **Thursday 3:30**

Operator Algebras Seminar, Prof. Dadarlat, **Tuesdays, 2:30**

PDE Seminar, Prof. Bauman, **Thursday, 3:30**

Probability Seminar (STAT 69100 CRN 40030), Tuesday, 3:30

Spectral and Scattering Theory Seminar, Prof. SaBarreto, **Thursday 4:30**

Symplectic Geometry Seminar, Profs. Albers and Lee

Topology Seminar, Profs. Kaufmann and McClure, **Thursdays 3:30**

Working Algebraic Geometry Seminar, Profs. Arapura and Matsuki, **Wednesday 3:30**