Courses and Seminars of Interest to Graduate Students
offered by the
Mathematics Department
Spring, 2009

MA 55400, CRN: 22179, LINEAR ALGEBRA
Instructor: Prof. J.-K. Yu, office: Math 604, phone: 49–67414, e-mail: `jyu@math.purdue.edu`
Time: MWF 8:30
Prerequisite: a basic course in linear algebra, familiarity with the notion of a field, the ring of integers, and the ring of polynomials over a field.
Description: The course will cover most material from chapters 3-6 in the text (various normal forms, modules over a principal ideal domain, bilinear and hermitian forms) together with additional topics.
References: Roman *Advanced linear algebra*
Text: W. Adkins and S. Weintraub *Algebra*

MA 58400, CRN: 34045, ALGEBRAIC NUMBER THEORY
Instructor: Prof. F. Shahidi, office: Math 650, phone: 49–41917, e-mail: `shahidi@math.purdue.edu`
Time: MWF 9:30
Prerequisite: MA 55300 adn 55400.
Description: Dedekind domains, norm, discriminant, different, finiteness of class numbers, Dirichlet unit theorem, quadratic and cyclotomic extensions, quadratic reciprocity, decomposition and inertial groups, completions and local fields.
References: S. Lang *Algebraic Number Theory*
Text: G. Janusz, *Algebraic Number Fields*, GSM7, MAS, 1996.

MA 59800, CRN: 34039, ACOUSTICS OF POROUS MEDIA: THEORY, NUMERICS AND APPLICATIONS
Instructor: Prof. J. Santos, office: Math 416, e-mail:`santos@math.purdue.edu`
Time: TTh 12:00-1:15
Description: The course will describe the theory of wave propagation in fluid-saturated porous media, with applications to detection and characterization of hydrocarbon reservoirs, monitoring of CO_2 sequestration after injection and characterization of partially frozen porous media among others. The equations describing wave propagation in saturated porous media will be solved using finite element techniques. Computer implementation willl be discussed. Numerical upscaling techniques used to represent highly heterogeneous fluid–filled porous media will also be presented.
Course Contents

1) Derivation of the constitutive relations and equations of motion for fluid-saturated porous media (Biot's media). Relation with Darcy's law and thermodynamic considerations.
2) Determination of the coefficients in the constitutive relations and the viscodynamic coefficients in Biot's equations of motion in terms of the properties of the individual solid and fluid phases. Introduction of viscoelasticity employing the Correspondence Principle.
3) Analysis of the phase velocities and attenuation coefficients for the different types of body waves propagating in Biot's media.
4) Review of the Finite Element Method. Description of some finite element spaces in 1D, 2D and 3D. Analysis of the interpolation error. Mixed finite element spaces methods for solving elliptic and Maxwell equations.
5) Solution of elliptic problems using finite element methods. Error analysis.
6) Numerical solution of Biot's equations of motion using the finite element method in the 1D and 2D cases. Global and iterative parallelizable domain decomposition finite element procedures.
7) Extension of Biot's theory for the case when the porous matrix is composed of weakly coupled solids. Plane wave analysis. Application to wave propagation in gas-hydrate bearing sediments.
8) Definition of numerical upscaling procedures in Biot's media to determine associated complex frequency dependent plane wave and shear moduli. Application to wave propagation in patchy-saturated porous media.
9) Numerical solution of the coupled Biot's and Maxwell's equation in Biot's media using the finite element method. Seismoelectric and Electroseismic applications.

Description of Homework Assignements

1: Calculation of the coefficients in Biot's equations of motion for some materials using fortran or similar computer language.
2: Calculation of the phase velocities and attenuation coefficients for some fluid-saturated porous materials.
3: Numerical simulation for 1D wave propagation in Biot's media. Application to the analysis of attenuation and dispersion effects in partially saturated porous media.
4: Numerical simulation of 2D wave propagation in Biot's media. Serial and parallel implementations.
5: Computer implementation of numerical upscaling procedures in Biot's media. Application to seismic monitoring in CO_2 sequestration sites.
6: Numerical modeling of coupled electromagnetic and seismic waves in 1D fluid-saturated porous media.

MA 59800, CRN: 34040, MATHEMATICS OF DISPERSIVE PROCESSES
Instructor: Prof. J. Cushman, office: Math 416, phone: 49–48040, e-mail: `jcushman@math.purdue.edu`
Time: TTh 1:30-2:45
Description: Dispersive processes take place in almost all natural and engineered environments that involve fluctuations in a field variable such as velocity in turbulence and porous media flows, topographic elevation in landscapes, conductance in a semiconductor, price in an option contract or stock, and etc. This course focuses on developing a unified framework for these apparently disparate fields. We will cover many of the standard approaches for developing theories of dispersion: continuous time random walks, projection operator methods of statistical mechanics, renormalization group and generalized central limit theorems as applied to stochastic differential equations and their associated Fokker–Planck equations, and stochastic perturbation and Green's functions approaches, to name a few. The field equations that result from the various approaches will all be shown to have a universal form. Applications will be stressed

MA 59800, CRN: 34041, MODULI SPACES AND STACKS
Instructor: Prof. R. Kaufmann, office: Math 710, phone: 49–41205, e-mail: `rkaufmann@math.purdue.edu`
Time: TTh 9:00-10:15
Content: We will discuss stacks in the topological, algebraic geometric and differentiable categories. Applications will include moduli spaces, orbifolds, gerbes and a dash of Gromov–Witten theory.
Abstract: When considering questions like what is the quotient of a space by a group action, which is not necesarily nice, or what kind of space classifies all objects of a certain type – these spaces are usually called moduli spaces – one realizes that the quotient or universal space are not of the same type as those one started with. For instance, taking the quotient of a manifold by a Lie group action might not lead to a manifold. Also the moduli space of all Riemann surfaces does not behave as expected if one considers it merely as a space. The solution to these problems are orbifolds or stacks.

There has been an avid interest in stacks from many different aspects in the last years and the literature is growing exponentially as classical results are being transferred and extended to this new setting. Stacks play a central role in algebraic geometry in Gromov–Witten theory, but they also appear in number theory, topology and as orbifolds in differential geometry. In particular, the derived algebraic geometry approach in topology heavily uses stacks. The interest has also been stoked by physics, where stacks naturally appear as quotients of systems by symmetries.

In the course, we will start with the motivating examples of group actions and moduli spaces. We will then define stacks in topology, differential and algebraic geometry and look at basic properties of them. As further examples we will study orbifolds in more detail and look at a dash of Gromov–Witten theory.

MA 59800, CRN: 34042, THE RADON AND X–RAY TRANSFORMS. AN INTRODUCTION TO INVERSE PROBLEMS
Instructor: Prof. P. Stefanov, office: Math 448, phone: 49–67330, e-mail: `stefanov@math.purdue.edu`
Time: MWF 2:30
Prerequisite: Familiarity with distributions and Fourier transform (MA 54200) is highly desired.
Description: The course will focus on the Radon transform (integrals over hyperplanes) and on the X–ray transform (integrals along lines). We will study questions of invertibility, stability estimates, description of the range, and Helgason type of support theorems. The attenuated ray trnasform and the Doppler transform (integrals of vector fields) will be covered as well. The last part of the course will be devoted to local tomography and recovery of the wave front set.

Radon types of transforms play fundamental role in applications like medical imaging and geophysics.

The course is intended as an introduction to Inverse Problems through Radon type of transforms. We will also briefly discuss the generalization of those problems to nonEuclidean geometries.

MA 59800, CRN: 34044, TOPICS IN COMPLEX ANALYSIS
Instructor: Prof. A. Eremenko, office: Math 450, phone: 49–41975, e-mail: `eremenko@math.purdue.edu`
Time: TTh 12:00-1:15
Description: This is a second graduate course in one-dimensional complex analysis. The prerequisits are MATH 53000 and MATH 54400. The textbook is Rudin, *Real and Complex Analysis*, I plan to cover most of the second part of this book.

MA 59800, CRN: 22196, BRIDGE TO RESEARCH SEMINAR
Instructor: Prof. S. Bell, office: Math 628, phone: 49–41967, e-mail: bell@math.purdue.edu
Time: M 4:30
Description: The seminar has two main goals, both aimed at helping students early in their graduate career find their place in the department. The first is to help students discover what area of mathematics they might be interested in researching, as well as who they might like to work with. The second is to provide students with an opportunity to interact with faculty in a casual setting. This is achieved by having professors from the department give brief talks about their research area at a level that is accessible to those in their first and second year of graduate study.

MA 61100, CRN: 34046, METHODS OF APPLIED MATHEMATICS I
Instructor: Prof. D. Danielli, office: Math 620, phone: 49–41920, e-mail: danielli@math.purdue.edu
Time: MWF 10:30
Prerequisite: MA 51100, 54400.
Description: The purpose of this course is to present the most fundamental theorems of functional analysis, keeping applications in mind. Topics covered include metric spaces; Banach spaces; linear transformations; the Fredholm-Riesz-Schauder theory and elements of spectral theory for compact operators; Hilbert spaces and spectral theory for self-adjoint operators. Applications to ordinary and partial differential equations, as well as to integral equations, will be discussed.
Text: A. Friedman, *Foundations of modern analysis*, Dover

MA 61500, CRN: 22198, NUMERICAL METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS I (meets with CS 61500)
Instructor: Prof. Z. Cai, office: Math 412, phone: 49–41921, e-mail: zcai@math.purdue.edu
Time: TTh 10:30-11:45
Prerequisite: MA 51400, MA 52300. Authorized equivalent courses or consent of instructor may be used in satisfying course prerequisites.
Description: Finite element method for elliptic partial differential equations; weak formulation; finite-dimensional approximations; error bounds; algorithmic issues; solving sparse linear systems; finite element method for parabolic partial differential equations; backward difference and Crank-Nicholson time-stepping; introduction to finite difference methods for elliptic, parabolic, and hyperbolic equations; stability, consistency, and convergence; discrete maximum principles.

MA 64300, CRN: 22201, METHODS OF PARTIAL DIFFERERENTIAL EQUATIONS II
Instructor: Prof. P. Bauman, office: Math 718, phone: 49–41945, e-mail: bauman@math.purdue.edu
Time: MWF 9:30
Prerequisite: MA 64200.
Description: Continuation of MA 642. Topics to be covered are Lp theory for solutions of elliptic equations, including Moser's estimates, Aleksandrov maximum principle, and the Calderon-Zygmund theory. Introduction to evolution problems for parabolic and hyperbolic equations, including Galerkin approximation and semigroup methods. Applications to nonlinear problems.

MA 66100, CRN: 22202, MODERN DIFFERENTIAL GEOMETRY
Instructor: Prof. H. Donnelly, office: Math 716, phone: 49–41944, e-mail: hgd@math.purdue.edu
Time: MWF 1:30
Prerequisite: MA 56200.
Description: A foundational course in Riemannian geometry. Topics include the Levi–Civita connection, geodesics, normal coordinates, Jacobi fields. Emphasis on curvature and its relation with topology. Familiarity with differentialble manifolds, tensor fields, and differential forms is assumed.
Text: John M. Lee, *Riemannian Manifolds: An Introduction to Curvature*, Springer–Verlag, 1997

MA 66500, CRN: 34676: ALGEBRAIC GEOMETRY II
Instructor: Prof. D. Arapura, office: Math 642, phone: 49–41983, e-mail: dvb@math.purdue.edu
Time: TTh 12:00-1:15
Description: Imagine starting an introductory course on integral calculus with the definition of Lebesgue measure. While logically correct, this would be a disaster pedagogically. The choices for a second course in algebraic geometry, such as this one, are analogous. I could go through chapters II and III of [H] or I and II of [GH] methodically, but by the end of the semester you may have no idea what this stuff is good for. Or, I could take certain technical constructions as black boxes and apply them in (hopefully) interesting ways. I prefer to do the latter.

To focus ideas, I will be concentrate on the so called Lüroth problem. The problem asks whether a field between $\mathbb{C}$ and a field of rational functions over $\mathbb{C}$ is also a field of rational functions. In one variable, this is true by Lüroth's theorem. Although stated algebraically, this is a basic result in the theory of algebraic curves (or Riemann surfaces). In two variables, the problem has a positive solution by Castelnuovo's rationality theorem for algebraic surfaces. This is a somewhat deeper result which will require considerable preparation (sheaves, cohomology, intersection theory for divisors...), and it will take a good part of the semester to set up everything. The story doesn't end here. By the 1970's, counterexamples were found in three or more dimensions. If time permits, I will say something about the Clemens-Griffiths example, which will require quick tour of Hodge theory, Abelian varieties, and intermediate Jacobians.

This is a second course in algebraic geometry. So I will assume that everyone knows basic algebraic geometry (for example from Prof. Włodarczyk's class from the fall).
References: 1. [GH] Griffiths, Harris *Principles of Algebraic Geometry*
2. [H] Hartshorne, *Algebraic Geometry*
3. [KSC] Kollár, Smith, Corti,, *Rational and Nearly Rational Varieties*

MA 69000, CRN: 22204, TOPICS IN ALGEBRA AND ALGEBRAIC GEOMETRY
Instructor: Prof. S. Abhyankar, office: Math 600, phone: 49–41933, e-mail: `ram@math.purdue.edu`
Time: TTh 3:00-4:15
Description: I shall discuss several topics in algebra and algebraic geometry. There are no prerequisites. All interested students are welcome. I shall use my new book Lectures on Algebra Volume I (published by World Scientific) as a text-book which the students are expected to purchase. Although this is an advanced book, the course will move much slower and can be taken as a basic course, understandable to scientists and engineers. During the course a softer version of the book will be produced. The students may also find it desirable to read my user-friendly book Algebraic Geometry For Scientists And Engineers (published by American Mathematical Society). Here is a list of some of the topics which may be covered:
(1) Expansions of polynomials of any degree in terms of sequences of other polynomials.
(2) Resultants, Discriminants, and solutions of higher degree polynomial equations in several variables.
(3) Newton's Theorem on Fractional Expansions.
(4) Implicit Function Theorem and Inverse Function Theorem.
(5) Intersection Theory and Bezout's Theorem.
(6) Classification and Resolution of Singularities of Curves, Surfaces, and Higher Dimensional Varieties.
(7) Divisors, Differentials, and Genus Formulas.

MA 69000, CRN: 22205, TOPICS IN COMMUTATIVE ALGEBRA
Instructor: Prof. Heinzer, office: Math 636, phone: 49–41980, e-mail: `heinzer@math.purdue.edu`
Time: MWF 3:30
Description: The course is planned to be a continuation of MA 690B of this fall. I hope to cover various topics in commutative algebra related to material in the text by W. Bruns and J. Herzog titled *Cohen-Macaulay Rings*, revised edition. Students enrolled in the course will be encouraged to actively participate by presenting material in class.

MA 69000, CRN: 22210, BRUHAT-TITS THEORY
Instructor: Prof. J-K Yu, office: Math 604, phone: 49–67414 e-mail: `jyu@math.purdue.edu`
Time: MWF 9:30
Description: Bruhat–Tits theory is the structural theory of p–adic reductive groups, developed by Bruhat and Tits. Such groups have extremely rich structures and a large part of them is encoded in a geometric object called the Bruhat–Tits building. The theory has many applications to number theory, representation theory and geometry. We will cover the theory for split and quasi–split groups, the Bruhat–Tits models, the descent theory and Moy–Prasad theory. Lecture notes will be handed out.

MA 69200, CRN: 22211, INTRODUCTION TO SPECTRAL METHODS FOR SCIENTIFIC COMPUTING
Instructor: Prof. J. Shen, office: Math 406, phone: 49–41923, e-mail: `shen@math.purdue.edu`
Time: TTh 1:30-2:45
Prerequisite: A good knowledge of calculus, linear algebra, numerical analysis and some basic programming skills are essential. Some knowledge of real analysis and functional analysis will be helpful but not necessary.
Description: This is an introduction course on spectral methods for solving partial differential equations (PDEs). We shall present some basic theoretical results on spectral approximations as well as practical algorithms for implementing spectral methods. We shall specially emphasize on how to design efficient and accurate spectral algorithms for solving PDEs of current interest.

The course is suitable for advanced undergraduate students in mathematics and graduate students in sciences and engineering.
Text: Jie Shen and Tao Tang *Spectraland High-Order Methods with Applications*

MA 69200, CRN: 22213, TOPICS IN IMAGING, SPECTRAL THEORY OF THE EARTH
Instructor: Prof. M. de Hoop, office: Math 422, phone: 49–66439, e-mail: `mdehoop@math.purdue.edu`
Time: TTh 10:30-11:45

MA 69300, CRN: 34047, COMMUTATIVE AND NON–COMMUTATIVE HARMONIC ANALYSIS
Instructor: Prof. L. Lempert, office: Math 728, phone: 49–41952, e-mail: `lempert@math.purdue.edu`
Time: TTh 12:00-1:15
Prerequisite: Mathematics as required on the Qualifier Examinations; basic general topology (MA 571 is more than enough); notions of a differentiable manifold, Hilbert space.
Description: The leitmotiv of this course is that a large part of mathematics can be understood in terms of symmetries (or: in terms of group representations). This will be illustrated historically starting with XVII-th century number theory and probability and concluding with XX-th century quantum mechanics; symmetries lurk behind all.

Topics touched upon: Early probabilities; representing integers by quadratic forms; Dirichlet's work on: Fourier series, Gauss sums, primes in arithmetic progressions; the birth of noncommutative representation theory; partial differential equations; Weyl's work on: representation theory of compact Lie groups, quantum mechanics; symmetries in quantum mechanics according to Neumann and Wigner.
Recommended Text: G. M. Mackey *The Scope and History of Commutative and Noncommutative Harmonic Analysis,* American Mathematical Society, 1992

MA 69300, CRN: 34048, ENTIRE FUNCTIONS
Instructor: Prof. de Branges, office: Math 800, phone: 49–46057, e-mail: `branges@math.purdue.edu`
Time: MWF 9:30
Description: The theory of entire function as it developed in the nineteenth and twentieth centuries intended applications to zeros of zeta functions. A proof of the Riemann hypothesis has been obtained which uses some of these now classical methods and also new methods which can be seen as extensions of the classical ones. A basic course in entire functions, which requires only information taught for qualifying examinations, is now offered for the purpose of presenting the proof of the Riemann hypothesis. Special techniques taught include weighted Hardy spaces and derived Hilbert spaces of entire functions as presented in *Hilbert Spaces of Entire Functions* (1968) on reserve in the library.

MA 69300, CRN: 34049, TOPICS IN C* ALGEBRAS AND GROUP REPRESENTATIONS
Instructor: Prof. M. Dadarlat, office: Math 708, phone: 49–41940, e-mail: `mdd@math.purdue.edu`
Time: MWF 2:30
Prerequisite: Math 54400 and some background in Functional Analysis such as Math 54600
Description: The first few lectures will be devoted to the basic theory of C*-algebras, including the Gelfand transform. Then we will cover topics such as group C*-algebras, the classic representation theory of compact groups, induced representations, Mackey's machine.
References:
Jacques Dixmier, *C*-algebras*
Barry Simon, *Representations of Finite and Compact Group*
I. Raeburn and D. P. Williams, *Morita Equivalence and Continuous-Trace C*-Algebras*
Gerald B. Folland, *A Course in Abstract Harmonic Analysis*
Text: We will follow no specific textbook.

MA 69400, CRN: 34103, TOPICS IN STOCHASTIC DIFFERENTIAL EQUATIONS
Instructor: Prof. F. Baudoin, office: Math 438, phone: 49–41406, e-mail: `fbaudoin@math.purdue.edu`
Time: MWF 11:30
Description: The purpose of this course is to propose an introduction to some advanced topics in the theory of stochastic differential equations and their applications. In a first part of the course, we will focus on small time behaviour properties and study the stochastic Taylor expansion for solutions of stochastic differential equations and show that it provides a powerful tool in:

- The study of heat kernels for parabolic equations in small times:
- Numerical approximation methods for solutions of parabolic and related functional inequalities.

In a second part, we will focus on long time behaviour properties and related functional inequalities.
A good knowledge in Stochastic processes is required and the course will partially follow the book [1].
References: F. Baudoin, *An introduction to the geometry of stochastic flows*, Imperial College Press, 2005.

MA 69600, CRN: 34038, INTRODUCTION TO KAEHLER GOEMETRY
Instructor: Prof. Yeung, office: Math 712, phone: 49–41942, e-mail: `yeung@math.purdue.edu`
Time: MWF 10:30
Prerequisite: MA 56200, 53000
Description: The purpose of the course is to introduce foundational materials for Kaehler geometry, bring in some analytic techniques and study their applications in complex manifolds and algebraic geometry. The final goal is to understand higher dimensional complex manifolds or algebraic varieties from an analytic or geometric point of view. The first one third of the course will be devoted to introductory materials, essentially with no prerequisite. We can find this in the first few chapters of books of Kodaira and Morrow, Griffiths and Harris, or Mok. Then we will go through some standard techniques from analysis or partial differential equations such as L^2 estimates and harmonic maps. Finally we will get into more advanced topics, including multiplier ideal sheaves and its applications such as invariance of plurigenera, and existence of canonical metrics such as metrics with constant curvature on a line bundle and extremal metrics.
References: 1. Kodaira, K., Morrow, J., *Complex manifolds*
2. Griffiths, P., Harris, J., *Principle of algebraic geometry*
3. Mok, N., *Metric rigidity theorems on locally hermitian symmetric spaces.*

Seminars

Algebra and Algebraic Geometry Seminar, Prof. Abhyankar
Time: Thursday 4:30–6:00

Applied Math Lunch Seminar, Prof. Buzzard
Time: Friday 11:30

Automorphic Forms and Representation Theory Seminar, Profs. Goldberg and Yu
Time: Thursdays, 1:30

Bridge to Research Seminar, Prof. Bell
Time: Mondays 4:30

Commutative Algebra Seminar, Profs. Heinzer and Ulrich
Time: Wednesdays 4:30

Computational and Applied Math Seminar, Prof. Shen
Time: Fridays 3:30

Computational Finance Seminar, Profs. Figueroa-Lopez and Viens
Time: Mondays 4:30

Function Theory Seminar, Prof. Eremenko
Time: flexible.

Geometric Analysis Seminar, Prof. Yeung
Time: Monday 3:30

Foundations of Analysis Seminar, Prof. de Branges
Time: Thursday 9:30

Number Theory Seminar, Prof. Goins
Time: Thursday, 3:30

Operator Algebras Seminar, Prof. Dadarlat
Time: Tuesdays, 2:30

PDE Seminar, Prof. Bauman
Time: Thursdays, 3:30

Probability Seminar, Prof. Sellke
Time: Mondays 3:30

Spectral and Scattering Theory Seminar, Prof. Sá Barreto
Time: Wednesday 4:30

Topology Seminar, Prof. Kaufmann
Time: Thursday 3:30

Working Algebraic Geometry Seminar, Profs. Arapura and Matsuki
Time: Wednesday 3:30-5:00