# Courses and Seminars of Interest to Graduate Students offered by the Mathematics Department Spring, 2005

**MA 545: Functions of Several Variables and Related Topics**
**Instructor:** Prof. L. Brown, office: Math 704, phone: 49–41938, e-mail: `lgb@math.purdue.edu`
**Time:** MWF 10:30
**Description:** The course will cover some basic topics in classical and harmonic analysis, including the Hardy–Littlewood maximal function, the Lebesgue differentiation theorem for functions of several variables, generalized derivatives and Fourier transforms.

**MA 558: Abstract Algebra I**
**Instructor:** Prof. Heinzer, office: Math 636, phone: 49–41980, e-mail: `heinzer@math.purdue.edu`
**Time:** MWF 3:30
**Description:** I plan to cover material from the text *Introduction to Commutative Algebra* by M. F. Atiyah and I. G. Macdonald. In particular the course will cover: rings and ideals, modules, rings and modules of fractions, primary decomposition, integral dependence and valuations, chain conditions, Noetherian rings, Artin rings, discrete valuation rings and Dedekind domains, completions and dimension theory.
**Text:** M. F. Atiyah and I. G. Macdonald, *Introduction to Commutative Algebra*

**MA 572: Introduction to Algebraic Topology**
**Instructor:** Prof. McClure, office: Math 714, phone: 49–42719, e-mail: `mcclure@math.purdue.edu`
**Time:** MWF 1:30
**Prerequisite:** Basic point-set topology, up to compactness and connectedness. Some knowledge of the fundamental group is also desirable.
**Description:** This spring I will be teaching MA 572 (Introduction to Algebraic Topology) in a way which is designed to be useful for students in other fields, especially algebraic geometry, commutative algebra, and several complex variables. The first part of the course will be about line integrals in open subsets of the plane and their paths of integration.
**Text:** 1. Fulton *Algebraic Topology*
2. Munkres *Elements of Algebraic Topology*

**MA 598A: Topics in Elliptic Curves**
**Instructor:** Prof. Goins, office: Math 612, phone: 49–41936, e-mail: `egoins@math.purdue.edu`
**Time:** MWF 10:30
**Prerequisite:** MA 425, 503; some number theory
**Description:**We will cover the various guises of elliptic curves. We will begin with the basic definition of elliptic curves, present the chord-tangent construction of the group law, and explain the classical connection using integrals. We will then present various applications of elliptic curves, including Koblitz/Rusin's theory of relating areas of rational triangles; Klein's theory of finding roots of polynomials of degree 5; Lenstra's theory of factoring large numbers; and Wiles' theory for proving Fermat's Last Theorem.
Grading Policy: There will only be weekly homework.
**References:** 1. Joseph Silverman *The Arithmetic of Elliptic Curves*, GTM 106, Springer–Verlag
2. Dale Husemoller, *Elliptic Curves*, GTM 111, Springer-Verlag
3. Various research papers which will be made available
**Text:** The instructor's course lecture notes will be made available.

**MA 598B: Upscaling Tools for Porous Media**
**Instructor:** Prof. Cushman, office: Math 816, phone: 49–48040, e-mail: `jcushman@math.purdue.edu`
**Time:** TTh 12:00-1:15
**Description:** Most natural and artificial materials are porous. Examples include animal and plant tissues, the earth's mantle, ceramics, soils, aquifers and reservoirs, household insulation, drug delivery substrates, etc. Processes taking place in porous materials occur over a hierarchy of scales; examples include blood flow in animal tissue, groundwater flow, heat flow in insulators, mantle convection, and controlled release of drugs to the body. This course focuses on developing and employing the tools used to upscale (homogenize) processes in, and properties of, porous media. As time permits, topics discussed will include matched asymptotics, method of moments, central limit and Martingale methods, stochastic Green's functions, hybrid mixture theory, projection operators, continuous time random walks, stochastic convective approaches, renormalization group, and fractals. We will illustrate why and how different tools are used for different types of hierarchies, functional, structural, discrete and continuous.

**MA 598D: Numerical Partial Differential Equations**
**Instructor:** Prof. Santos, office: Math 808, phone: 49-41925, e-mail: `santos@math.purdue.edu`
**Time:** TTh 10:30-11:45
**Prerequisite:** MA 523 or consent of instructor
**Description:** The objective of the course is to teach the methodology for developing efficient and accurate algorithms for the numerical solution of partial differential equations in problems arising in applied mathematics, physics, geophysics and engineering.

The course will begin with polynomial approximation theory in Sobolev spaces. Then the theory and application of mixed finite element method for the solution of second order elliptic and parabolic problems will be covered, including the computer implementation of the algorithms to simulate groundwater flow and contaminant transport in porous media. Inverse problems related to the estimation of hydrogeological parameters in soils will also be formulated and discussed.

The last part of the course will cover the theory of wave propagation in fluid-saturated poroviscoelastic materials and the analysis of iterative procedures for the approximate solution of the equations of motion in this type of media.

Applications to wave propagation in composite porous materials such as wave propagation in permafrost, use of seismic methods in shaley-sandstones and evaluation of freezing conditions of foods by ultrasonic techniques will be covered at the discretion of the instructor and depending on time limitations.
**Reference Textbooks:** 1. S. C. Brenner and L. R. Scott, *The Mathematical Theory of Finite Element Methods*, Springer, 1994.
2. P. G. Ciarlet, *The Finite Element Method for Elliptic Problems*, North-Holland, 1980.
3. H. Banks and K. Kunish, *Estimation Techniques for Distributed Parameter Systems*, Birkhauser, 1989.
4. W. Jury, W. R. Gardner and W. H. Gardner, *Soil Physics*, 5th Edition, J. Wyley, 1991.
5. A. Tarantola, *Inverse Problem Theory*, Elsevier, 1987.

**MA 598E: Topics in Geometric Function Theory**
**Instructor:** Prof. Eremenko, office: Math 450, phone: 49–41975, e-mail: `eremenko@math.purdue.edu`
**Time:** TTh 10:30-11:45
**Prerequisite:** MA 530, 544; 531 is desirable but not absolutely necessary.
**Description:** List of Topics:
- Method of extremal length
- Riemann surfaces
- Hyperbolic geometry
- Quasiconformal mappings
- Applications to holomorphic dynamics

**Texts:** 1. Ahlfors, *Conformal Invarians*, out of print
2. Ahlfors, *Quasiconformal mappings*, out of print
3. Milnor, *Dynamics in one complex variable*, Vieweg, 1999.

**MA 598F: Toric Varieties, an Introduction**
**Instructor:** Prof. Walther, office: Math 746, phone: 49–41959, e-mail: `walther@math.purdue.edu`
**Time:** TTh 12:00-1:15
**Prerequisite:** Interest in commutative algebra. Knowledge of the material of the algebra core course. Having taken Commutative Algebra is a plus but not required. Some ideas (like "singular homology" or "manifold" or "CW–complex") of Algebraic Topology are helpful but not required.
**Description:** The first steps into toric geometry can be taken in several ways, most notably through semigroup rings or through torus actions. We shall take the semigroup ring approach which is very concrete, and develop a theory of varieties which look locally like the spectrum of a semigroup ring. A semigroup in general is a set with a multiplication that behaves reasonably (has associativity, and if you are lucky an identity). We will look at semigroups that are subsets of $\mathbb{Z}^n$, the n–fold product of the integers. From a semigroup one can make a ring, by interpreting the elements of the semigroup as exponents. The semigroup being Abelian this gives a commutative ring. Toric varieties arise as the geometric objects that come with such a ring through taking the spectrum.

The semigroup may be sometimes viewed as the integral points of the positive real cone over some polyhedron, and this opens the road for combinatorial inspections which turn out to be very fruitful. Triangulations and similar constructions lead to resolutions of singularities and other blow–up operations that are familiar from algebraic geometry.

Topologically, a toric variety is a space that arises by gluing together a bunch of things along common tori, that is, spaces that look like $\mathbb{C}^n$ without its coordinate axes. These spaces allow actions by another torus which gives the name. Since a torus is a very nice object, algebraically and combinatorially, one can mimic ideas like CW–complexes (with torus building bricks), determine Picard groups in combinatorial fashion, link them to the geometry of the toric variety and so on.

If time permits, we will also investigate how toric varieties can be viewed in a way that generalizes projective space and its homogeneous coordinate ring, we will study how modules arise over a toric variety, and maybe say some things about Batyrev's mirror construction. (This is supposed to explain why lots of physicists are interested in toric varieties and their cohomology.)
Contents:

- varieties, schemes, sheaves and some general setup of algebraic geometry.
- semigroups of $\mathbb{Z}^n$.
- fans, and their associated toric varieties (schemes).
- Weil and Cartier divisors, torus invariant or otherwise, and the Picard group. The Cox coordinate ring.
- Cohomology of toric varieties with coefficients in line bundles and otherwise.
- The moment map, a way of viewing a toric variety over $\mathbb{C}$ as a real compact set.
- Some intersection theory on toric varieties.
- Some remarks on mirror symmetry.
- Cohomology of toric varieties and local cohomology of Stanley–Reisner rings.
- Some algebra of semigroup rings: Bass numbers, local cohomology, resolutions, multiplier ideals.

**Text:** We will use the book by Fulton (Princeton series), hopefully some notes of Mustata and Fulton rewriting this book, and some lecture notes by Sturmfels on toric varieties.

**MA 598G/EAS 591F: Introduction to Continuum Mechanics**
**Instructor:** Prof. Gabrielov, office: Math 648, phone: 49–47911, e-mail: `agabriel@math.purdue.edu`
**Time:** MWF 12:30
**Description:** This course is an introduction to the fundamental ideas and methods of continuum mechanics, with a view towards geophysical applications. The goal is to provide beginning graduate students with the physics background and mathematical tools necessary for more advanced, special topics courses in continuum mechanics and its applications in the geosciences. Prerequisites include standard undergraduate calculus sequence: linear algebra, differential equations, and multivariate calculus.
The topics to be covered include:

- Essential Mathematics
- Stress Principles
- Kinematics of Deformation and Motion
- Fundamental Laws and Equations
- Linear Elasticity
- Linear Viscoelasticity
- Rock Rheology
- Friction and Fracture
- Classical Fluids
- Geophysical Fluid Dynamics

The course will be based on the book *Continuum Mechanics for Engineers* by G. T. Mase and G. E. Mase, with additional material on Friction and Fracture, Rock Rheology, and Geophysical Fluid Dynamics from other sources.

**MA 598K: Algebraic Coding Theory**
**Instructor:** Prof. Moh, office: Math 638, phone: 49–41930, e-mail: `ttm@math.purdue.edu`
**Time:** MWF 11:30
**Prerequisite:** None. MA 554 will help.
**Description:** Code means "self-correcting code" in the literature. It is widely used in industry for telephone, e-mails, CD, photo from Mars etc. It is one of the greatest discoveries in 20th century. The course will consist of three parts:
(1) Hamming code based on Linear Algebra (two weeks).
(2) BCH, Reed-Solomon, Classical Goppa codes based on Polynomial Algebra (four weeks).
(3) Geometric Goppa code based on Algebraic Geometry (five weeks) over a finite field (nine weeks totally).
In this course, some "pure mathematics" will be shown as "applied mathematics".
**Text:** Lecture Notes.

**MA 598S: Representation Theory of Lie Groups**
**Instructor:** Prof. Shahidi, office: Math 650, phone: 49–41917, e-mail: `shahidi@math.purdue.edu`
**Time:** MWF 9:30
**Prerequisite:** MA 544 and 553.
**Description:** In this course we will study the representation theory of compact Lie groups. The theory is very similar to the finite dimensional representation theory of semisimple Lie algebras. We shall mainly follow Bröcker–Dieck's book on compact groups. We will cover:
1. Basic facts about Lie groups and algebras.
2. Basic facts about representation theory.
3. Peter–Weyl theorem and its consequences.
4. Weyl integration formula.
5. Review of root systems.
6. Irreducible characters and weights.
7. Remarks about non–compact groups.

**Text:** T. Bröcker and T. Dieck, *Representations of Compact Lie Groups*; GTM 98, Springer–Verlag.
**Recommended Reading:** 1. J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory*, GTM 9, Springer–Verlag.
2. J. Duistermaat and J. Kolk, *Lie Groups*, Universitext, Springer.

**MA 598T: Introduction to Microlocal Analysis**
**Instructor:** Prof. P. Stefanov, office: Math 742, phone: 49–67330, e-mail: `stefanov@math.purdue.edu`
**Time:** MWF 1:30
**Prerequisite:** MA 504, MA 547.
**Description:** This course is intended as an introduction to Microlocal Analysis. The basic idea of Microlocal Analysis is that we should study functions not only near a point $x$ but also near a point and a direction (in the phase space). We will develop the basic principles of the theory of pseudodifferential operators, construction of parametrix of an elliptic partial differential operator, the local theory of Fourier integral operators, and applications. Pseudodifferential operators appear naturally in solving linear partial differential equations: the inverse of an elliptic linear partial differential operator is not a differential operator, because the inverse of a polynomial is not a polynomial. Instead, it is a pseudodifferential operator. One can use this calculus to define functions of differential operators, for example, fractional derivatives, etc. Microlocal Analysis is the modern way of understanding linear PDEs and is becoming increasingly important for some questions in the theory of non–linear PDEs.

Basic knowledge of Fourier Transform and Distributions is required. Also general knowledge of Functional Analysis (Hilbert spaces and linear operators) is needed.

**MA 598U/STAT 598W: Design and Analysis of Financial Algorithms**
**Instructor:** Prof. Viens, office: Math 504, phone: 49–46035, e-mail: `viens@stat.purdue.edu`
**Time:** Arrange Hours
**Prerequisite:** The student must be interested in financial mathematics and must have a prior knowledge of Excel.
**Description:** Information technology (IT) has become a major function in the financial industry. The industry has been employing different software and programming languages to process and maintain the data, to price equity and fixed income derivatives and to predict the stock movement. With good programming skills, one can excel in his/her job performance. In this course, we expect to learn Excel VBA, C/C++, MATLAB and GAMS/CPLEX which are some of most useful programming tools in financial firms.

**MA 598W: Introduction to Algebraic K-theory**
**Instructor:** Prof. Wilkerson, office: Math 700, phone: 49–41955, e-mail: `wilker@math.purdue.edu`
**Time:** MWF 12:30
**Description:** K-theory originated with the study of vector bundles over topological spaces or varieties. Swan observed that this was a special case of studying projective modules over a ring. This has grown over the years to include applications to number theory and manifolds.

The text is a Milnor classic - concise but gentle on the reader. If time permits, slightly later work by Quillen will be sketched in the course of describing "higher" K-theories, and the Milnor, Lichtenbaum, and other conjectures discussed.

Some homological algebra would be helpful to a student, but advanced topics therein should not be required.
**Text:** John Milnor, *Introduction to K–theory*

**MA 611: Methods of Applied Mathematics I**
**Instructor:** Prof. SaBarreto, office: Math 604, phone: 49–41965, e-mail: `sabarre@math.purdue.edu`
**Time:** MWF 3:30
**Prerequisite:** MA 511, 544.
**Description:** Banach and Hilbert spaces; linear operators; spectral theory of compact linear operators; applications to linear integral equations and to regular Sturm-Liouville problems for ordinary differential equations.

**CS 615: Numerical Solution of Partial Differential Equations**
**Instructor:** Prof. Lucier, office: Math 634, phone: 49–41979, e-mail: `lucier@math.purdue.edu`
**Time:** MWF 10:30
**Description:** The two-point boundary value problem: existence and uniqueness, finite element approximations. The finite element method for elliptic and parabolic problems in two dimensions. Iterative methods for linear systems: the conjugate gradient method and multigrid. Introduction to finite difference methods for elliptic, parabolic, and hyperbolic partial differential equations.

Each student will contribute to a group computing project, which will extend an object-oriented framework for the finite element method for elliptic and parabolic partial differential equations in two dimensions.
**Text:** Peter Knabner and Lutz Angermann, *Numerical Methods for Elliptic and Parabolic Partial Differential Equations*, Springer, 2003.

**MA 631: Several Complex Variables**
**Instructor:** Prof. Catlin, office: Math 744, phone: 49–41958, e-mail: `catlin@math.purdue.edu`
**Time:** MWF 10:30
**Prerequisite:** MA 530
**Description:** Power series, holomorphic functions, representation by integrals, extension of functions, holomorphically convex domains. Local theory of analytic sets (Weierstrass preparation theorem and consequences). Functions and sets in the projective space $P^n$ (theorems of Weierstrass and Chow and their extensions).

**MA 639: Stochastic Processes II**
**Instructor:** Prof. Viens, office: Math 504, phone: 49–46035, e-mail: `viens@stat.purdue.edu`
**Time:** T 5:30 pm – 8:00 pm
**Prerequisite:** MA/STAT 638 or a solid course in measure-theoretic stochastic calculus.
**Description:** This is the continuation of MA/STAT 638. We will concentrate on specific chapters from the textbook, including Ch VI-IX (Local Times, Generators, Girsanov's theorem, Stochastic Differential Equations). Some material from another textbook (Karatzas and Shreve, *Brownian Motion and Stochastic Calculus*), and the instructor's own work, may also be used, especially to cover Feynman-Kac formulas and the connection to PDEs and Stochastic PDEs. New topics not treatable using martingales will also be investigated, including: stochastic integration with respect to Fractional Brownian Motion and other, more irregular Gaussian processes; anticipative stochastic calculus; Gaussian and non-Gaussian regularity theory.
**Text:** Daniel Revuz and Marc Yor, *Continuous Martingales and Brownian Motion*, 3rd ed., Springer V., 1998.

**MA 643: Methods of Linear and Nonlinear Partial Differential Equations II**
**Instructor:** Prof. N. Garofalo, office: Math 616, phone: 49–41971, e-mail: `garofalo@math.purdue.edu`
**Time:** TTh 10:30-11:45
**Description:** This course will be a continuation of MA 642, only in the sense that the material covered in the Fall semester constitutes a useful exposure to some of the tools and ideas used in pde's. In reality, the course will have a self-contained character, and can be profitably attended also by students who have not taken MA 642 in the Fall, provided that they are willing to make a serious effort. We will continue the study of some of the main trends in pde's of the second order, with special emphasis on topics which lie at the interface of classical analysis and geometry. Topics to be covered are $L^p$ theory for solutions of elliptic equations, including Moser's estimates, Alexandrov maximum principle, and the Calderon-Zygmund theory. We will discuss the Yamabe problem, its solution, and its ramifications. We will also give an introduction to the theory of minimal surfaces and generalized perimeters, and present some of the basic results in this subject.

**MA 644: Calculus of Variation**
**Instructor:** Prof. Danielli, office: Math 802, phone: 49–41920, e-mail: `danielli@math.purdue.edu`
**Time:** TTh 1:30-2:45
**Prerequisite:** The course will be essentially self-contained. The only prerequisites are familiarity with the Lebesgue integral and the first properties of $L^p$ spaces, as well as a basic knowledge of Sobolev spaces.
**Description:** The fundamental problem of the calculus of variations is the minimization of the integral functional

$$\mathcal{F}(u,\Omega) = \int_\Omega F(x,u,Du(x))\,dx$$

among all the functions $u$ taking prescribed boundary values on the boundary $\Omega$. In this course we will focus on its solution via the so-called *direct methods*, which consist in proving the existence of the minimum of the integral functional $\mathcal{F}$ directly, rather than resorting to its Euler equation. The central idea is to consider $\mathcal{F}$ as a real-valued mapping on the manifold of functions taking on $\partial\Omega$ the given boundary values and applying to it a generalization of Weierstrass' theorem on the existence of the minimum of a continuous function. There are two main issues in this approach. The first one is that semicontinuity, rather than continuity, is the key assumption to apply Weierstrass theorem to the functional $\mathcal{F}$, whereas the second one is to identify the Sobolev spaces as the proper function spaces for compactness results to hold. On the other hand, the solution of the existence problem in the Sobolev class opens up a series of questions about the regularity of the minimizers. This was a long-standing open problem, until the way to its solution (in a non-direct fashion, since it involves the Euler equation) was opened by the celebrated De Giorgi-Nash-Moser result concerning the Hölder continuity of solutions to uniformly elliptic PDEs in divergence form with bounded and measurable coefficients. A first step towards the use of direct methods in the regularity issue came from a 1982 paper by Giaquinta and Giusti, who proved the Hölder continuity of *quasi-minima*, that is functions $u$ for which

$$\mathcal{F}(u,K) \le Q\mathcal{F}(u+\varphi,K)$$

for every $\varphi$ with compact support $K \subset \Omega$. The notion of quasi-minima reduces of course to the one of minimum when $Q = 1$, but it is substantially more general, since it includes solutions of linear and nonlinear elliptic equations and systems.

The course will follow the path outlined above, and it will combine the classical approach to the subject with its latest developments. In particular, emphasis will be placed on presenting an unified treatment of the regularity of the minima of functionals in the calculus of variations, and of the solutions to elliptic equations and systems in divergence form. We will also explore the connections with the study of minimal surfaces and applications to optimal control theory.
**Text:** E. Giusti, *Direct methods in the calculus of variations*, World Scientific, 2002. (Note: The library does not have this book yet, but you are welcome to borrow my copy if considering taking this class)

**MA 646: Banach Algebras and $C^*$-algebras**
**Instructor:** Prof. L. Brown, office: Math 704, phone: 49–41938, e-mail: `lgb@math.purdue.edu`
**Time:** MWF 12:30
**Prerequisite:** MA 546 or equivalent
**Description:** $C^*$-algebras originally arose for use in quantum mechanics and also have important relationships with geometry, topology, and probability. Although the course will not cover these advanced applications, it will provide the background needed for them. The course will begin with Banach algebras and the Gelfand theory for commutative Banach algebras. Then comes the basic theory of $C^*$-algebras and representations, including the non-commutative Gelfand-Naimark theorem. The course will also include some of the basic theory of von Neumann algebras.
No text will be formally used, but here are some good references:

1. J. Conway, *A Course in Functional Analysis, Chapters VII,VIII*
2. J. Dixmier, *$C^*$-algebras and Their Representations*
3. John von Neumann, *Collected Works, Vol. 3*
4. G. Pedersen, *$C^*$-algebras and Their Automorphism Groups*
5. S. Sakai, *$C^*$-algebras and $W^*$-algebras*
6. P. Fillmore, *A User's Guide to Operator Algebras*
7. K. Davidson, *$C^*$-algebras by Example*

**MA 690A: Transcendental Algebraic Geometry**
**Instructor:** Prof. Arapura, office: Math 642, phone: 49–41983, e-mail: `dvb@math.purdue.edu`
**Time:** TTh 3:00-4:15
**Description:** This will be a continuation of my similarly titled course from the fall. Rather than telling you what topics I plan to cover, which may or not be meaningful, let me explain the guiding philosophy of the subject. Given a complex algebraic variety, consider its (singular) cohomology. This is a discrete invariant, which is fine enough to tell a rational curve apart from an elliptic curve, but it can't distinguish two nonisomorphic elliptic curves. But now suppose, we enhance the cohomology by keeping track of the Hodge decomposition on it, then we get a continuous invariant, which will distinguish nonisomorphic elliptic curves. The basic dream is that the cohomology of a variety with its Hodge structure should give a rather complete portrait of the variety in general. A rather specific instance of this philosophy is given by the Hodge conjecture which predicts that subvarieties, and maps between varieties, can be detected just from the above structure. There is one other aspect of this subject which is, at first glance, surprising and that is its connection to arithmetic. Let me explain this by way of an example. Suppose that $X_1$ and $X_2$ are smooth projective varieties and that $X_1$ can be cut up into pieces and reassembled to get $X_2$, then it is known that the Betti numbers are same. This is not as easy as it sounds. In fact, there are only two ways to prove this (1) reduce to finite fields and apply the Weil conjectures or (2) apply Deligne's theory of mixed Hodge structures...

So that's the subject in a nutshell. If we get half as a far as I hope to get, then we will already have gotten into fairly deep water. There won't be an official textbook for this class. I am planning to follow/write up notes as we proceed.

**MA 690G: Multiplicities and Chern Classes**
**Instructor:** Prof. Lipman, office: Math 750, phone: 49–41994, e-mail: `lipman@math.purdue.edu`
**Time:** MWF 11:30
**Description:** A continuation of MA 690G, fall, 2004.
**Text:** Paul Roberts, *Multiplicities and Chern classes in Local Algebra*, Cambridge.

**MA 692F: Mathematical Models in Biology**
**Instructor:** Prof. Feng, office: Math 814, phone: 49–41915, e-mail: `zfeng@math.purdue.edu`
**Time:** MWF 2:30
**Description:** This course is an introduction to the application of mathematical methods and concepts to the description and analysis of biological processes. The mathematical contents consist of difference and differential equations and elements of stochastic processes. The topics to be covered include dynamical systems theory motivated in terms of its relationship to biological theory, deterministic models of population processes, epidemiology, coevolutionary systems, structured population models, nonlinear dynamics and chaos, stochastic processes, and introduction to Mathematica and MATLAB (computer packages). Bio–mathematical research projects (in small group) may be carried out.
**Text and class materials:** 1. Class notes and Handouts and Articles on reserve.
2. Brauer and Castilli–Chavez, *Mathematical Models in Population Biology and Epidemiology* (optional)
3. Kot, *Elements of Mathematical Ecology* (optional)
4. Thieme, *Mathematics in Population Biology* (optional)

**MA 692S: Analysis and Implementaion of Spectral Methods**
**Instructor:** Prof. Shen, office: Math 806, phone: 49–41923, e-mail: `shen@math.purdue.edu`
**Time:** TTh 9:00-10:15
**Prerequisite:** A good knowledge of advanced numerical analysis, elementary functional analysis and applied PDE theory are essential. The course is suitable for graduate students in mathematics and those in science and engineering with a solid mathematical background.
**Description:** This is an advanced course on the analysis and implementation of spectral methods for solving partial differential equations (PDEs). We shall present theoretical results on spectral approximations as well as practical algorithms for implementing spectral methods. We shall specially emphasize on how to combine the spectral methods with suitable time discretization schemes for solving PDEs of current interest.
**Reference books:** 1. Jie Shen, Tao Tang and Li-Lian Wang, *Lecture notes*
2. Walter Gautschi, *Orthogonal Polynomials: Computation and Approximation*, Oxford University Press, Oxford, 2004.
3. C. Bernardi & Y. Maday, *Spectral Method*, in Handbook of Numerical Analysis, V. 5 (Part 2) eds. P. G. Ciarlet and L. L. Lions, North-Holland, 1997.

**MA 693A: Sheaf Theory and its Applications in Complex and Algebraic Geometry**
**Instructor:** Prof. Lempert, office: Math 728, phone: 49–41952, e-mail: `lempert@math.purdue.edu`
**Time:** MWF 9:30
**Prerequisite:** Qualifier level algebra and analysis, MA 631 or some course in algebraic geometry.
**Description:** Sheaves are a powerful tool/formalism in complex and algebraic geometry. In this course we start by introducing sheaves and sheaf cohomology groups on general topological spaces, then focus on coherent sheaves on complex and algebraic manifolds. We shall prove Cartan's basic Theorems A and B concerning sheaves on Stein manifolds, Oka's coherence theorem, the finiteness theorems of Cartan–Serre and Gravert, and delve into Serre's GAGA on the relation between analytic and algebraic objects in projective manifolds.
**References:** 1. R. Gunning, *Introduction to holomorphic functions of several variables*, I – III., Wadsworth & Brooks/Cole, 1990.
2. H. Gravert, R. Remmert, *Coherent analytic sheaves*, Springer, 1984
3. H. Gravert, R. Remmert, *Theory of Stein spaces*, Springer, 1979 or 2004.
4. P. Griffith, J. Adams, *Topics in algebraic and analytic geometry*, Princeton Univ. Press, 1974
5. J.-P. Serre, *Oeuvres (Collected papers)*, Springer, 1986.

**MA 693B: Quantum Mechanics**
**Instructor:** Prof. de Branges, office: Math 800, phone: 49–46057, e-mail: `branges@math.purdue.edu`
**Time:** MWF 10:30
**Description:** The symmetries of the three–dimensional space which serves as a model for the physical universe are explored. According to quantum mechanics the motion of an electron is described by self–adjoint transformations, representing position and momentum, in the Hilbert space of square integrable functions with respect to Lebesgue measure. Since these transformations satisfy commutative identities, their relationship is clarified by the Fourier transformation. Since the rotation group on three–dimensional space commutes with the Fourier transformation, the space decomposes into irreducible invariant subspaces which are left fixed by the Fourier transformation. The harmonic polynomials of a given degree determine the structure of these subspaces. There results a theory of atomic spectra which is presented by Hermann Weyl in his classical treatise on *The Theory of Groups and Quantum Mechanics*. The aim of the present course is to present a relationship to number theory. The rotations of three–dimensional space which leave fixed a cube with center at the origin form a group of order twenty–four. Zeta functions are constructed from those spherical harmonics which are left fixed by the group. An Euler product and a functional identity are obtained. An unconfirmed proof of the Riemann hypothesis applies to these functions. These constructions require no information about electrons other than symmetries and the vanishing of wave functions in a neighborhood of the origin in the position and momentum spaces. Physical forces presumably create these conditions, but no knowledge of them is required. Students need preparation in pure and applied mathematics.

**MA 694D: Topics on Differential Equations**
**NOTE: This course was mistakenly advertised as MA 694A. It should be MA 694D**
**Instructor:** Prof. Garofalo, office: Math 616, phone: 49–41971, e-mail: garofalo@math.purdue.edu
**Time:** TTh 12:00-1:15
**Description:** This is intended as the second part of a one year long course which aims at developing some recent trends in PDE's arising in diverse areas, such as sub-Riemannian, or CR geometry, several complex variables, control theory and mathematical finance. In this part of the course we will focus on various nonlinear and fully nonlinear PDE's which are the object of intense recent attention. For instance, we will develop a theory of convexity in a sub-Riemannian setting introducing different notions such as geometric convexity and convexity in the viscosity sense, and we will prove their equivalence. We will also establish several basic properties of the relevant convex functions, such as for instance their Lipschitz continuity with respect to the control metric and the analogue of the celebrated theorem of Busemann-Feller and A. D. Alexandrov on the existence a.e. of the second derivatives of a convex function. We will then connect such notions of convexity to a fundamental open question, the validity of an Alexandrov-Bakelman-Pucci type maximum principle, and its usefulness in problems in geometry and mathematical finance. In another direction we will develop a sub-Riemannian calculus on hypersurfaces in sub-Riemannian groups (for instance, in the Heisenberg group), introduce a notion of mean curvature and of minimal surface, and study several basic questions such as first and second variation of the relevant "area functional", local and global monotonicity. We will also develop a regularity theory for the minimal surfaces and discuss some results on the flow by mean curvature. The latter naturally arises, and plays an important role in, e.g., modeling neuronal perception in the cerebral cortex. The physiology of the problem will also be discussed. We will also prove the existence of the isoperimetric profiles in sub-Riemannian groups, and show that, under suitable restrictions, at least for the Heisenberg group they can be competely characterized and are suitable $C^2$ compact hypersurfaces of positive constant mean curvature. If time allows we will also discuss the CR Yamabe problem and present its complete solution due to Jerison-Lee, Gamara-Yacoub.

Many basic open problems will be discussed in detail.

Although exposure to the first part of the course is undoubtedly helpful, the course will have a self-contained character and can be profitably followed also by students who have a taste for pde's and geometry and who have not taken 694D in the Fall.

**MA 696A: Topics in Algebra and Algebraic Geometry**
**Instructor:** Prof. Abhyankar, office: Math 600, phone: 49–41933, e-mail: ram@math.purdue.edu
**Time:** TTh 1:30-2:45
**Description:** I shall survey many aspects of algebra and algebraic geometry. These will include several possible "Advanced Topics" themes as well as numerous possible "Thesis" problems. There are no prerequisites and all interested students are welcome. The following aspects may be discussed.
Contents:

- Theorems of Newton, Hensel, and Weierstrass
- Theorems of Zariski, Serre, and Suslin
- Analysis and Resolution of Singularities
- Rational and Polynomial Parametrization
- Intersections of Curves and Surfaces
- Polynomial maps and Jacobian Conjecture
- Constructive Galois Theory
- Calculation of Fundamental Groups
- Enumerative Combinatorics
- Invariant Theory and Young Tableaux

**NOTE: The course will start three weeks late, i.e., on 1 February 2005.**
**Text:** 1. Shreeram S. Abhyankar, *Algebraic Geometry for Scientists and Engineers* Amer Math Soc
2. Shreeran S. Abhyankar, *Enumerative Combinatorics of Young Tableaux*, Marcel Dekker

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## Seminars

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**Algebra and Algebraic Geometry Seminar,** Prof. Abhyankar
**Time: Thursday 4:30–6:00**

**Applied Analysis Seminar,** Prof. Petrosyan
**Time: Tuesday or Thursday afternoon**

**Automorphic Forms and Representation Theory Seminar,** Prof. Goldberg
**Time: Thursdays, 1:30**

**Commutative Algebra Seminar,** Prof. Heinzer
**Time: Wednesdays 4:30-5:20**

**Computational and Applied Math Seminar,** Prof. Shen
**Time: Fridays 3:30**

**Computational Finance Seminar,** Prof. Viens
**Time: Fridays 2:30**

**Function Theory Seminar,** Prof. Eremenko
**Time: Tuesday, time to be determined.**

**Geometric Analysis Seminar,** Prof. Lempert
**Time: Thursday 12:45-1:45**

**Foundations of Analysis Seminar,** Prof. de Branges
**Time: Thursday 10:30-11:20**

**Mathematical Biology,** Prof. Feng
**Time: MWF, 1:30**

**Operator Algebras Seminar,** Prof. Dadarlat
**Time: Tuesdays, 2:30-4:20**

**PDE Seminar,** Prof. Garofalo
**Time: Thursdays, 3:30**

**Scattering and Spectral Theory Seminar,** Prof. SaBarreto
**Time: Wednesday 4:30**

**Topology Seminar,** Prof. McClure
**Time: Tuesday 1:30-2:20**

**Working Algebraic Geometry Seminar,** Profs. Arapura and Matsuki
**Time: Wednesday 3:30-5:00**