Advanced Discrete Mathematics

Instructor: Prof. Donu Arapura
Course Number: MA 51800
Credits: Three
Time: T Th 12:00 – 1:15 P.M.
Location: UNIV 117

Description

This course uses the book "Concrete Mathematics," which introduces the mathematics that supports advanced computer programming and the analysis of algorithms. The primary aim of its well-known authors (Ron Graham, Don Knuth, and Oren Patashnik) is to provide a solid and relevant base of mathematical skills - the skills needed to solve complex problems, to evaluate horrendous sums, and to discover subtle patterns in data. It is an indispensable text and reference for serious users of mathematics in virtually every discipline.

Introduction to Partial Differential Equations

Instructor: Prof. Harold Donnelly
Course Number: MA 52300
Credits: Three
Time: M W F 10:30 – 11:20 A.M.
Location: REC 123

Description

First order quasi-linear equations, the Cauchy Kovalevsky theorem, characteristics, classification and canonical forms of linear equations, equations of mathematical physics, study of Laplace, wave and heat equations, methods of solution.

Textbook: Introduction to Partial Differential Equations, Zachmanoglou and Thoe.

Probability Theory II
Instructor: Prof. Samy Tindel
Course Number: MA 53900
Credits: Three
Time: T Th 4:30 – 5:45 P.M.
Location: MSEE B010

Description

This course introduces various crucial notions concerning discrete and continuous time random functions. It has to be seen as a continuation of MA 538.

We begin by a quick review of conditional expectation. Then we will introduce the notion of martingale, which is a class of stochastic processes arising in the description of fair games. We will study the convergence properties of this kind of object, when the time index is discrete. The next topic to be covered concerns ergodic theorems, which can be seen as another tool allowing to get limit theorems for sequences of random variables. Eventually we construct and analyze the most important continuous time stochastic process, namely Brownian motion.

Prerequisites: MA 53800

As we did for MA 538, we will mostly follow Durrett's book: Probability - Theory and Examples.

Introduction to Differential Geometry and Topology
Instructor: Prof. Chi Li
Course Number: MA 56200
Credits: Three
Time: M W F 2:30 – 3:20 P.M.
Location: REC 313

Description

1. Differentiable manifolds; Tangent spaces; Submanifolds; Integral Curves; Differential forms; De Rham cohomology groups.
2. Riemannian Metrics; Geodesics; Laplace Operator; Harmonic Forms.
3. Geometry of Surfaces in E3; Gaussian curvature; Mean Curvature; Gauss-Bonnet and Poincare theorems on vector fields.
4. Vector Bundles; Connections; Parallel Transport; Curvatures.

References:

1. Introduction to Differentiable Manifolds and Riemannian Geometry, by William Boothby, Revised second edition, 2003.
2. Riemannian Geometry and Geometric Analysis, by Jurgen Jost, Sixth edition.
3. Vector Bundles; Connections; Parallel Transport; Curvatures.

Prerequisites: None

Algebraic Number Theory
Instructor: Prof. Prof. Edray Goins
Course Number: MA 58400
Credits: Three
Time: M W F 12:30 – 1:20 P.M.
Location: REC 307

Description

We will study the properties of orders in number fields. In the process, we will consider localizations, Dedekind domains, discrete valuation rings, ideles, adeles, the Zeta function of a number field, and L-series. There will be weekly homework assignments due every other Friday at the start of class. There will be final presentations at the end of the course; each will be graded on the combination of an oral presentation and a written paper. We will cover Part I of Lang's Algebraic Number Theory, although we will supplement the material via lecture notes.

Mathematical Logic I
Instructor: Prof. Leonard Lipshitz
Course Number: MA 58500
Credits: Three
Time: T Th 9:00 – 10:15 A.M.
Location: UNIV 101

Description

Formal theories for propositional and predicate calculus with study of models, completeness, and compactness. Formalization of elementary number theory; Turing machines, halting problem, and the undecidability of arithmetic.

Introduction to Iwasawa Theory
Instructor: Prof. Chung Pang Mok
Course Number: MA 59800ANT
Credits: Three
Time: MWF 10:30 – 11:20 A.M.
Location: MATH 215

Description

Iwasawa theory has its roots in the works of Kummer in the 19th century on the study of cyclotomic fields, Fermat's Last Theorem, and higher power reciprocity laws, and which was revitalized by the works of Iwasawa since the 1960's. The main theme of Iwasawa theory is the p-adic interpolation of various arithmetic quantities, for example special values of L-functions, and ideal class groups (and more generally Selmer groups). It has been a very important tool in algebraic number theory and arithmetic geometry for the past few decades.

In this course, we will start with the construction of the Kubota-Leopoldt p-adic L-function, and then study various aspects of the cyclotomic Z_p extension, following Iwasawa. Time allowed, we would like to give the proof of the Iwasawa main conjecture for cyclotomic fields.

References: Lectures on p-adic L-functions by K. Iwasawa and Introduction to cyclotomic fields by L. Washington.

Methods Of Linear And Nonlinear Partial Differential Equations I.
Instructor: Prof. Donatella Danielli-Garofalo
Course Number: MA 64200
Credits: Three
Time: MWF 11:30 A.M.– 12:20 P.M.
Location: MATH 215

Description

This is the first semester of a one-year course in the theory of second order elliptic and parabolic PDEs. The aim of the course is to study the solvability of boundary value problems and regularity properties of solutions. The first semester will focus on linear elliptic equations, both in divergence and non-divergence form. The starting point for the study of classical solutions will be the theory of Laplace's and Poisson's equations. The emphasis here will be on:

1. Existence of solutions to the Dirichlet problem for harmonic functions via the Perron method (based on the maximum principle)
2. Holder estimates for Poisson's equation derived from the analysis of the Newtonian potential. The crowning achievement of the theory of classical solutions is Schauder's theory, which extends the results of potential theory to a general class of non-divergence form equations with Holder-continuous coefficients.

In the second part of the semester we will consider a more general - and modern - approach to linear problems, based not on potential theory, but on Hilbert space methods for so-called "weak" solutions. Our main goal will be to prove the celebrated De Giorgi-Nash-Moser theorem on the regularity of weak solutions. The relevant tools from the theory of Sobolev spaces will be developed concurrently.

Textbook: Elliptic Partial Differential Equations of Second Order by D. Gilbarg and N. S. Trudinger, Second Edition, Springer.

Commutive Algebra.
Instructor: Prof. Bernd Ulrich
Course Number: MA 65000
Credits: Three
Time: MWF 4:30 – 5:20 P.M.
Location: REC 307

Description

This is an intermediate course in commutative algebra. The course is a continuation of MA 55700/55800, but should be accessible to any student with basic knowledge in commutative algebra (localization, Noetherian and Artinian modules, associated primes and primary decomposition, integral extensions, dimension theory, completion, Ext and Tor).

The topics of this semester will include: Regular sequences, depth and Cohen-Macaulayness, structure of finite free resolutions, regular rings and normal rings, canonical modules and Gorenstein rings, local cohomology and local duality.

Prerequisites: Basic knowledge of commutative algebra (such as the material of MA 55700/55800).

Textbook: No specific text will be used, but possible references are:

(1) Commutative ring theory, by H. Matsumura, Cambridge University Press

(2) Cohen-Macaulay rings by W. Bruns and J. Herzog, Cambridge University Press.
(3) Commutative algebra with a view toward algebraic geometry by D. Eisenbud, Springer.

Introduction to Algebraic Geometry.
Instructor: Prof. Kenji Matsuki
Course Number: MA 66500
Credits: Three
Time: MWF 11:30 A.M.– 12:20 P.M.
Location: UNIV 219

Description

The purpose of this course is to give a concise introduction to algebraic geometry, discussing the materials covered in Chapters I, II, III, IV of Hartshorne's textbook Algebraic Geometry. I would like to make the course as accessible to the begin- ning graduate students as possible. The student does not have to have Algebraic Geometry as his/her major future interest of research, and hence the student whose prospective major is Number Theory or any other subject, where the basic knowledge in Algebaric Geometry would be immensely helpful, is welcome. I would like to make the atmosphere of the class to be relaxed and freindly, but at the same time to make it a training camp where the emphasis is on "getting one's hand dirty" by working on the exercise problems as much as possible.

Textbook: Algebraic Geometry by R. Hartshorne, GTM 52 Springer

Reference Book(s):

- Basic Algebraic Geometry by Shafarevich, Springer
- Algebraic Geometry by S. Iitaka, GTM 76 Springer
- Algebraic Geometry by Joe Harris, GTM 133
- Commutative Ring Theory, H. Matsumura, Cambridge studies in advanced mathematics 8
- Commutative Algebra (with a view toward Algebraic Geometry) by D. Eisenbud, GTM 150 Springer

Classes:

- Every other Friday is dedicated to the presentation of the solution to some of the exercise problems by STUDENTS

Grading scheme:

- In order to get A, a student has to give at least 2 presentations of the exercise problems during the semester.

Rough Outline of Schedule

Chapter I. Varieties

§1. Affine varieties

1.1 Topology

- Usual topology vs. Zariski topology

1.1.1. Basic properties of Zariski topology

- Non-Hausdorff
- noetherian
- quasi-compact

1.2. Hilbert Nullstellensatz
1.3. Smoothness

1.3.1. Criterion for smoothness
1.3.2. Tangent and derivative
1.3.3. Generic Smoothness

1.4. Dimension
1.5. Morphisms between affine varieties
§2. Varieties in general
2.1. Projective varieties
2.2. Gluing affine varieties to obtain varieties in general
2.3. Hilbert polynomial

Chapter II. Schemes

§1. Can we recover an affine variety from its functions ?

- points
- topology
- When do we identify two affine varieties ?

§2. Sheaf

3.1. Definition
3.2. Morphisms between sheaves

§3. A locally ringed space

4.1. Definition
4.2. Morphisms between locally ringed spaces

§4. Affine scheme
§5. Scheme
§6. R vs. Spec R

Chapter III. Sheaves of modules on schemes (X, O_X)

§1. Quasi-coherent O_X-modules

- Coherent O_X-modules

§2. Why do we care ?

2.1. Line bundles and free O_X-module of rank 1
- Cartier divisors

2.2. Vector bundles and free O_X-module of rank r
2.3. Sheaf of differentials

§3. Exact sequence
§4. Cohomology

Chapter IV. Curves

§1. Riemann-Roch Theorem

- Characterization of P^1

§2. Embedding into a projective space

- Every complete nonsingular curve is projective.
- Canonical embedding
- Hyperelliptic curves

§3. Morphisms between curves
§4. Elliptic curves

Topics on Automorphic L-functions.
Instructor: Prof. Freydoon Shahidi
Course Number: MA 69000
Credits: Three
Time: MWF 9:30 – 10:20 A.M.
Location: MATH 215

Description

After the study of intertwining operators, Eisenstein series and their constant terms which are and will be treated in the course this semester, we will study non-constant Whittaker coefficients of Eisenstein series and the local theory attached to it. We expect to apply this theory to prove interesting cases of functoriality. Hopefully, we will also discuss the recent interests on L-functions by Ngo(Braverman-Kazhdan/Godement-Jacquet) as time permits.

References: F.Shahidi: Eisenstein series and L-functions, AMS Colloquium Pub., Vol 58, 2010, complemented by some related papers. The standard references for Eisenstein series such as: Moeglin-Waldspurger, Cambridge Tracts in Mathematics 113, Cambridge U. Press, 1995, will also be used as needed.

Modeling Comp Wave Propagation
Instructor: Prof. Peijun Li
Course Number: MA 69200
Credits: Three
Time: T Th 12:00–1:15 PM
Location: REC 122
Course website: www.math.purdue.edu/∼lipeijun/math692.html

Description

This course addresses some recent developments on the mathematical modeling and the numerical computation of problems in optics and electromagnetics. The fundamental importance of the fields is clear, since they are related to technology with significant industrial and military applications. The recent explosion of applications from optical and electromagnetic scattering technology has driven the need for modeling the relevant physical phenomena and developments of fast, efficient numerical algo- rithms. As the applied mathematics community has begun to address a few of these challenging problems, there has been a rapid development of the theory, analysis, and computational techniques in these areas. The course will provide introductory material to the areas in optics and electromagnetics that offer rich and challenging mathematical problems. It is also intended to convey some up-to-date results to students in applied and computational mathematics, and engineering disciplines as well.
Particular emphasis of this course is on the formulation of the mathematical models and the design and analysis of computational approaches. Topics are organized to present model problems, physical principles, mathematical and computational approaches, and engineering applications corresponding

to each of these problems.

Prerequisites: Basic knowledge of functional and numerical analysis, and partial differential equations.

Text: No textbook required. Lecture notes will be made available to students.

Course grade: No exams. Students are required to present course-related material in class.

References:

1. G. Bao, L. Cowsar, and W. Master, Mathematical Modeling in Optical Science
2. D. Colton and R. Kress, Inverse Acoustic and Electromagnetic Scattering Theory
3. J. Jin, The Finite Element Method in Electromagnetics
4. P. Monk, Finite Element Methods for Maxwell's Equations
5. J.-C. Nedelec, Acoustic and Electromagnetic Equations: Integral Representations for Harmonic Problems

Topics in Applied Mathematics
Instructor: Prof. Jianlin Xia
Course Number: MA 69200B
Time: T Th 10:30–11:45 am
Location: MATH 215

Introduction to Noncommutative Geometry
Instructor: Prof. Marius Dadarlat
Course Number: MA 69300
Time: MWF 3:30–4:20pm
Location: UNIV 117

Description

We aim for a friendly introduction based on examples to the ideas of noncommutative geometry.

Topics will include:

1. Review of differential forms and de Rham cohomology
2. Hochschild cohomology and noncommutative differential forms
3. Cyclic cohomology
4. Review of K-theory
5. Characteristic numbers and the Connes-Chern character
6. spectral triples and noncommutative Riemannian manifolds

Prerequisites: Some prior exposure to algebraic topology, functional analysis and differential geometry would be helpful but not absolutely required.

References:

1. A. Connes, Noncommutative Geometry, Academic Press, San Diego, CA, 1994, 661p.
2. A. Connes, Non commutative differential geometry, Publ. Math. IHES no. 62 (1985), 41-144 (both available on line: http://www.alainconnes.org/en/bibliography.php)
3. M. Khalkhali,Â Basic Noncommutative Geometry, EMS Series of Lectures in Mathematics

Topics in Analysis
Instructor: Prof. Victor Lie
Course Number: MA 69300B
Credits: Three
Time: M W F 12:30–1:20 PM
Location: REC 123

Description

This course is intended as a modern introduction in Harmonic Analysis with specific focus on the time-frequency area.

We will start with several basic facts about Fourier Series (properties of the Fourier coefficients, Riemmann localization, norm-convergence) and then pass to the proof of the celebrated theorem of L. Carleson regarding the almost everywhere convergence of the Fourier Series for L^2-functions.

The next topic starts as a warm up with the classical result of David and Journe - the so called "$T(1)$ theorem". From here we pass to the proof of the boundedness of the bilinear Hilbert transform (result due to C. Thiele and M. Lacey) a fundamental result that besides positively answering a conjecture of A. Calderon, generated tools that proved influential in other areas including ergodic theory.

Finally, if time allows, we will discuss recent advancements on a conjecture of A. Zygmund concerning the question of the boundedness of the Hilbert transform along vector fields.

Prerequisites: Real Analysis, Integration theory and Hilbert spaces; Complex Analysis helpful but not required.

Books:

- Introduction to Harmonic Analysis, Y. Katznelson;
- Classical and Multilinear Harmonic Analysis, C. Muscalu, W. Schlag;
- Wave Packet Analysis, C. Thiele
- Harmonic Analysis: Real variable methods, orthogonality, and oscillatory integrals, E. Stein;
- Functional Analysis, W. Rudin;

Papers:

- Michael Bateman, Single annulus ï¿œL^p estimates for Hilbert transforms along vector fields, (Rev. Mat. Iberoam. , 2013)
- Michael Bateman and Christoph Thiele, ï¿œL^p estimates for the Hilbert transforms along a one-variable vector field, (Anal. PDE, 2013)
- Lennart Carleson: On convergence and growth of partial sumas of Fourier series (Acta Math., 1966).
- Charles Fefferman: Pointwise convergence of Fourier series (Ann. of Math., 1973.)
- Michael Lacey and Christoph Thiele: A proof of boundedness of the Carleson operator. (Math. Res. Lett., 2000.)
- Michael Lacey and Christoph Thiele: L^p estimates on the bilinear Hilbert transform for $2 < p < \infty$. (Ann. of Math., 1997).

- Michael Lacey and Christoph Thiele: On Calderon's conjecture. (Ann. of Math., 1999.)
- Michael T. Lacey and Xiaochun Li: Maximal theorems for the directional Hilbert transform on the plane (Trans. Amer. Math. Soc., 2006)
- Michael Lacey and Xiaochun Li, On a conjecture of E. M. Stein on the Hilbert transform on vector fields, Mem. Amer. Math. Soc. 2010)

Motion by Mean Curvature - from an applied analysis perspective

Instructor: Prof. Aaron N. K, Yip
Course Number: MA 69400
Credits: Three
Time: T Th 9:00–10:15 AM
Location: MATH 215

Description

This course introduces basic properties of motion by mean curvature (MMC) from an applied analysis point of view. Besides its geometric interest, MMC appears often in many applications, in particular, surface motions in materials science. Its analysis has lead to a broad range of useful and versatile mathematical techniques and machinery. The course roughly consists of three components:

(1) Basic differential geometry of surfaces, geometric properties of MMC, self-similarity, monotonicity formula, and blow-up phenomena;
(2) Methods of analyzing MMC: classical PDE techniques, level set formulation, (viscosity solution), singular perturbations (asymptotic expansion), varifolds, and thresholding schemes;
(3) Extensions and related geometric motions: motion of filaments (curves in space), triple junctions, higher order flows such as surface diffusion and Willmore flow.

Prerequisites: Basic Partial Differential Equations (MA 52300).

Text: There is no official textbook but a good starting point is the book by Klaus Ecker, "Regularity Theory for Motion by Mean Curvature". This and most of the other materials are all available online.

Topics in Topology

Title: Bundles and classifying spaces
Instructor: Prof. Jeremy Miller
Course Number: MA 69700
Credits: Three
Time: T Th 12:00–1:15 PM
Location: MATH 215

Description

In this course, we will discuss various types of fiber bundles such as vector bundles, covers, and principal bundles. We will prove that the set of isomorphism classes of bundles is in bijection with the set of homotopy classes of maps to a space called the classifying space. We will describe ways of constructing classifying spaces, including the simplicial bar construction. The homology of classifying spaces of discrete groups is called group homology. We will compute the group homology of cyclic groups and use this to show that a group with torsion cannot act freely on finite dimensional Euclidean space. An emphasis will be placed on geometric models of classifying spaces such as Grassmannians and configuration spaces. Time permitting, we will discuss characteristic classes, Eilenberg-MacLane spaces, fibrations, and the Serre spectral sequence.

Prerequisites: MA 57200.