INTRODUCTION TO FUNCTIONAL ANALYSIS

Instructor: Prof. Antônio Sá Barreto (sbarre@math.purdue.edu, 4-1909)
Course Number: MA 54600 CRN: 23409
Credits: Three
Time: MWF 9:30am-10:20am

Description

Main Topics: Fundamentals of functional analysis: Hilbert Spaces, Banach spaces, Fréchet Spaces. Bounded operators, the principle of uniform boundedness, the closed graph and open mapping theorems, the Hahn-Banach theorem. Compact operators and analytic Fredholm theory, the spectral theorem for bounded self-adjoint operators. Unbounded operators, self-adjointness and the spectral theorem for unbounded self-adjoint operators. Prerequisites: MA544. Some basic knowledge of complex analysis (holomorphic and meromorphic functions) at the undergraduate level is also important.

References

1. E.B. Davies. Spectral theory and differential operators. Cambridge Studies in Advanced Mathematics, 42. Cambridge University Press, Cambridge, 1995.
2. M. Reed and B. Simon. Functional Analysis, Volume 1. Academic press

ABSTRACT ALGEBRA I

Instructor: Prof. Shreeram S. Abhyankar (ram@math.purdue.edu, 4-1933)
Course Number: MA 55700 CRN: 23413
Credits: Three
Time: TTh 3:00pm-4:15pm

Description

This course and its Spring sequel MA55800 will cover the basic algebra needed for studying advanced algebra, algebraic geometry, number theory, complex analysis, and applications to computer aided design and geometric modelling. I shall use my book Lectures on Algebra Volume I (published by World Scientific) as a text-book which the students are expected to buy. The students may also find it desirable to read my user-friendly book Algebraic Geometry For Scientists And Engineers (published by American Mathematical Society). As a useful supplement to the course, the students are strongly urged to participate in my Algebra and Algebraic Geometry Seminar meeting at 4:30 PM on Thursdays. Here is a list of possible topics to be dealt with:

1. Group Theory including Sylow Theorem and Burnside's Theorem.

2. Rings and Modules incluing, Euclidean Domains, Principal Ideal Domains, and Unique factorization Domains.
3. Fundamental Theorems of Galois Theory.
4. Polynomials and Power series incluing Hensel's Lemma and Newton's Theorem as well as the Preparaton Theorem of Weierstrass.
5. Valuation Theory and Integral Dependence.
6. Resultants and Discriminants leading to solutions of higher degree polynomial equations in several variables.
7. Primary decomposition in noetherian rings and noetherian modules.
8. Artinian rings and lengths of modules.
9. Local Rings and Graded Rings.
10. Algebraic Varieties including their spectral and modelic versions.
11. Hilbert Nullstellensatz and Noether Normaliztion.
12. Cohen-Macaulay Rings and Gorenstein Rings.
13. Hibert Syzygies and Unique Factorization in Regular Local Rings.
14. Resolution of Singularities by means of Quadratic and Monoidal Transformations.

References

1. Lectures on Algebra Volume I by Shreeram S. Abhyankar, Published by World Scientific
2. Algebraic Geometry for Scientists and Engineers by Shreeram S. Abhyankar, Published by Amer Math Soc

A POSTERIORI ERROR ESTIMATION AND ADAPTIVE FINITE ELEMENT METHODS

Instructor: Prof. Zhiqiang Cai (zcai@math.purdue.edu, 4-1921)
Course Number: MA 59800 CRN: 45583
Credits: Three
Time: MWF 8:30am-9:20am

Description

All computational results for simulating physical phenomena in engineering applications and scientific predictions contain numerical error. Discretization error could be large and unpredictable by classical heuristic means and could invalidate numerical predictions. A posteriori error estimation is a rigorous mathematical theory for estimating and quantifying discretization error in terms of the error's magnitude and distribution. This information provides a basis for error control or verification and for adaptive meshing. The theory of the a posteriori error estimation has become an important area of research and has found application in an increasing number of commercial software products and scientific programs.

This is an introductory course on the a posteriori error estimation and adaptive finite element method. The course will introduce basic principles on how to design viable a posteriori error estimators and basic tools for analyzing estimators.

Prerequisite: CS/MA 514 and CS/MA 615 or equivalent or consent of instructor.

References

1. M. Ainsworth and J. T. Oden, A Posteriori Error Estimation in Finite Element Analysis, John Wiley and Sons, Inc., 2000.
2. I. Babuska and T. Strouboulis, The Finite Element Method and Its Reliability, Oxford Science Publication, New York, 2001.
3. R. Becker and R. Rannacher, An optimal control approach to a posteriori error estimation in finite element methods, Acta Numer., 1-102, 2001.
4. R. Verfurth, A Review of A-Posteriori Error Estimation and Adaptive Mesh Refinement Techniques, John Wiley and Teubner Series. Advances in Numerical Mathematics., 1996.
5. research articles.

INTRODUCTION TO THE BASIC THEORY OF ALGEBRAIC CURVES AND BEYOND

Instructor: Prof. Kenji Matsuki (kmatsuki@math.purdue.edu, 4-1970)
Course Number: MA 59800 CRN: 23432
Credits: Three
Time: MWF 11:30am-12:20am

Description

The purpose of this course is to give an introduction to the basic theory of algebraic curves, and to discuss some more advanced topics as the time allows.

We will start with Chapter IV of Hartshorne, which covers the basic theory of algebraic curves: Riemann-Roch Theorem, Projective Embedding, Elliptic

Curves, Canonical Embedding etc. Even though it is Chapter IV, one does NOT have to know the material of the previous chapters extensively. Actually if one takes it as "a black box", he can understand most of the essential ingredients of Chapter 4. Chapters I, II, and III are rather tasteless, if you are reading Hartshorne for the first time. Only after you find how they are applied to the theory of curves, you learn the motivation behind Chapters I, II,and III ... So my hidden goal is to provide that motivation even to those who have not read Chapters I, II, or III.

Then we would like to go slightly deeper, using "Geometry of Algebraic Curves volume I" written by 4 authors. We would like to cover the topics such as: Jacobian of a curve, Theta functions, Brill-Noether theory, and possibly Moduli of curves.

Prerequisite: We use the language of schemes and sheaves. However, one just needs the minimum to start understanding the theory of curves. One does not have to have the extensive knowledge as discussed in Chapters I, II, and III of "Algebraic Geometry" by Hartshorne. Some familiarity with the basics of complex manifolds and theory of Riemann surfaces will help, but not necessary. What is required is your willingness and enthusiasm to learn.

References

1. Algebraic Geometry by R. Hartshorne, GTM 52 Springer
2. Geometry of Algebraic Curves Volume I by E. Arbarello, M. Cornalba, P.A. Griffiths, J. Harris, A series of comprehensive studies in mathematics 267

TAME GEOMETRY

Instructor: Prof. Andrei Gabrielov (agabriel@math.purdue.edu, 4-7911)
Course Number: MA 59800 CRN: 23431
Credits: Three
Time: MWF 2:30am-3:20am

Description

In his famous (rejected) proposal "Esquisse d'un Programme" (1984) Alexander Grothendieck argued that "tame topology" suitable for algebraic geometry, as opposed to analysis, should be developed. Such a theory should not allow "wild" geometric objects, such as Cantor sets and nowhere continuous functions. The basic (and most important in applications) example of such a theory is real semialgebraic geometry. A general setting for tame geometry and topology is provided by o-minimal theory, based on interactions between geometry, topology, and mathematical logic. It has many applications in mathematics (including analysis) and other areas, such as robotics, control theory, and computational complexity. An introduction to real semialgebraic geometry and o-minimal theory will be given, based mainly on "Tame topology and o-minimal structures" by Lou van den Dries.

Prerequisites: Basic set-theoretic and algebraic topology. No preliminary knowledge of mathematical logic or real algebraic geometry is expected.

References

1. Lou van den Dries, Tame topology and o-minimal structures, Cambridge University Press, 1998.

2. Michel Coste, An introduction to semialgebraic geometry; An introduction to o-minimal geometry (Lecture notes downloadable from the Web).

APPLIED INVERSE PROBLEMS

Instructor: Prof. Maarten De Hoop (mdehoop@math.purdue.edu, 6-6439)
Course Number: MA 59800 CRN: 59251
Credits: Three
Time: TTh 12:00pm-1:20pm

Description

References

STOCHASTIC INTEGRATION AND STOCHASTIC DIFFERENTIAL EQUATIONS WITH A VIEW TOWARD FINANCIAL MATHEMATICS AND PDE THEORY

Instructor: Prof. Fabrice Baudoin (fbaudoin@math.purdue.edu, 4-1406)
Course Number: MA 59800 CRN: 52201
Credits: Three
Time: MWF 11:30am-12:20pm

Description

This course is intended to provide a rigorous account on stochastic calculus and some of its applications. The topics to be covered include:

1. Reminder about Brownian motion
2. Stochastic Integration
3. Semimartingales
4. Ito's formula
5. Stochastic differential equations
6. Arbitrage free theory

7. Diffusion processes

Prerequisites for this class are basic probability theory and basic theory of stochastic processes as taught in MA538/MA539 or MA532.

References

1. Lecture Notes written by the instructor will be made available to the students

2. Continuous martingales and Brownian motion by D. Revuz and M. Yor.

SEMICLASICAL MICROLOCAL ANALYSIS

Instructor: Prof. Plamen Stefanov (stefanov@math.purdue.edu, 6-7330)
Course Number: MA 59800 CRN: 23437
Credits: Three
Time: MWF 1:30pm-2:20pm

Description

This course is an introduction into Semiclassical (SC) Microlocal Analysis. We will introduce the SC Fourier Transform, and the SC pseudo-differential operators calculus. If the time permits, an introduction to the theory of the SC Fourier Integral Operators will be presented.

Semiclassical (Microlocal) Analysis, roughly speaking, provides PDE tools based on the classical-quantum (particle-wave) correspondence. It is microlocal in nature, in the sense that it studies distributions in the phase space. Important applications and a motivation for its existence is Quantum Mechanics, and spectral asymptotics. It is closely related to the "standard" microlocal analysis. The students are expected to have basic knowledge of PDEs and Functional Analysis, including distributions and Fourier Transform. We will follow closely the online book by M. Zworski.

References

1. Maciej Zworski, Semiclassical Analysis, available at
http://math.berkeley.edu/ zworski/semiclassical.pdf

REPRESENTATION THEORY OF REAL LIE GROUPS

Instructor: Prof. Freydoon Shahidi (shahidi@math.purdue.edu, 4-1917)
Course Number: MA 59800 CRN: 58565
Credits: Three
Time: MWF 9:30pm-10:20am

Description

This course is aimed to cover some of the basic aspects of representation theory of real Lie groups: Some examples of representations of low rank groups, a brief survey of representation of compact Lie groups, Universal enveloping algebra and C1vectors, examples of discrete series, Induced representations, Admissible representations. Prerequisite: Some knowledge of manifolds, tangent spaces, Lie groups and Lie algebras.

References

1. Text: A. Knapp; Representation Theory of Semisimple Groups, Princeton University Press, 1986.
2. Reference for prereqs and compact groups: T. Brocker and T. Dieck. Representations of Compact Lie Groups, GTM98, Springer, 2003.

METHODS OF LINEAR AND NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS I

Instructor: Prof. Donatella Danielli (danielli@math.purdue.edu, 4-1920)
Course Number: MA 64200 CRN: 38721
Credits: Three
Time: MWF 12:30pm-1:20pm

Description

Prerequisite: MA 54400 and MA 61100, or instructor's approval. Description: This is the first semester of a one-year course in the theory of second order elliptic and parabolic PDEs. The aim of the course is to study the solvability of boundary value problems and regularity properties of solutions. The first semester will focus on linear elliptic equations, both in divergence and non-divergence form. The starting point for the study of classical solutions will be the theory of Laplace's and Poisson's equations. The emphasis here will be on: 1. Existence of solutions to the Dirichlet problem for harmonic functions via the Perron method (based on the maximum principle); 2. Holder estimates for Poisson's equation derived from the analysis of the Newtonian potential. The crowning achievement of the theory of classical solutions is Schauder's theory, which extends the results of potential theory to a general class of non-divergence form equations with Holder-continuous coefficients. In the second part of the semester we will consider a more general - and modern - approach to linear problems, based not on potential theory, but on Hilbert space methods for so-called "weak" solutions. Our main goal will be to prove the celebrated De Giorgi-Nash-Moser theorem on the regularity of weak solutions. The relevant tools from the theory of Sobolev spaces will be developed concurrently.

References

1. D. Gilbarg and N. S. Trudinger, Elliptic Partial Differential Equations of Second Order, Second Edition.

CARDINALITY

Instructor: Prof. Louis De Branges (branges@math.purdue.edu, 4-6057)
Course Number: MA 69000 CRN: 58345
Credits: Three
Time: MWF 9:30am10:20am

Description

Cardinality, the comparability of sets, is a fundamental hypothesis of mathematical analysis which is usually stated in the equivaent form of the axiom of choice. An argument due to Cantor shows that the cardinality of a set is less than the cardinality of the class of its subsets. The existence of infinite sets which are not countable results. Such sets are fundamental to mathematical analysis, for example in proving that a Cartesian product of compact sets is compact. The course is offered to graduate students who would like to know more about cardinality (including research issues) than they have learned in preparation for qualifying examinations.

APPLICATIONS OF MATHEMATICS IN EPIDEMIOLOGY AND ECOLOGY

Instructor: Prof. Zhilan Feng (zfeng@math.purdue.edu, 4-1915)
Course Number: MA 69200 CRN: 45590
Credits: Three
Time: MWF 3:30pm-4:20pm

Description

This special topic course focuses on recent advances in modeling studies for biological systems including both mathematical methods and modeling approaches and frameworks. The mathematical contents consist of difference and differential equations and elements of stochastic processes. The topics to be covered include dynamical systems theory motivated in terms of its relationship to biological theory, deterministic models of population processes, epidemiology, coevolutionary systems, structured population models, nonlinear dynamics, and stochastic simulations. Bio-mathematical research projects (in small group) may be carried out.

References

1. Class notes, Handouts and Articles
2. Brauer and Castillo-Chavez, Mathematical Moldes in Population Biology and Epidemiology (optional)
3. Kot, Elements of Mathematical Ecology (optional)
4. Thieme, Mathematics in Population Biology (optional)

TOPICS ON SPARSE AND STRUCTURED MATRIX COMPUTATIONS

Instructor: Prof. Jianlin Xia (xiaj@math.purdue.edu, 4-1922)
Course Number: MA 69200 CRN: 45593
Credits: Three
Time: TTh 10:30am-11:45am

Description

This course includes both introductions to sparse matrix computations and advanced sparse and structured matrix theories and techniques. Classical direct and iterative solvers for sparse linear systems, least squares, and eigenvalue problems are reviewed, including related graph theory and convergence analysis. Topics related to the fast multipole method and semiseparable matrices are discussed in detail. Rank structures and modern fast structured solvers are shown, including recent developments on randomized techniques. Their applications to PDE solutions, imaging problems, etc. are also included.

Prerequisite: Numerical linear algebra or similar, Math 514, CS 51500, or consent of instructor

References

1. Lecture notes, handouts, slides, and research papers
2. Steffen Boerm, Efficient Numerical Methods for Non-Local operators, H^2-matrix compression, algorithms and analysis, European Mathematical Society, 2010.
3. Vandebril, Van Barel, and Mastronardi, Matrix Computations and Semiseparable Matrices,v1-2, Johns Hopkins.
4. Gene Golub and Charles Van Loan, Matrix Computations, John Hopkins.

INTRODUCTION TO SPECTRAL METHODS FOR SCIENTIFIC COMPUTING

Instructor: Prof. Jie Shen (shen@math.purdue.edu, 4-1923)
Course Number: MA 69200 CRN: 58617
Credits: Three
Time: TTh 10:30am-11:45am

Description

This is an introduction course on spectral methods for solving partial differential equations (PDEs). We shall present some basic theoretical results on spectral approximations as well as practical algorithms for implementing spectral methods. We shall specially emphasize on how to design efficient and accurate spectral algorithms for solving PDEs of current interest.

The course is suitable for advanced undergraduate students in mathematics and graduate students in sciences and engineering.

Prerequisite: A good knowledge of calculus, linear algebra, numerical analysis and some basic programming skills are essential. Some knowledge of real analysis and functional analysis will be helpful but not necessary.

References

1. Jie Shen and Tao Tang. Spectraland High-Order Methods with Applications, Science Press of China, 2006.

2. Jie Shen, Tao Tang and Li-Lian Wang. Spectral Methods: Algorithms, Analysis and Applications, Springer Series in Computational Mathematics, Vol. 41, Springer, 2011.

MODELING AND COMPUTATION IN OPTICS AND ELECTROMAGNETICs

Instructor: Prof. Peijun Li (lipeijun@math.purdue.edu, 4-0846)
Course Number: MA 69200 CRN: 57856
Credits: Three
Time: TTh 9:00am-10:15am

Description

This course addresses some recent developments on the mathematical modeling and the numerical computation of problems in optics and electromagnetics. The fundamental importance of the fields is clear, since they are related to technology with significant industrial and military applications. The recent explosion of applications from optical and electromagnetic scattering technology has driven the need for modeling the relevant physical phenomena and developments of fast, efficient numerical algorithms. As the applied mathematics community has begun to address a few of these challenging problems, there has been a rapid development of the theory, analysis, and computational techniques in these areas. The course will provide introductory material to the areas in optics and electromagnetics that offer rich and challenging mathematical problems. It is also intended to convey some up-to-date results to students in applied and computational mathematics, and engineering disciplines as well. Particular emphasis of this course is on the formulation of the mathematical models and the design and analysis of computational approaches. Topics are organized to present model problems, physical principles, mathematical and computational approaches, and engineering applications corresponding to each of these problems. No textbook is required. Lecture notes will be made available to students. Course grade: No exams. Students are required to present course-related material in class.

Prerequisite: Basic knowledge of functional and numerical analysis, and partial differential equations.

References

1. G. Bao, L. Cowsar, and W. Master, Mathematical Modeling in Optical Science
2. D. Colton and R. Kress, Inverse Acoustic and Electromagnetic Scattering Theory
3. J. Jin, The Finite Element Method in Electromagnetics
4. P. Monk, Finite Element Methods for Maxwell's Equations
5. J.-C. Nedelec, Acoustic and Electromagnetic Equations: Integral Representations for Harmonic Problems

TOPICS IN ANALYSIS

Instructor: Prof. Nicola Garofalo (garofalo@math.purdue.edu, 4-1971)
Course Number: MA 69300 CRN: 57862
Credits: Three
Time: TTh 12:00pm-1:15pm

Description

References

TOPICS IN COMPLEX GEOMETRY

Instructor: Prof. Sai Kee Yeung (yeung@math.purdue.edu, 4-1942)
Course Number: MA 69600 CRN: 57858
Credits: Three
Time: MWF 11:30am-12:20pm

Description

We would spend some time on basic topics in complex geometry, followed by more specialized directions as well as connections to other areas. Here are the tentative topics to be discussed.

1. Introduction to Kaehler geometry
2. Introduction to complex hyperbolicity
3. Introduction to diophantine geometry
4. For the last topic, we hope that we may explain some of the work of Faltings and Vojta on Mordell Conjecture

Prerequisite: MA 56200, MA 52500

SEMINARS

1. Operator Algebras Seminar
   Tuesday 2:30pm - MATH 215
2. Function Theory Seminar
   Wednesday 2:30pm - BRNG B261
3. Working Algebraic Geometry Seminar
   Wednesday 3:30pm - MATH 211
4. Student PDE Seminar
   Wednesday 3:30pm - MATH 215
5. PDE Seminar
   Thursday 3:30pm - MATH 215
6. Topology Seminar
   Thursday 3:30pm - REC 303
7. Topics in Algebra and Algebraic Geometry Seminar
   Thursday 4:30pm - MATH 211
8. Computational and Applied Math Seminar
   Friday 3:30pm - REC 103
9. Number Theory
   Friday 4:30pm - MATH 211