## Probability Theory II (Managed As STAT 53900)

Instructor: Professor Jing Wang

Course Number: MA 53900

Credits: Three

Time: 10:30–11:45 AM TTh

### Description

This course is the continuation of Math 538 in probability theory. The goal is to understand the basic theory of stochastic calculus. We will start from Brownian motion and continues martingale. Then we will cover the topics of stochastic integrals, Ito's calculus, representation of martingales, Girsanov theorem, stochastic differential equations, etc. We will use the following references:

(1) R. Durrett: Probability Theory and Examples, 4th Edition

(2) F. Baudoin: Diffusion Processes and Stochastic Calculus

## Real Analysis Measure Theory

Instructor: Professor Antonio Sa Barreto

Course Number: MA 54400

Credits: Three

Time: 3:30–4:20 PM MWF

### Description

1. Introduction, the Riemann integral versus the Lebesgue integral
2. Topology of metric spaces (properties of open, closed, compact, and connected sets)
3. Continuity, semi-continuity, sequences of functions, pointwise and uniform convergence
4. The Riemann integral, sets of measure zero
5. Cantor sets
6. Construction of the Lebesgue measure and integral, abstract measure spaces and integral properties of measurable functions
7. When is $\lim_{n\to\infty} \int f_n \, dx = \int \lim_{n\to\infty} f_n \, dx$ for the Lebesgue integral? Convergence theorems, Fatou's Lemma
8. $L^p$- spaces, Hölder and Cauchy-Schwatz inequalities. Inner product spaces.
9. The Fubini-Tonelli Theorem, convolutions in $\mathbb{R}^n$
10. Applications: Approximations of the identity, the Fourier transform and its inverse
11. Completeness of $L^p$ spaces and the Fourier transform of $L^2$ functions
12. Vitali Covering Theorem
13. Functions of bounded variation, differentiation of monotone functions, absolute continuity, the Helly Selection Theorem
14. The Lebesgue Differentiation Theorem

**References:**

1. I will write my own lecture notes, which I do not claim are original, which will be posted in Blackboard (actually in D2L).
2. Textbooks, but them if you wish: A. Torchinsky, Real Variables; W. Rudin, Principles of Mathematical Analysis

**Grade:**

1. One set of homework problems per week. Average of the homework scores= 100 points
2. Two midterm evening exams, 100 points each
3. Final exam, 150 points

**Functions of Several Variables and Related Topics**

Instructor: Professor Nung Kwan Yip

Course Number: MA 54500

Credits: Three

Time: 9:00–10:15 AM TTh

**Description**

This course is a continuation of 544. The emphasis will be on the "applications" of function spaces. Topics to be covered include:

(i) reivew of Lp spaces;

(ii) Fourier transforms and distributions;

(iii) Sobolev spaces;

(iv) Applications to solution of Laplace, heat, and wave equations;

(v) Functions with bounded variation and sets of finite perimeter;

(vi) Functional Inequalities with applications in geometry and analysis (for example, isoperimetric, Brunn-Minkowski, and log-Sobolev inequalities).

**Prerequisites:** MA 544 (or measure theory)

**Textbook:** Analysis (2nd edition), by Elliott H. Lieb and Michael Loss, Graduate Studies in Mathematics, Volume 14, American Mathematical Society;

Measure Theory and Fine Properties of Functions (revised edition), by Lawrence C. Evans and Ronald F. Gariepy, CRC Press

## Mathematical Logic I

Instructor: Professor Margaret Thomas
Course Number: MA 58500
Credits: Three
Time: 1:30–2:45 PM TTh

### Description

This course will serve as an introduction to various topics on which the modern field of mathematical logic is founded.

We will begin with a formal study of (mathematical) argument, focussing on two frameworks: first propositional calculus, and then predicate calculus (also known as first-order logic). This introduces useful concepts such as models, theories, satisfiability, completeness and compactness. Thereafter, we will look at a formalization of elementary number theory and the beginnings of computability, including the incompleteness theorems, Turing machines, the halting problem and the undecidability of arithmetic. Notions from set theory will be introduced as needed.

Students are welcome to participate not only if they are interested in a deeper study of logic itself, but also if they are keen to learn a formal perspective from logic that might be applicable to other areas of mathematical study, such as algebraic geometry, number theory, combinatorics, algebra, or analysis.

**Textbook:** E. Mendelson, Introduction to Mathematical Logic, Sixth edition, Textbooks in Mathematics, CRC Press, Boca Raton, FL, 2015.

There will be regular assignments, as well as a final examination.

## Analytic Number Theory: a first course

Instructor: Professor Trevor Wooley
Course Number: MA 59800AANT
Credits: Three
Time: 10:30–11:20 AM MWF

### Description

This course serves as an introduction to analytic number theory. Its focus is the theory of the Riemann zeta-function and Dirichlet L-functions, the distribution of prime numbers, and Dirichlet's theorem on primes in arithmetic progressions. The development of the theory and application of Dirichlet series in number theory leads to surprisingly powerful results on the distribution of prime numbers, and today motivates a complex and beautiful body of research which aims to describe and explain arithmetic phenomena. This is a basic introduction to the theory of prime numbers and L-functions. Students interested in more advanced topics, or in preparing to undertake research in this area, will find this a useful first course ... and there are many beautiful results and theoretical developments along the way to keep the competent enthusiast interested. Prerequisites in number theory will be confined to such topics as the Chinese remainder theorem and primitive roots (a basic first course will suffice). An analytic prerequisite is the calculation of contour integrals by summing residues. Further analytic material will be developed in the course ... Riemann-Stieltjes integration, Euler-Maclaurin summation, Jensen's inequality, the Borel-Caratheodory lemma, Hadamard products, and the Poisson summation formula.

**Assessment:** Six or seven problem sets will be offered through the semester, and class participants can demonstrate engagement with the course by any written and/or in-class presentations featuring a reasonable subset of these problems – three levels of difficulty: short problems testing basic skill-sets, extended problems integrating the essential methods of the course, and more challenging problems for enthusiasts with detailed hints available on request.

The course will be based on the instructor's lecture notes, which in turn are based on: H. Davenport, Multiplicative Number Theory, 2nd ed., Springer-Verlag, GTM 74, 1980; H. L. Montgomery and R. C. Vaughan, Multiplicative Number Theory, I. Classical Theory, Cambridge Studies in Advanced Mathematics 97, Cambridge University Press, 2007.

**Basic Topics.** (i) Basic properties of Dirichlet series; (ii) Mean values of arithmetic functions, Renyi's theorem; (iii) Elementary prime number estimates; (iv) Simplest sieve estimates, the Brun-Titchmarsh theorem; (v) Dirichlet characters; (vi) Dirichlet L-functions; (vii) Primes in arithmetic progressions; (viii) The Mellin transform; (ix) Zeros of the zeta function; (x) The prime number theorem; (xi) Equivalent forms of the prime number theorem; (xii) Further comments on the prime number theorem; (xiii) Primitive characters, Gauss sums; (xiv) The Polya-Vinogradov inequality, primitive roots in short intervals; (xv) The Poisson summation formula; (xvi) Theta functions; (xvii) The functional equation of the zeta function and L-functions; (xviii) Zero-free regions for L-functions; (xix) Siegel's theorem, Landau's variant; (xx) The prime number theorem for arithmetic progressions and applications.

**Advanced topics,** depending on demand and available time, may include explicit formulae, applications of the Riemann Hypothesis, the large sieve and applications, estimates for prime number sums, the Bombieri-Vinogradov theorem.

**Prerequisites:** Elementary number theory and basic real and complex analysis.

**Mathematical Theory and Applications of Deep Learning**
Instructor: Professor Haizhao Yang
Course Number: MA 59800ADL
Credits: Three
Time: 12:00–1:15 PM TTh

**Description**

Part I: machine learning basics; deep feedforward networks; convolutional networks; advanced network design;

Part II: approximation theory of deep neural networks; stochastic optimization methods; regularization for deep learning; generalization error of deep neural networks;

Part III: sparse and structured computation; sequence modeling: recurrent and recursive nets; deep reinforcement learning; deep generative models; distributed and decentralized learning.

**Loop Spaces, Configuration Spaces, and Operads**
Instructor: Professor Jeremy Miller
Course Number: MA 59800ALCSO
Credits: Three
Time: 10:30–11:45 AM TTh

**Description**

This class will focus on the relationship between configuration spaces and loop spaces in algebraic topology. We prove the group-completion theorem and the recognition principle for iterated loop spaces. We will also prove Poincare duality using the Dold-Thom theorem.

**Introduction to Spectral Methods and Computational Fluid Dynamics**
Instructor: Professor Jie Shen
Course Number: MA 59800ASMFD
Credits: Three
Time: 1:30–2:20 PM MWF

**Description**

**Topology and Its Applications**
Instructor: Professor Ralph Kaufmann
Course Number: MA 59800ATOPA
Credits: Three
Time: 9:30–10:20 AM MWF

**Description**

The course is an introduction to topological concepts and their applications. It is intended to be interdisciplinary in nature. The aim is to build on known concepts and and connect them through theory. The pace and presentation will be adjusted to the participants in the course. The course will start with basic examples of topological phenomena known from multi–variable calculus. There are numerous application of this for instance in form of the Gauss law in Electrodynamics and other fields, where central potentials play a role. The analog in 2d is known as the winding number and appears in covering theory, holomorphic functions and in several applied contexts such as the Aharonov-Bohm effect and the Berry phase. Analyzing the ingredients of this we will proceed to discuss fundamental groups, homotopy groups, homology and cohomology. These are introduced in a practical way directed at understanding topological charges such as Chern classes. Physical examples would come from Bloch theory in condensed matter or families of Hamiltonians more generally. Further subjects may include bundles, K-theory, index theory, Chern-Simons classes, topological field theory and gauge theory. These would provide examples of Kitaev's periodic table classifying the various possible topological charges. Further applications would be in Quantum Field theory possibly including more detailed discussions of Dirac and Majorana Fermions and Anyons.

**Commutative Algebra**
Instructor: Professor William Heinzer
Course Number: MA 65000
Credits: Three
Time: 3:30–4:20 PM MWF

**Description**

This course will cover topics in commutative algebra as in the book written by Irena Swanson and Craig Huneke titled Integral Closure of Ideals, Rings, and Modules.

**Prerequisites:** basic concepts in abstract algebra

**Introduction to Deformation Theory**
Instructor: Professor Kenji Matsuki
Course Number: MA 69000
Credits: Three
Time: 2:00–3:15 PM MF

**Description**

The purpose of this course is to give an easy introduction to the local and global Deformation Theory so that the students, who have learned the basics of the language of schemes, sheaves, and cohomology, can see how they are used in the face of the important problems such as the construction of the moduli and infinitesimal deformation of a given variety and/or smoothing of a singularity.

We use the textbook "Deformation Theory" written by Hartshorne to cover the local aspect of the deformation theory. Hartshorne is an excellent expositor, who has written a well-known textbook "Algebraic Geometry", providing an introductory account of the schemes, sheaves, and cohomology, among others. However, the students often get tired of and lost in the myriads of the technicalities required. We hope that this course may offer a practice ground so that the students can revive and refresh their understanding by seeing how they are actually used in the real problems. In this sense, this course is a continuation of the introductory course on "Algebraic Geometry".

We will aso give a concise discussion of the Hilbert scheme and the construction of the moduli space (e.g., of the curves and of the vector bundles over a fixed variety) via Geometric Invariant Theory in the global aspect of the deformation theory. Both of them have acquired the reputation of the formidable subjects to master, whether justifiably or not. However, the underlying ideas are marvelously simple, as is almost always the case with any important subjects and theories. We would like to present these basic ideas in the course, while reading the standard textbooks to master these subjects might require **an unsurmountable amount of time.**

(1.) **Background:** Given an object of investigation, the idea of studying it not only as a seperate and individual entity but also as a member of a certain family it belongs to, is old and classical. When Riemann studied the complex curve, which now bears his name "a Riemann surface", he was already counting the dimension of the "moduli space" to be $3g - 3$, which parametrizes the varying complex structures of a smooth compact orientable surface of geneus $g$. Or even more classical, one can classify the isomorphism classes of cubic curves by the $j$-invariant, as a precursor of the moduli space of the elliptic curves. It is probably fair to say that the modern treatment of the local deformation theory in terms of sheaves and cohomology started by the work of Kodaira-Spencer, where they discuss the deformation of the complex structures for complex manifolds. This led to the development of the corresponding theory in the algebraic setting by Grothendieck and others, which is the subject matter of the local aspect of this course. Simplifying the original construction of the Hilbert scheme by Grothendieck, Mumford then constructed the moduli space of curves via Geometric Invariant Theory. The Hilbert scheme provides a naive parametrization of the curves with lots of duplications for one isomorphic class of curves. We identify then all the curves in an isomorphic class and crash them into to a point, by taking the group quotient, so that we have the nice moduli space with points parametrizing the isomorphic classes of the curves. This simple idea is the subject matter of the global aspect of this course.

(2.) **Textbooks:**

- "Deformation Theory" by Robin Hartshorne, Graduate Texts in Mathematics, Springer
- "Lectures on curves on an algebraic surface" by David Mumford, Annals of Mathematics Studies No. 59
- "Complex Manifolds and Deformation of Complex Structures" by Kunihoko Kodaira, Grundlehren der mathematischen Wissenshaften 283, Springer-Verlag

(3.) **Prerequisites:** Basic knowledge of the scheme, sheaves and cohomology at the level of Chapters I, II, III, in "Algebraic Geometry" by Hartshorne.

WARNING: I am NOT requiring the students to have the mastery of the subjects metioned above. Rather, I am very much aware that the students usually start having indigestions by swallowing so much of technical details, without seeing them actually used in the real problems. This course is intended, to some extent, as a remedy to this trouble so that the students can start disgesting the materials in a more lively manner, seeing some of the motivations behind the development of the schemes, shaeves, and cohomology in regard to the deformation theory.

(4.) **Time:** We will meet twice a week on Mondays and Fridays, **for 75 minutes each**.

(5.) **Place:** To be announced.