Advanced Graduate Courses offered by the Mathematics Department Spring, 2000

MA 598A/GEOS 591B	MA 598B	MA/STAT 598G
MA 611	MA 631	MA 643	MA 646	MA 661
MA 690A	MA 690B	MA 690C	MA 690D	MA 690E
MA 692C/AGRY 598M	MA 693A	MA 694A	MA 696A	MA 697A

MA 598A/GEOS 591B: Fractals and Chaos with Applications to Earth Sciences
Instructor: Prof. A. Gabrielov, office: Math 648, phone: 49--47911, e-mail: agabriel@math.purdue.edu
Time: TTh 12:00-1:15
Description: The goal of this course is to give an introduction to the theory and phenomenology of nonlinear dynamics, chaos, self-similarity, and fractal geometry, for advanced undergraduate and beginning graduate students. The course includes geophysical applications of this theory.

The topics covered in the course: Self-similarity, fractals, scaling, renormalization. Pattern formation, self-organization, critical phenomena. Continuous and discrete dynamical systems. Attractors. Regular vs chaotic dynamics. Strange attractors. Instability and unpredictability. Bifurcations and onset of chaos.

Applications: Convection, Atmospheric circulation, Geomagnetism. Turbulence, Percolation, Self-organized criticality. Geomorphology, Geotectonics, Seismicity. Distribution of ore and sediment deposits, floods and droughts.

Familiarity with ordinary differential equations and linear algebra (MA 262 or 265/266) is expected.
Texts:
1. S.H.Strogatz, *Nonlinear Dynamics and Chaos With Applications to Physics, Biology, Chemistry, and Engineering*, Addison-Wesley, 1994.
2. D.L.Turcotte, *Fractals and Chaos in Geology and Geophysics*, 2nd Ed., Cambridge Univ. Press, 1997.

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MA 598B: The Mathematical Theory of Age Structured Population Dynamics
Instructor: Prof. M. Iannelli, office: Math 712, phone: 49--41942, e-mail: miannel@math.purdue.edu
Time: MWF 11:30
Prerequisite: advanced analysis
Description: The purpose of the course is to present the basic phenomenology and the mathematical methods for the modelization of age structured populations.

We will discuss the following topics:
The basic linear theory 1. Definition of the basic parameters. 2. The Lotka--McKendrick equation. 3. The renewal equation. 4. Analysis of the Lotka--McKendrick equations. 5. Asymptotic behavior of solutions.
Further developements of the linear theory. 1. The age profile. 2. Time dependent rates. 3. Strong and weak ergodicity. 4. Infinite maximum age.
Numerical methods for the linear model 1. Approximation of the are--density. 2. Approximation of the basic reproduction rate. 3. Finite difference methods. 4. The

method of characteristics. 5. Higher order methods. 6. Numerical simulations. 7. Comments and references.
Non-linear models 1. A general nonlinear model. 2. Existence and uniqueness. 3. Existence of equilibria. 4. The Allee--logistic model with a single size. 5. Two size models.
Numerical methods for the non-linear model 1. Approximation of the age--density. 2. Approximation of the basic reproduction rate. 3. Numerical simulations.
Stability of equalibria 1. Definitions and assumptions. 2. The basic characteristic equation. 3. Stability and instability. 4. Some results about the characteristic equation. 5. Revisiting the Allee--logistic model. 6. Bifurcations.
Global behavior 1. A general approach to a special class of models. 2. The purely logistic model. 3. Separable models. 4. The case of infinite maximum age.
Epidemics in an age structured population 1. A general unstructured model for epidemics. 2. Endemic states for the age structured S-I-S model. 3. Asymptotic behavior in the intra-cohort case. 4. Endemic states for the age structured S-I-R model. 5. Asymptotic behavior.
Class-age structure for epidemics 1. The Kermack-McKendrick model. 2. Reduction of the system. 3. Behavior of the solutions. 4. On the constitutive form of the infection rate. 5. Introducing the vital dynamics. 6. Endemic states.

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MA/STAT 598G: Advanced Probability and Financial Options
Instructor: Prof. Protter, office: Math 602, phone: 49--41964, e-mail: protter@math.purdue.edu
Time: TTh 9:00-10:15
Prerequisite: Undergraduate probability theory (MA/STAT 519 or STAT 516), Ordinary Differential Equations (any of MA 360, 364, or 366), and Elementary Analysis (e.g. MA 440 or MA 504).
Description: We develop the basic theory of Stochastic Finance. This will include the Binomial Model, Stochastic Integrals, Stochastic Differential Equations, the Black-Scholes model, Portfolio Dynamics, Arbitrage Pricing, Completeness and Hedging, Interest Rate Models. The mathematical tools used will be stochastic calculus.
Text: Tomas Björk *Arbitrage Theory in Continuous Time*, Oxford University Press, 1998.

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MA 611: Methods of Applied Mathematics I
Instructor: Prof. D. Phillips, office: Math 706, phone: 49--41939, e-mail: phillips@math.purdue.edu
Time: MWF 1:30
Prerequisite: MA 511, 544
Description: This course develops the functional analysis needed to study differential equations and applied math. Examples and applications will be given using Sobolev spaces and Sturm-Liouville theory.Topics include Banach and Hilbert spaces; weak topologies; linear operators; Lax-Milgram theorem; compact operators; Riesz-Fredholm theory; spectral theory of compact operators.
Text: Avner Friedman, *Foundations of Modern Analysis*
Reference: Haim Brezis, *Analyse Fonctionelle Theorie et Applications*. Students are encouraged to order the reference directly from the publisher at http://www.dunod.com/cgi-bin/booke.pl?179:1:'au=(Brezis\%2CH*)'

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MA 631: Several Complex Variables
Instructor: Prof. D. Catlin, office: Math 744, phone: 49--41958, e-mail: catlin@math.purdue.edu
Time: TTh 9:00-10:15
Prerequisite: MA 530
Description: Power series, holomorphic functions, representation by integrals, extension of functions, holomorphically convex domains. Local theory of analytic sets

(Weierstrass preparation theorem and consequences). Functions and sets in the projective space P^n (theorems of Weierstrass and Chow and their extensions.)

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MA 643: Methods of Partial Differential Equations II
Instructor: Prof. P. Bauman, office: Math 718, phone: 49--41945, e-mail: bauman@math.purdue.edu
Time: MWF 10:30

Description: This is a continuation of MA 642. Topics to be covered include L^p theory for elliptic equations. Moser's estimates, Calderon-Zygmund theory, Aleksandrov maximum principle. Introduction to evolution problems. Parabolic and hyperbolic equations. Galerkin approximation, semigroup techniques. Applications to nonlinear problems.

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MA 646: Banach Algebras and C^*-algebras
Instructor: Prof. L. Brown, office: Math 704, phone: 49--41938, e-mail: lgb@math.purdue.edu
Time: MWF 10:30
Prerequisite: MA 646 or equivalent
Description: The subject of operator algebras originated from models for quantum physics and has since been related to many other fields. Some aspects of the theory can be described as "non-commutative topology", "non-commutative geometry", "non-commutative probability theory", etc. For more on this, see: http://msri.org/activities/programs/0001/opalg/. Although all of these topics are beyond the scope of the course, the intent is to provide the prerequisites for such advanced topics.

Topics will include Banach algebras, Gelfand theory, the commutative Gelfand-Naimark theorem and applications to normal operators, C^*-algebras and representations, the non-commutative Gelfand-Naimark theorem, von Neumann algebras, and Murray-von Neumann equivalence. Some operator theory or other topics may be included as time permits.
References:

1. W. Arveson, *An Invitation to C^*-algebras*
2. J. Conway, *A Course in Functionsl Analysis*, chaps VII, VIII, and IX
3. R. Kadison and J. Ringrose, *Fundamentals of Operator Algebras*
4. J. von Neumann, *Collected Works*, vol. III, chaps 2,3,4,5
5. G. Pedersen, *C^*-algebras and their Automorphism Groups*
6. G. Simmons, *Introduction to Topology and Modern Analysis*
7. M. Takesaki, *Theory of Operator Algebras*

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MA 661: Modern Differential Geometry
Instructor: Prof. H. Donnelly, office: Math 716, phone: 49--41944, e-mail: hgd@math.purdue.edu
Time: MWF 3:30
Prerequisite: MA 544, 554 and 562
Description: An introduction to Riemannian geometry. Tensors, manifolds, and vector bundles reviewed. Primary topics include Riemannian metrics, connections, geodesics, curvature, second fundamental form of submanifolds, Gauss--Bonnet theorem, Jacobi fields, curvature and topology.

Text: John M. Lee *Riemannian Manifolds*.

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MA 690A: Topics in Algebraic Geometry
Instructor: Prof. S. Abhyankar, office: Math 432, phone: 49--41933, e-mail: ram@math.purdue.edu
Time: TTh 3:00-4:15
Description: We shall discuss various topics of current interest. There are no prerequisites. All interested persons are welcome.

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MA 690B: Categories of Modules
Instructor: Prof. L. Avramov, office: Math 640, phone: 49--41978, e-mail: avramov@math.purdue.edu
Time: TTh 1:30-2:45
Description: The first part of the course will study categories of modules over Cohen-Macaulay rings that have, up to isomorphism, only finitely many classes of maximal Cohen-Macaulay modules. This will be based on (the easier parts of) the book of Y. Yoshino, *Cohen-Macaulay Modules over Cohen-Macaulay Rings*, Cambridge Univ. Press, 1990. Some results from remining parts of the book, covered in this semester's MA 651, will be useful, but not crucial for understanding.

The second part of the course will cover some classical module theory (e.g., Morita equivalence) as well as more recent results on modules over (not necessarily) commutative artinian rings. The basis for this will be the book of M. Auslander, S. Smalo, and I. Reiten, *Representation of Artin Algebras*, Cambridge Univ. Press, 1996.

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MA 690C: Singularities in Symplectic and Contact Spaces
Instructor: Prof. Zakalyukin
Time: MWF 2:30
Prerequisites: 4 semesters of Calculus.
Description: The goal of the course is to give an elementary introduction into singularity theory and its applications to differential equations, wave theory, differential geometry, and control theory.

We start with the classification of simple singularities of functions, describe their relation to Coxeter groups generated by reflections.

Caustics and wavefronts provide most common singularities arising in many areas of science: quantum physics, wave theory, biology, computer vision, and mathematical economy. We describe this theory in more details, including recent results on Lagrangian and Legendrian varieties.
Text:
1. (recommended) V.I.Arnold, A.N.Varchenko, S.M. Gussein-Zade, *Singularities of differentiable mappings*, vol. I, II, Birkhauser,1985 (second ed.)
2. (recommended) V.I.Arnold, *Singularities of caustics and wavefronts*, Kluwer, 1990
3. (recommended) V.I.Arnold, *Mathematical methods of classical mechanics*, Springer-Verlag, 1978 (second ed.)

NOTE: All textbooks in this list contain much more material than will be used in the course. Consider them as texts/references.

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MA 690D: Zeta Functions and L-series of Number Fields
Instructor: Prof. J. Lipman, office: Math 750, phone: 49--41994, e-mail: lipman@math.purdue.edu
Time: TTh 10:30-11:45
Prerequisite: Basics of Algebraic Number Theory, such as can be found, for example, in Samuel's *Algebraic Theory of Numbers*. Also, MA 530 and MA 544, or the equivalent.
Description: From the introduction to Neukirch's Chap. 7 (paraphrased): "Among the most remarkable features of number theory is that many deep arithmetic properties of a number field lie hidden within a single analytic function, its **zeta-function**. This function, while simple in appearance, is jealous of yielding up its mysteries. Whenever we succeed in stealing one of these well-guarded truths, we may expect to be rewarded by the revelation of some surprising and significant relationship. This is why zeta functions and their generalizations, the L-series, have moved to the forefront of number-theoretic research."

We will cover, initially, the Riemann zeta function and Dirichlet L-series, with applications to the prime number theorem and primes in arithmetic progressions; and then generalize to algebraic number fields-Dedekind zeta functions, Hecke and Artin L-series, analytic continuation and functional equations, special values, and connections with class field theory (reasonably self-contained), density of primes, class numbers, cyclotomic fields and Kummer's results on Fermat's Last Theorem for regular primes.

The course will be run as a seminar, i.e., lectures will be prepared and given by the participants (with my help, if desired).
Texts: (on reserve in library):
1. J. Neukirch, *Algebraic Number Theory*, Chapter 7.
2. Borevich and Shafarevich: *Number Theory*, Chapter 5.

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MA 690E: Topics in Commutative Algebra
Instructor: Prof. W. Heinzer, office: Math 636, phone: 49--41980, e-mail: heinzer@math.purdue.edu
Time: MWF 12:30
Prerequisite: MA 650
Description: A continuation of MA 650, Fall, 1999 semeseter.
Text: Ernst Kunz *Introduction to Commutative Algebra and Algebraic Geometry*

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MA 692C/AGRY 598M: The Physics of Nanofilms
Instructor: Prof. J. Cushman, office: Math 816, phone: 49--48040, e-mail: jcushman@math.purdue.edu
Time: TTh 1:30-2:45
Description: Films of a few molecular layers in thickness play a principle role in many modern technologies and natural processes. They play a role in nanotribology, nonochromatography, the atomic force microscope, the molecular--scale shear forces apparatus, and biopolymers, just to name a few.

The purpose of this course is to present the mathematical and computational tools required to study these phenomena. Topics covered include equilibrium and nonequilibrium statistical mechanics with emphasis on density functional theory, phase transitions, and wave--vector/frequency dependent phenomena. In addition, some quasi--static equilibrium statistical mechanical computation techniques will be presented in novel ensembles. Lessor emphasis will be placed on nonequilibrium simulators.

The physical background of the course can be found in:
1. M. Schmidt, *Freezing in confined geometrics*, Shaker--Verlag, 1997.
2. J. Israelachvili, *Intermolecular and surface forces*, Academic, 1992.
3. J. H. Cushman, *The physics of fluids in hierarchical porous media: Angstroms to miles*, Kluwer, 1997.

4. J. S. Rowlinson and B. Widom, *Molecualr theory of capillarity*, Oxford, 1989.
5. J. Boon and S. Yip, *Molecular hydrodynamics*, 1980.

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MA 693A: Constructive Complex Analysis
Instructor: \rm Prof. S. Bell, office: Math 740, phone: 49--41956, e-mail: bell@math.purdue.edu
Time: TTh 1:30-2:45
Description: I will cover recent developments in complex analysis arising from the remarkable discovery made in 1978 by N. Kerzman and E. M. Stein that the centuries old Cauchy Transform is nearly a self adjoint operator when viewed as an operator on L^2 of the boundary. This new but fundamental result represented a shift in the bedrock of complex analysis. It has allowed the classical objects of potential theory and conformal mapping in the plane to be constructed and analyzed in new and very concrete terms.

Among the topics that I shall cover will be
1. Boundary behavior of the Cauchy transform.
2. The Kerzman-Stein integral equation and applications.
3. The Szeg\H o kernel function and projection.
4. The Ahlfors mapping and its importance in complex analysis and potential theory.
5. The Bergman kernel function and projection.
6. The Poisson kernel and the Dirichlet problem.
7. A constructive approach to the classical objects of conformal mapping and potential theory.
8. Complexity in complex analysis.

The only prerequisite for the course is MA 530. There will be some overlap with the material of MA 531, but not much.

I hope that at the end of this course a student will be ready to start work on a thesis problem in either pure complex analysis or numerical conformal mapping and potential theory.

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MA 694A: Pseudodifferential Operators and Applications
Instructor: Prof. A. SaBarreto, office: Math 604, phone: 49--41965, e-mail: sabarre@math.purdue.edu
Time: MWF 2:30
Description: As the name suggests, pseudodifferential operators are a generalization of differential operators. For example, the inverse of certain differential operators fall in this class. These operators have been extensively studied and have been applied to many problems in pure and applied mathematics, from topology, as in the celebrated Atiyah-Singer index theorem, to fluid dynamics.

In this course we won't be able to cover so much, but we plan to develop the basic calculus of these operators and apply it to problems in solvability of certain partial differential equations, spectral theory and fluid dynamics.

The pre-requisites are a certain familiarity with Distribution Theory, like Sobolev spaces, and with partial differential equations.

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MA 696A: Moduli of Vector Bundles on Curves
Instructor: Prof. P. Sastry, office: Math 616, phone: 49--41971, e-mail: pramath@math.purdue.edu
Time: TTh 9:00-10:15
Description: I will assume knowledge of Hartshorne's *Algebraic Geometry*, up to Cohomology of Projective Space (and the resulting calculus of twists). We will construct Hilbert Schemes, do some Geometric Invariant Theory and construct the relevent moduli spaces. Properties of this moduli space will be explored. I will essentially follow Le Potier's book on the subject.

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MA 697A: Homotopy Theory
Instructor: Prof. J. Smith, office: Math 720, phone: 49--47910, e-mail: jhs@math.purdue.edu
Time: MWF 11:30
Prerequisite: A course in general topology and a course in basic algebra.
Description: We will begin by defining the set of homotopy classes of maps from one topological space to another. There is no good general procedure for computing the set of homotopy classes of maps. However, there are many techniques that work well in special cases. Those are the topics we will cover.

We will begin by defining homotopy groups and cell complexes. We will then consider the set of homotopy classes of maps from a cell complex to a space with a single non-trivial homotopy group. This gives a result about covering spaces and one definition of cohomology groups.

Next we will introduce the category simplicial sets. These are a kind of abstract cell complex. We will see that one can do homotopy theory just using simplicial sets. In particular, questions about the homotopy theory of spaces can be changed into equivalent questions about the homotopy theory of simplicial sets.

Next we will look at homotopy colimits. This is a general technique for chopping a space up into pieces that are understood and then gluing back together to learn something about the space.

Further topics will depend on time and the interest of the participants.

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