# Seminars and Advanced Graduate Courses
# offered by the
# Mathematics Department
# Fall, 2011

## Courses

**MA 55700: CRN:23413 TITLE**
**Instructor:** Prof. Heinzer, office: Math 636, phone: 49–41980, e-mail: heinzer@math.purdue.edu
**Time:** MWF 3:30
**Description:** I plan to cover material from the text *Introduction to Commutative Algebra* by M. F. Atiyah and I. G. Macdonald. In particular, the course will cover: Rings and Ideals, Modules, Rings and Modules of Fractions, Primary Decomposition, Integral Dependence and Valuations, Chain Conditions, Noetherian Rings, Artinian Rings, Discrete Valuation Rings and Dedekind Domains, Completions and Dimension Theory. I hope to encourage active student participation in the class by having students present in class exercises and/or selected material from the text.

**MA 59800: CRN:45583 Geometric Measure Theory**
**Instructor:** Prof. Donatella Danielli, office: Math 620, phone: 49–41920, e-mail: daneilli@math.purdue.edu
**Time:** MWF 11:30
**Prerequisite:** MA 54400, or equivalent (with instructor's consent)
**Description:** The purpose of this course is to provide an overview of an analyst's essential toolbox. We will explore the measure theoretic structure of $R^n$, with particular emphasis on integration and differentiation. Topics presented in the class will include: Hausdorff measure; Rademacher's Theorem (asserting the a.e. differentiability of Lipschitz functions); Area and Coarea Formulas (which yield change-of-variable rules); Traces, extensions, and pointwise properties of Sobolev and BV functions; Capacity; Isoperimetric inequalities; The reduced and the measure theoretic boundary; Alexandrov's Theorem (asserting the twice differentiability of convex functions a.e.).
**Text:** L.C. Evans and R.F. Gariepy, *Measure Theory and Fine Properties of Functions,* CRC Press.

**MA 59800: CRN:23431 Elliptic Curves and Cryptography**
**Instructor:** Prof. Goins, office: Math 612, phone: 49–41936, e-mail: egoins@math.purdue.edu
**Time:** MWF 12:30
**Prerequisite:** MA 45300, MA 50300, or an equivalent.
**Description:** An elliptic curve $E$ is an arithmetic-algebraic object: It is simultaneously a nonsingular projective curve with an affine equation, which allows one to perform arithmetic on its points; and a finitely generated abelian group, which allows one to apply results from abstract algebra. The goal of this course will be to learn about elliptic curves and give applications by using their properties to study problems in cryptography.

There are two aspects when discussing secrets: how to safely encrypt a message so that an unauthorized party cannot read it, and how to effectively decrypt a message so that an authorized party can. The method of encryption using elliptic curves is a type of Public Key cryptosystem, an idea first put forth by Whitfield Diffie and Martin Hellman in 1976. One takes a message, expresses each character as a positive integer (using say UTF-8), then encodes the message as a positive integer. One can then perform mathematics on this integer using a shared elliptic curve, and then send this encrypted message to a friend. This process is known as Elliptic Curve Diffie-Hellman (ECDH). The method of decrypting using elliptic curves is a variant of Pollard's $(p-1)$ Algorithm, an idea first put forth by Hendrik Lenstra in 1987. One wishes to solve the Elliptic Curve Discrete Logarithm Problem (ECDLP) by factoring an integer using elliptic curves. One chooses a random point on a random elliptic curve, then uses the group law to eventually find a factor which allows one to invert the discrete logarithms. This process is known as Elliptic Curve Factorization Method (ECM).

In this course, we will begin with the basic definition of elliptic curves, present the chord-tangent construction of the group law, and discuss on properties of curves over finite fields. We will then focus on applications to questions in cryptography, such as the discrete logarithm problem, Diffie-Hellman key exchange, and factorization methods. Previous knowledge of the arithmetic of elliptic curves is not necessary, but knowledge of abstract algebra is a prerequisite.
**Texts** 1. L. Washington's *Elliptic Curves: Number Theory and Cryptography*
2. Cohen and Frey's *Handbook of Elliptic and Hyperelliptic Curve Cryptography.*
3. Stein's *Elementary Number Theory: Primes, Congruences, and Secrets* will be used as a supplemental textbook.

**MA 59800: CRN:23437 Introduction to Algebraic Geometry with Focus on Cohomology and Theory of Algebraic Curves**
**Instructor:** Prof. Matsuki, office: Math 614, phone: 49–41970, e-mail: kmatsuki@math.purdue.edu
**Time:** MWF 1:30
**Prerequisite:** PREREQUISITE We use the language of schemes and sheaves. However, one just needs the minimum to understand the cohomology theory and the theory of curves. One does not have to have the extensive knowledge as discussed in Chapters I and II of *Algebraic Geometry* by Hartshorne. Some familiarity with the basics of complex manifolds and theory of Riemann surfaces will help, but not necessary. What is required is your willingness to learn.
**Description:** The purpose of this course is to give an introduction to the subject of algebraic geometry, focused on the following two topics:

- cohomology, and
- theory of algebraic curves. In the introductory course in algebraic geometry run in the semester of Fall 2010, we roughly covered Chapters I and II of the textbook *Algebraic Geometry* by Hartshorne. Since the above two topics are the titles of Chapters III and IV of the same textbook, our course could be considered a continuation of the course in Fall 2010. However, cohomology and the theory of curves can be learned with the minimum knowledge of schemes and sheaves. Therefore, we try to provide the material digestible to the students who did not take the course in Fall 2010 so that they can learn the basics of the above two topics almost from scratch. The subject matters in the theory of curves covered in Chapter IV of *Algebraic Geometry* by Hartshorne is close to the absolute minimum. We would like to go slightly deeper and broader by using *Geometry of Algebraic Curves* volume I written by 4 authors.

The student participation is a vital part of this course. Every other Friday is designated as the problem session, where the students present their solutions to the assigned exercise problems. The students' solutions presented in Fall 2010 were much better than the ones I anticipated or prepared for myself, and we had fun. I expect we will have much fun again learning algebraic geometry by getting our hands dirty and not by just attending lectures and/or reading books.
**Text:** 1. R. Hartshorne, *Algebraic Geometry*, GTM 52 Springer
2. E. Arbarello, M. Cornalba, P. A. Griffiths, J. Harris, *Geometry of Algebraic Curves*, volume 1, A series of comprehensive studies in mathematics 267.

**MA 59800: CRN:23432 Zeta Functions and Quadratic Number Fields**
**Instructor:** Prof. Lipman, office: Math 750, phone: 49–41994, e-mail: lipman@math.purdue.edu
**Time:** MWF 9:30
**Prerequisite:** MA 55300, MA 53000
**Description:** The theory of quadratic number fields, established in the first half of the nineteenth century, is historically one of the cornerstones of the subsequent more abstract theory of algebraic numbers, and can still serve as a strong motivator for study of the latter. Some of the timeless results of Gauss and Dirichlet (inspired by Fermat, Euler, Legendre, Lagrange) will be treated by means of both analytic and algebraic methods.

Topics: Arithmetic in quadratic number fields, application to Diophantine equations such as $x^3 + y^3 = z^3$ (no solutions with $xyz \neq 0$), $x^2 + 2 = y^3$ (just one solution in positive integers), and representation of integers as $x^2 + ny^2$ for small $n$; Dirichlet series, Riemann's zeta function, infinitude of primes in an arithmetic progression; binary quadratic forms and their relation to ideals in quadratic number fields, equivalence classes, binary quadratic representation of integers, class number formula, genus theory.

**MA 59800: CRN:52201 The Radon and the X-Ray Transforms**
**Instructor:** Prof. Stefanov, office: Math 448, phone: 49–67339, e-mail: stefanov@math.purdue.edu
**Time:** MWF 3:30
**Description:** This course is intended as an introduction to Integral Geometry and X- ray tomography in the Euclidean space $R^n$. We will study in detail the integral transforms of functions or distributions over lines or planes. The main topics that will be covered are the proper way to define them in the appropriate distribution spaces, their mapping properties, injectivity, invertibility, reconstruction formulas, stability theorems, support theorems, and range conditions. The X-ray transforms of vector fields and tensor fields will be considered, too. The attenuated X-ray transform and its inversion will be covered, as well. We will begin with a review of distributions and some facts about homogeneous distributions.

The Radon and the X-ray transform are an essential tool in many medical imaging methods. They also appear in other fields.

Prerequisites include theory of distributions and the Fourier Transform. We will follow the Helgason and the Natterer books; as well as instructor's notes that will be distributed among the students.
**References:** S. Helgason, *The Radon Transform*, Birkhauser Boston, 1999; free downloadable version available on the author's home page
**Text:** F. Natterer, *The Mathematics of Computerized Tomography*, John Wiley, 1986

**MA 59800: CRN:52202 Application of Weak Convergence in Nonlinear Partial Differential Equations**
**Instructor:** Prof. Yip, office: Math 432, phone: 49–41941, e-mail: yip@math.purdue.edu
**Time:** MWF 9:30
**Prerequisite:** MA 54400 (real analysis) and MA 52300 (partial differential equations, preferred)
**Description:** This course is an introduction to some "versatile" techniques in the study of nonlinear partial differential equations. The emphasis will be on elliptic equations with a variational structure. They arise in many applications such as minimal surfaces, elasticity, and homogenization. Tentative topics include:
(1) direct method of calculus of variations;
(2) compensated compactness;
(3) concentration compactness;
(4) Gamma-convergence.
The main purpose is to see how "real analysis" and "linear theory" are used in nonlinear situations.

This course is intended for students with interests in analysis and applications.
**Text:** No official textbooks. But I will basically follow:
(1) Lawrence C. Evans: Weak *Convergence Methods for Nonlinear Partial Differential Equations* (available in library)
(2) Michael Struwe: *Variational Methods* (Chapter I) (available online through Purdue)
(3) selected journal papers.

**MA 59800: CRN:52203 Elliptic Curves**
**Instructor:** Prof. Jiu-Kang Yu, office: Math 604, phone: 49–67414, e-mail: jyu@math.purdue.edu
**Time:** MWF 10:30 **NOTE NEW TIME**
**Prerequisite:** Experience with algebraic number theory or algebraic curves
**Description:** Elliptic curves are almost the simplest kind of algebraic curves, yet the number theory of elliptic curves is amazingly rich and an active area of research. In this introductory course, we will cover: the geometry of elliptic curves, elliptic functions, elliptic curves over a finite field or a local field, Neron models, the Mordell-Weil theorem.
**Text:** J. Silverman, *Arithmetic of elliptic curves*, GTM 106, Springer-Verlag

**MA 59800: CRN:52204 Introduction to p-adic Galois Representation**
**Instructor:** Prof. Tong Liu, phone: 49–41946, e-mail: tongliu@math.purdue.edu
**Time:** TTh 10:30-11:45
**Prerequisite:** MA58400
**Description:** This course provides the rudiments of theory of p-adic Galois representations. I plan to start with basic propeties of p-adic representations then cover the following topics: l-adic representations and Grothendiecks l-adic monodromy Theorem, C-representations and Sen's theory, semi-stable representations and Fontaine's theory. If time allows, we will discuss some topics of integral p-adic Hodge theory and Fontaine-Mazur conjecture.
**References:** Jean-Marc Fontaine and Yi Ouyang *Theory of p-adic Galois representations*
**Text:** Jean Pierre Serre *Abelian l-adic Representations and Elliptic Curves*

**MA 59800: CRN:52205 Geometric Measure Theory and Conservation Laws**
**Instructor:** Prof. Torres, office: Math 634, phone: 49–41969, e-mail: torres@math.purdue.edu
**Time:** MWF 2:30
**Prerequisite:** This class is most suitable for graduate students who have already taken MA54400 or equivalent.
**Description:** In this class we will present an introduction to geometric measure theory including Hausdorff measures, rectifiable sets, coarea formulas, the analysis of fine properties of functions of bounded variation, sets of finite perimeter, etc. In the second part of the class, we will study divergence measure fields and the corresponding Gauss-Green formulas. Finally, we will connect the theory of geometric measures to the study of hyperbolic conservation laws. We will discuss some known results and will present some open questions in the field.
**References:** 1. Ambrosio, Fusco and Pallara, *Functions of Bounded Variation and Free Discontinuity Problems*
2. Evans and Gariepy, *Measure Theory and Fine Properties of Functions*
3. Giusti, *Minimal Surfaces and Functions of Bounded Variation*
4. Ziemer, *Weakly Differentiable Functions*

**MA 64200: CRN:38721 Methods of Linear and Nonlinear Partial Differential Equations I**
**Instructor:** Prof. Nicola Garofalo, office: Math 616, phone: 49–41971, e-mail: garofalo@math.purdue.edu
**Time:** TTh 1:30-2:45
**Description:** This course will focus on equations of elliptic and parabolic type. Prerequisites for this course are a knowledge of Lebesgue integration theory, an introductory course in pde's, such as for instance MA 52300, and the basic facts about Sobolev spaces, such as the material covered in MA 54500.

The content of the course will be as follows:

1. Start with a discussion of various maximum principles for elliptic and parabolic equations. The presentation will include a review of the classical method of Perron-Wiener-Brelot for solving the Dirichlet problem.
2. The problem of regularity of boundary points in the Dirichlet problem for both elliptic and parabolic equations. Wiener type criteria.
3. Elliptic and parabolic equations in variational form. Local a priori estimates for weak solutions.
4. Regularity theory: Nirenberg's method of difference quotients
5. Schauder theory
6. Estimates in $L^p$ spaces
7. De Giorgi's solution of Hilbert's XIX problem for elliptic equations
8. Nash's approach for parabolic equations
9. Harnack inequalities and Gaussian estimates
10. Elliptic and parabolic equations in nonvariational form
11. The Alexandrov-Bakelman-Pucci maximum principle and its parabolic counterpart
12. If time allows we will discuss viscosity solutions to nonlinear pde's

**MA 65000: CRN:52206 Commutative Algebra**
**Instructor:** Prof. Ulrich, office: Math 618, phone: 49–41972, e-mail: ulrich@math.purdue.edu
**Time:** MWF 3:30
**Prerequisite:** Basic knowledge of commutative algebra (such as the material of MA 55700/55800).
**Description:** Description: This is an intermediate course in commutative algebra. The course is a continuation of MA 557/558, but should be accessible to any student with basic knowledge in commutative algebra (localization, Noetherian and Artinian modules, associated primes and primary decomposition, integral extensions, dimension theory, completion, Ext and Tor).

The topics of this semester will include: Graded rings and modules, depth and Cohen-Macaulayness, structure of finite free resolutions, regular rings and normal rings, canonical modules and Gorenstein rings, local cohomology and local duality.

**Texts:** No specific text will be used, but possible references are:
- H. Matsumura, *Commutative ring theory*, Cambridge University Press
- W. Bruns and J. Herzog, *Cohen-Macaulay rings*, Cambridge University Press
- D. Eisenbud, *Commutative algebra with a view toward algebraic geometry*, Springer.

**MA 66500: CRN:53577 Algebraic Geometry**
**Instructor:** Prof. Abhyankar, office: Math 600, phone: 49–41933, e-mail: ram@math.purdue.edu
**Time:** TTh 3:00-4:15
**Description:** Algebraic geometry is concerned with solutions of systems of polynomial equations, and their graphical representations. This is aided by field theory, ideal theory, valuation theory, and local algebra. In the complex domain, analysis and topology also play a significant role. This course is intended as an introduction to various topics in algebraic geometry such as:

- Analysis and resolution of singularities
- Rational and polynomial parametrization
- Intersections of curves and surfaces
- Polynomial maps
- Fundamental Groups and Galois groups

The lectures will be expository in nature and so will be accessible to everyone. Thus there are no formal prerequisites and all interested students are welcome. In particular the required algebra, analysis, and topology will be developed simultaneously. The course will continue with its second part in the Spring.
**Texts:** 1. Shreeram S. Abhyankar, *Algebraic Geometry for Scientists and Engineers*, Amer Math Soc
2. Shreeram S. Abhyankar, *Ramification Theoretic Methods in Algebraic Geometry*, Princeton University Press

**MA 69000: CRN:45591 Polytops, Rings and K-Theory**
**Instructor:** Prof. Caviglia, office: Math 608, phone: 49–41973, e-mail: gcavigli@math.purdue.edu
**Time:** TTh 1:30-2:45
**Description:** I am planning to discuss Part 1) and part 2) of the following book by Bruns and Gubalaze: *Polytopes, Rings, and K-Theory*, Volume 978, Issues 0-76352 The course is intended for gradute students interested in commutative algebra. Knowledge of the material covered in 55700 and 55800 will be helpful although not strictly required.

**MA 69000: CRN:23442 Introduction to Finite Element Methods for Maxwell's Equation**
**Instructor:** Prof. Peijun Li, office: Math 440, phone: 49–40846, e-mail: lipeijun@math.purdue.edu
**Time:** TTh 12:00-1:15 **NOTE NEW TIME**
**Prerequisite:** Basic knowledge of functional and numerical analysis, and partial differential equations.
**Description:** Over the last two decades, the dramatic growth of computational capability and the de- velopment of fast algorithms have transformed the methodology for scientic investigation and industrial applications in the field of computational electromagnetism. Reciprocally, the practical applications and scientic developments have driven the need for more advanced mathematical models and numerical meth- ods to describe the scattering of complicated structures, and to compute electromagnetic fields and thus to predict the performance of a given structure, as well as to carry out optimal design of new structures. The aim of this course is to provide an up-to-date and sound theoretical foundation for finite element methods in computational electromagnetism. The emphasis is on finite element methods for a class of scattering problems that involve the solution of Maxwell's equations.

The course is intended to be self-contained and will provide introductory material to the areas in electromagnetism that offer rich and challenging mathematical problems. Tentative topics will include: Sobolev spaces, conforming finite edge elements, variational formulations, finite element approximations, adaptivity with a posterior error estimates, perfectly matched layer techniques, and inverse scattering. As applications, the following problems will be covered: biperiodic grating problem, cavity scattering problem, exterior or open domain scattering problem, unbounded rough surface scattering problem, and inverse medium scattering problem.
**Text:** No textbook is required. Lecture notes will be made available to students.
Course grade: No exams. Course grades will be based on homework assignments and projects.
**References:** 1. P. Monk, *Finite Element Methods for Maxwell's Equations*
2. J. Jin, *The Finite Element Method in Electromagnetics*
3. J.-C. Nedelec, *Acoustic and Electromagnetic Equations: Integral Representations for Harmonic Problems*
4. D. Colton and R. Kress, *Inverse Acoustic and Electromagnetic Scattering Theory*

**MA 69200: CRN:45590 Special Topics in Mathematical Biology**
**Instructor:** Prof. Feng, office: Math 414, phone: 49–41915, e- mail: zfeng@math.purdue.edu
**Time:** TTh 1:30-2:45
**Description:** This course is an introduction to the application of mathematical methods and concepts to the description and analysis of biological processes. The mathematical contents consist of difference and differential equations and elements of stochastic processes. The topics to be covered include dynamical systems theory motivated in terms of its relationship to biological theory, deterministic models of population processes, epidemiology, coevolutionary systems, structured population models, nonlinear dynamics, and stochastic simulations. Bio-mathematical research projects (in small group) may be carried out.
**References:** 1.) Brauer and Castillo-Chavez *Mathematical Moldes in Population Biology and Epidemiology* (optional);
2.) Kot *Elements of Mathematical Ecology* (optional);
3.) Thieme *Mathematics in Population Biology* (optional)
**Text:** *Class notes, Handouts and Articles*

**MA 69200: CRN:45593 Topics in Structured Matrix Computations**
**Instructor:** Prof. Xia, office: Math 442, phone: 49–41922, e-mail: xiaj@math.purdue.edu
**Time: NOTE NEW TIME** TTh 10:30-11:45
**Prerequisite:** numerical linear algebra or similar, or CS 51500, or consent of instructor
**Description:** This course is intended for graduate students with basic knowledge in matrix computations and also interested in doing research in numerical algebra. The course studies the theoretical background of various structures arising in mathematics and engineering, and discusses the solution techniques. We focus on structures such as Toeplitz, semiseparable, hierarchical, and polynomial representations. Several important structured techniques are discussed, including the fast multipole method (FMM), treecode algorithms, the multilevel summation method (MSM), fast HSS methods, and structured multifrontal algorithms. The application of the methods to PDE solution, imaging, optimization, control, electrical engineering, etc. will also be shown.
**References:** 1. Olshevsky, *Fast Algorithms for Structured Matrices, Theory and Applications*, AMS.
2. Vandebril, Van Barel, and Mastronardi, *Matrix Computations and Semiseparable Matrices*, volume 1-2, Johns Hopkins
3. Pan, *Structured Matrices and Polynomials: Unified Superfast Algorithms*, Birkhauser Boston.
**Text:** lecture notes and research papers

**MA 69300: CRN:45587 Conformal Mapping**
**Instructor:** Prof. de Branges, office: Math 800, phone: 49–46057, e-mail: branges@math.purdue.edu
**Time:** MWF 9:30
**Description:** The proof in 1984 of the Bieberbach conjecture is an estimation theory for injective analytic functions applied in the Riemann mapping theorem. The proof is an outgrowth of a theory of square summable power series which has been successful in teaching complex analysis in its relationship to Hilbert space. The original text of *Square Summable Power Series*, published in 1966 with James Rovnyak, has been updated to include the application to the proof of the Bieberbach conjecture. A second course in complex analysis is offered to students who have passed qualifying examinations and would like to write a doctoral thesis in complex analysis.

**MA 69700: CRN:52209 Cobordisms, Genera, Characteristic Classes and Toplogical K-Theory**
**Instructor:** Prof. Ralph Kaufmann, office: Math 710, phone: 49–41205, e-mail: rkaufman@math.purdue.edu
**Time:** MWF 2:30
**Description:** In this course we will discuss several generalizations of cohomology theories. We will start with the theory of cobordisms and discuss its relation to topological field theories. A related topic which we will treat are characteristic classes and numbers. We then go on to discuss general, with an emphasis on the elliptic genus and the Witten genus. Finally we will treat topological K-theory.

## Seminars

**Algebra and Algebraic Geometry Seminar,** Prof. Abhyankar, **Thursday 4:30–6:00**

**Applied Math Lunch Seminar,** Prof. Buzzard, **Fridays 11:30**

**Automorphic Forms and Representation Theory Seminar,** Profs. Goldberg and Yu, **Thursdays 1:30**

**Bridge to Research Seminar,** Prof. Bell, **Mondays 4:30**

**Commutative Algebra Seminar,** Profs. Heinzer and Ulrich, **Wednesdays 4:30**

**Computational and Applied Math Seminar,** Prof. Shen, **Fridays 3:30**

**Function Theory Seminar**, **Time To Be Determined**

**Geometric Analysis Seminar,** Prof. Lempert, **Monday 3:30**

**Number Theory Seminar,** Prof. Goins, **Thursday 3:30**

**Operator Algebras Seminar,** Prof. Dadarlat, **Tuesdays, 2:30**

**PDE Seminar,** Prof. Bauman, **Thursday, 3:30**

**Spectral and Scattering Theory Seminar,** Prof. SaBarreto, **Wednesday 4:30**

**Topology Seminar,** Profs. Kaufmann and McClure, **Thursdays 3:30 - 5:00**

**Working Algebraic Geometry Seminar,** Profs. Arapura and Matsuki, **Wednesday 3:30**