Courses and Seminars of Interest to Graduate Students
offered by the
Mathematics Department
Spring, 2010

MA 53100: CRN:45634 FUNCTIONS OF A COMPLEX VARIABLE II
Instructor: Prof. Eremenko, office: Math 450, phone: 49–41975, e-mail: `eremenko@math.purdue.edu`
Time: TTh 1:30-2:45
Prerequisite: 53000
Description: Stein and Sharkachi book that were not covered in Math 530, and as many of other topics as time permits. Fourier analysis in complex domain, Meromorphic functions, Elliptic, gamma, zeta and theta functions, analytic continuation, Riemann surfaces and linear differential equations.
Text: Stein and Shakarchi, *Complex Analysis*

MA 54500: CRN:22178 FUNCTIONS OF SEVERAL VARIABLES AND RELATED TOPICS
Instructor: Prof. Bañuelos, office: Math 830, phone: 49–41908, e-mail: `banuelos@math.purdue.edu`
Time: MWF 10:30
Prerequisite: Math 544. But, we will review, depending on the need, some 544 topics including differentiation of monotone functions, the Radon–Nikodym theorem, duality of L^p spaces, and Fubini's theorem.
Description: This course will cover some of the basic tools of analysis which are useful in many areas of mathematics including PDE's, stochastic analysis, modern harmonic analysis and complex analysis. Specific topics include: Covering and decomposition geometric theorems (Vitali, Wiener, and Whitney); the Hardy–Littlewood maximal function; convolutions; approximations to the identity and applications to density theorems in L^p and to the basic boundary value problems in the upper half space of $\mathbb{R}^n$ (the Dirichlet problem for the heat equation and the Laplacian with L^p–data); the Fourier transform and its basic properties on L^1 and L^2 (including Plancherel's theorem); interpolation of linear operators; singular integral theory and its applications to the Beltrami and Cauchy Riemann equations in several dimensions; fractional integrals; the Hardy-Littlewood-Sobolev and Nash inequalities, from the point of view of the heat semigroup.
Text: No text book is required. The course follows my (unpublished) lecture notes *"A Second Semester Course in Analysis".*
References: 1. E. M. Stein *Singular Integrals and Differentiability Properties of Functions*
2. E. H. Lieb and M. Loss, *Analysis*hfill

MA 55800: CRN:22181 ABSTRACT ALGEBRA II
Instructor: Prof. Abhyankar, office: Math 600, phone: 49–41933, e-mail: `ram@math.purdue.edu`
Time: TTh 3:00-4:15
Description: This course will cover the basic algebra which is needed for studying advanced algebra, algebraic geometry, number theory, complex analysis, and applications to computer aided design and geometric modelling. The lectures will be so designed that they will be understandable without having taken MA55700 which I taught in Fall 2009. I shall use my book *Lectures on Algebra* Volume I (published by World Scientific) as a text-book which the students are expected to buy. The students may also find it desirable to read my book *Algebraic Geometry For Scientists And Engineers* (published by American Mathematical Society). Here is a list of possible topics to be dealt with:
(1) Group Theory including Sylow Theorem and Burnside's Theorem.
(2) Rings and Modules incluing, Euclidean Domains, Principal Ideal Domains, and Unique factorization Domains.
(3) Fundamental Theorems of Galois Theory.
(4) Polynomials and Power series incluing Hensel's Lemma and Newton's Theorem as well as the Preparaton Theorem of Weierstrass.
(5) Valuation Theory and Integral Dependence.
(6) Resultants and Discriminants leading to solutions of higher degree polynomial equations in several variables.
(7) Primary decomposition in noetherian rings and noetherian modules.
(8) Artinian rings and lengths of modules.
(9) Local Rings and Graded Rings.
(10) Algebraic Varieties including their spectral and modelic versions.
(11) Hilbert Nullstellensatz and Noether Normaliztion.
(12) Cohen-Macaulay Rings and Gorenstein Rings.
(13) Hibert Syzygies and Unique Factorization in Regular Local Rings.
(14) Resolution of Singularities by means of Quadratic and Monoidal Transformations.

MA 58600: Mathematical Logic II MA 58600 was CANCELLED on November 12.
Instructor: Prof. Lipshitz, office: Math 722, phone: 49–46525, e-mail: `lipshitz@math.purdue.edu`
Time: TTh 10:30-11:45
Description: Topics to be covered will include some of the following, depending on the interests of the class and the instructor:

1. Beginning model theory (including saturated and homogeneous models, omitting types theorems, complete and model-complete theories).
2. Decidable theories (Elimination of quantifiers: $\mathbb{N}$ with addition, $\mathbb{R}$ with semi-algebraic and subanalytic structures, p-adic fields).
3. Undecidable theories (Hilbert's tenth problem. Other undecidable problems).
4. Beginning nonstandard analysis.
5. Models of computation.

MA 59800: CRN:43028 ABELIAN VARIETY
Instructor: Prof. Tong Liu, office: Math 738, phone: 49–41946, e-mail: `tongliu@math.purdue.edu`
Time: MWF 1:30
Prerequisite: Algebraic geometry (66500) or equivalent
Description: Definitions and basic properties, abelian variety over the complex numbers, the theorem of cube, isogenies, quotients by finite group schemes, the dual abelian variety, Rosati Involution, Weil Pairing, l-adic representations of abelian variety.

MA 59800, CRN: 34041, ACOUSTICS OF POROUS MEDIA: THEORY, NUMERICS AND APPLICATIONS
Instructor: Prof. J. Santos, e-mail: `santos@math.purdue.edu`
Time: TTh 1:30-2:45
Description: The course will describe the theory of wave propagation in fluid-saturated porous media, with applications to detection and characterization of hydrocarbon reservoirs, seismic monitoring of CO_2 sequestration after injection and characterization of partially frozen porous media among others. The equations describing wave propagation in fluid–saturated porous media will be solved using finite element techniques. Computer implementation will be discussed. Numerical upscaling techniques used to represent highly heterogeneous fluid–filled porous media will also be presented.
Course Contents

1. Derivation of the constitutive relations and equations of motion for fluid-saturated porous media (Biot's media). Relation with Darcy's law and thermodynamic considerations.
2. Determination of the elastic and viscodynamic coefficients in Biot's equations of motion in terms of the properties of the individual solid and fluid phases. Introduction of viscoelasticity employing the Correspondence Principle.
3. Analysis of the phase velocities and attenuation coefficients for the different types of body waves propagating in Biot's media.
4. Review of the Finite Element Method. Description of some finite element spaces in 1D, 2D and 3D. Analysis of the interpolation error. Mixed finite element spaces methods for solving elliptic and Maxwell equations.
5. Solution of elliptic problems using finite element methods. Error analysis.
6. Numerical solution of Biot's equations of motion using the finite element method in 1D and 2D bounded domains. Global and iterative domain decomposition finite element procedures.
7. Extension of Biot's theory for the case when the porous matrix is composed of weakly coupled solids. Plane wave analysis. Application to wave propagation in gas-hydrate bearing sediments.
8. Definition of numerical upscaling procedures in Biot's media to determine associated complex frequency dependent plane wave and shear moduli. Application to wave propagation in patchy-saturated porous media.
9. Numerical solution of the coupled Biot's and Maxwell's equation in Biot's media using the finite element method. Seismoelectric and Electroseismic applications.

Description of Homework Assignements

1: Calculation of the coefficients in Biot's equations of motion for some materials using fortran or similar computer language.
2: Calculation of the phase velocities and attenuation coefficients for some fluid-saturated porous materials.
3: Numerical simulation for 1D wave propagation in Biot's media. Application to the analysis of attenuation and dispersion effects in partially saturated porous media.
4: Numerical simulation of 2D wave propagation in Biot's media. Serial and parallel implementations.
5: Computer implementation of numerical upscaling procedures in Biot's media. Application to seismic monitoring in CO_2 sequestration sites.
6: Numerical modeling of coupled electromagnetic and seismic waves in 1D fluid-saturated porous media.

MA 59800: CRN:43026 ALGEBRAIC GROUPS
Instructor: Prof. David Goldberg, office: Math 640, phone: 49–41919, e-mail: goldberg@math.purdue.edu
Time: TTh 9:00–10:15
Description: This course will discuss the structure theory of algebraic groups (essentially closed subgroups of $GL_n(F)$) over an algebraically closed field, F. If time permits, topics for other fields, and in particular, rationality questions, may be addressed. The main goal is classification of reductive groups over algebraically closed fields. Prerequisites: MA 55300, MA 55400. Topics covered include some rudimentar algebraic geometry, definitions of algebraic groups, Jordan Decomposition, elementary representation theory, classification of tori, parabolic subgroups, Borel subgroups, root data. As mentioned above, time permitting, which may depend on student backgrounds) the classification of reductive groups over arbitrary fields may be discussed. Text for this course will be T.A. Springer's *Linear Algebraic Groups*, or Borel's book of the same title.

MA 59800, CRN: 22196, BRIDGE TO RESEARCH SEMINAR
Instructor: Prof. S. Bell, office: Math 628, phone: 49–41967, e-mail: bell@math.purdue.edu
Time: M 4:30
Description: The seminar has two main goals, both aimed at helping students early in their graduate career find their place in the department. The first is to help students discover what area of mathematics they might be interested in researching, as well as who they might like to work with. The second is to provide students with an opportunity to interact with faculty in a casual setting. This is achieved by having professors from the department give brief talks about their research area at a level that is accessible to those in their first and second year of graduate study.

MA 59800, CRN: 45729, MICROPOLAR FIELD THEORIES OF CONTINUUM PHYSICS
Instructor: Prof. J. Cushman, office: Math 416, phone: 49–48040, e-mail: jcushman@math.purdue.edu
Time: TTh 12:00-1:15
Description: A polar body is a material possessing substructure that cannot be represented as a classical continuum. Polar fluids and solids are common in many areas of science; examples include the earth's lithosphere, blood in micro capillary beds, and colloids in natural soils and aquifers. This course will provide the mathematical/physical tools necessary to study polar materials. The basic premise is that every point in the body has 12 degrees of freedom as opposed to the classical description with three degrees (position for solids or velocity for fluids). The additional nine degrees of freedom that polar materials possess are micro shear, micro stretch and micro rotation. These additional degrees of freedom can be represent by a second order tensor and require additional field equations beyond mass, linear and angular momentum, energy and entropy.

MA 59800, CRN: 43025, INTRO TO INVERSE PROBLEMS
Instructor: Prof. P. Stefanov, office: Math 448, phone: 49–67330, e-mail: stefanov@math.purdue.edu
Time: MWF 2:30
Prerequisite: MA-523, functional analysis methods for PDEs, pseudo-differential operators.
Description: This course will serve as an introduction to Inverse Problems through a few fundamental examples. The first one is the mathematical model of the Electrical Impedance Tomography - a problem posed first by Calderon. We will also study inverse scattering problems and inverse problems for hyperbolic equations.

The goal of the course is to demonstrate the analysis of some of the most important questions in Inverse Problems: uniqueness, stability, and reconstructive algorithms through those specific problems. In contrast to classical tomography, Calderon's problem is non-linear and severely ill posed. Inverse Problems for hyperbolic equations are usually not so ill posed. We will discuss the reasons for that. We will also discuss the proper way to linearize an ill-posed non-linear problem.

The course is based on the progress made in Inverse Problems in the last 20 years, including some very recent results.

MA 61100, CRN: 34046, METHODS OF APPLIED MATHEMATICS I
Instructor: Prof. N. Garofalo, office: Math 616, phone: 49–41971, e-mail: garofalo@math.purdue.edu
Time: TTh 10:30-11:45
Prerequisite: MA 51100, 54400.
Description: The purpose of this course is to present the most fundamental theorems of functional analysis, keeping applications in mind, especially to the solvability of the basic problems of second order partial differential equations, such as the Dirichlet and Neumann problems, and the solvability of the Cauchy problem for evolution equations such as the heat and the wave equations.

Topics covered include metric spaces; Banach spaces; linear transformations; the Fredholm-Riesz-Schauder theory and elements of spectral theory for compact operators; Hilbert spaces and spectral theory for self-adjoint operators. We will also cover the Hille-Yosida theorem and the basics of the theory of Sobolev spaces. Applications to ordinary and partial differential equations, as well as to integral equations, will be discussed.
Suggested textbooks: 1) H. Brezis, *Analyse Fonctionnelle*, Dunod (paperback)
2) A. Friedman, *Foundations of modern analysis*, Doveri (paparback)

MA 61500, CRN: 43130, NUMERICAL METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS I (meets with CS 61500)

Instructor: Prof. B. Lucier, office: Math 400, phone: 49–41979, e-mail: `lucier@math.purdue.edu`

Time: MWF 12:30

Prerequisite: MA 51400, MA 52300. Authorized equivalent courses or consent of instructor may be used in satisfying course prerequisites.

Description: Finite element method for elliptic partial differential equations; weak formulation; finite-dimensional approximations; error bounds; algorithmic issues; solving sparse linear systems; finite element method for parabolic partial differential equations; backward difference and Crank-Nicholson time-stepping; introduction to finite difference methods for elliptic, parabolic, and hyperbolic equations; stability, consistency, and convergence; discrete maximum principles.

MA 64300, CRN: 22201, METHODS OF PARTIAL DIFFERERENTIAL EQUATIONS II

Instructor: Prof. D. Danielli, office: Math 620, phone: 49–41920, e-mail: `danielli@math.purdue.edu`

Time: MWF 9:30

Prerequisite: MA 64200.

Description: This course is a continuation of MA 64200, but can also be taken independently with the instructor's consent. Topics to be covered are: 1. L^p theory for solutions of elliptic equations, including Moser's estimates, Aleksandrov maximum principle, and the Calderon-Zygmund theory. 2. Applications to nonlinear equations, with special focus on equations of mean curvature type. 3. Introduction to evolution problems for parabolic equations, including Galerkin approximation and semigroup methods.

References: 1. D. Gilbarg and N. S. Trudinger *Elliptic Partial Differential Equations of Second Order*
2. C. E. Kenig *Harmonic Analysis Techniques for Second Order Elliptic Boundary Value Problems*
3. E. Giusti *Minimal Surfaces and Functions of Bounded Variations*
4. L. C. Evans *Partial Differential Equations*
5. G. M. Lieberman *Second Order Parabolic Differential Equations*

MA 66100, CRN: 43124, MODERN DIFFERENTIAL GEOMETRY

Instructor: Prof. H. Donnelly, office: Math 716, phone: 49–41944, e-mail: `hgd@math.purdue.edu`

Time: MWF 11:30

Prerequisite: MA 56200.

Description: A foundational course in Riemannian geometry. Topics include the Levi–Civita connection, geodesics, normal coordinates, Jacobi fields. Emphasis on curvature and its relation with topology. Familiarity with differentialble manifolds, tensor fields, and differential forms is assumed.

Text: John M. Lee, *Riemannian Manifolds: An Introduction to Curvature*, Springer–Verlag, 1997

MA 69000, CRN: 22210: ETALE COHOMOLOGY

Instructor: Prof. D. Arapura, office: Math 642, phone: 49–41983, e-mail: `dvb@math.purdue.edu`

Time: TTh 10:30-11:45

Description: In 1949 Weil made a remarkable conjecture that given an equation with integer coefficients, the number of solutions over a finite field is governed in a precise way by the topology of the set of solutions over the field of complex numbers. Since these objects live in different worlds, one would need some kind of bridge between. In the early 1960's, Grothendieck and his school developed the etale topology and the associated cohomology theory, which provided the necessary bridge and more. By now, this has become a standard tool for algebraic geometers, and certain kinds of number theorists and topologists. My goal in this class is to provide an introduction to these ideas. I will assume that everyone knows some algebra (Galois theory, localizations and completions, regular rings, basic diagram chasing) and some algebraic geometry (quasiprojective varieties and perhaps schemes), but I'll develop whatever else we need from scratch. Rather than attempting to rush through this, my goal is to make things understandable. Here's a rough outline of what I think we can realistically do:

(1) Quick explanation/review of schemes.
(2) Flat, etale and smooth maps.
(3) Sheaves on a site.
(4) H^1 and torsors.
(5) Higher cohomology for sites with "enough points".
(6) Calculations of etale cohomology in some examples.
(7) Discussion of main results (probably) without proofs.

There won't be an official textbook, but I've listed some references below. Also I should mention that Milne's notes, which are similar to his book, can be downloaded for free off his website.

References:

Deligne et al, *Cohomologie Etale*, (SGA 4 1/2)
Freitag and Kiehl, *Etale cohomology*
Milne, *Etale cohomology*

MA 69000, CRN: 22204, REAL ALGEBRAIC GEOMETRY
Instructor: Prof. S. Basu, office: Math 742, phone: 49–48798, e-mail: `sbasu@math.purdue.edu`
Time: TTh 12:00-1:15
Description: Classically, algebraic geometry is studied over algebraically closed fields. Many theorems of algebraic geometry take their most satisfactory form when stated over algebraically closed fields – for example, the fact that a polynomial of degree d has exactly d roots counted with multiplicities. This fact is clearly false over the reals. However, for various reasons we are often interested in only the real solutions to polynomial equations. Real algebraic geometry is the study of sets defined by real polynomial equalities and inequalities– also known as semi-algebraic sets. Its roots go back to Hilbert's sixteenth and seventeenth problems, and it is now a very active area of research with connections to many other areas of mathematics such as model theory of o-minimal structures, complex geometry, optimization theory etc.

The course will cover the basics of real algebra and geometry including the definition and properties of real closed fields, real spectrum, positivstellensatz, geometric and topological properties of semi-algebraic sets, effective algorithms and notions of complexity. The course will be aimed at first/second year graduate students.
Textbooks: We will follow in parts the following three books.
1. *Real Algebraic Geometry* by Bochnak, Coste and Roy.
2. *Positive polynomials and sums of squares* by M. Marshall.
3. *Algorithms in Real Algebraic Geometry* by Basu, Pollack and Roy.

MA 69000, CRN: 22205, TOPICS IN COMMUTATIVE ALGEBRA
Instructor: Prof. W. Heinzer, office: Math 636, phone: 49–41980, e-mail: `heinzer@math.purdue.edu`
Time: MWF 12:30
Description: The course is planned to be a continuation of MA 69000 of this fall, but the course of this fall is not a prerequisite. I hope to cover various topics in commutative algebra related to material in the text by I. Swanson and C. Huneke titled *Integral Closure of Ideals, Rings and Modules* and the text by W. Bruns and J. Herzog titled *Cohen-Macaulay Rings*, revised edition. Students enrolled in the course will be encouraged to actively participate by presenting material in class.

MA 69200, CRN: 22211, SELECTED TOPICS IN SPECTRAL METHODS AND COMPUTATIONAL FLUID DYNAMICS
Instructor: Prof. J. Shen, office: Math 406, phone: 49–41923, e-mail: `shen@math.purdue.edu`
Time: TTh 1:30-2:45
Prerequisite: Advanced knowledge of numerical analysis and applied mathematics.
Description: This is an advanced course on selected optics in the numerical analysis of spectral methods and computational fluid dynamics. I shall present numerical schemes and their analysis for Navier-Stokes equations, phase-field models for incompressible multiphase flows, Helmholtz and Maxwell equations, and some high dimensional PDEs.

MA 69300, CRN: 34049, BIEBERBACH CONJECTURE
Instructor: Prof. de Branges, office: Math 800, phone: 49–46057, e-mail: `branges@math.purdue.edu`
Time: MWF 9:30
Description: An introduction to complex analysis is given as preparation for the proof of the Riemann Hypothesis. Complex analysis is treated as a theory of functions analytic on the unit disk. The traditional treatment of the Cauchy formula is preceded by a structural anlysis of the Hardy space of functions analytic in the unit disk. These functions are represented in the unit disk by square summable power series. The factorization of functions which are analytic and bounded by one in the unit disk is treated as a computation of invariant subspaces of contractive transformations of a Hilbert space into itself which are nearly isometric. The compostion of functions which are analytic and bounded by one in the unit disk is an underlying concept in the proof of the Bieberbach conjecure. The use of the Hardy space for the unit disk simplifies the estimation theory for coefficients of Riemann mapping functions which was obtained in 1984.

MA 69400: CRN:34103 DIFFUSION SEMIGROUPS AND APPLICATIONS
Instructor: Prof. F. Baudoin, office: Math 438, phone: 49–41406, e-mail: fbaudoin@math.purdue.edu
Time: MWF 10:30
Prerequisite: RECOMMENDED BACKGROUND:

- Partial differential equations: Laplace operator, the heat equation in $\mathbb{R}^n$.
- Functional analysis: Some elemenary aspects of the theory of unbounded operators on Hilbert spaces or Banach spaces.
- Differential geometry: Basic definitions and properties of manifolds.

Description: The purpose of this course is to provide to the student a self–contained aaccount on the theory of the semigroups associated to diffusion operators.
The course will focus on:

1. Diffusion semigroups: Diffusion operators, Hypoellipticity, L^2 theory, L^p theory, Hypercontractivity properties.
2. Applications of diffusion semigroups: Heat semigroup on a complete Riemannian manifold, Li–Yau inequality, Gaussian bounds for the heat kernel. Proof of the Bonnet–Myers theorem by semigroup techniques.

Lecture notes will be made available to the students.
References: 1. M. Reed, B. Simon: *Fourier Analysis, Self–adjointness*, Vol. II, 1975, Academic Press
2. E. B. Davies: *Heat kernels and spectral theory*, Cambridge Tracts in Mathematics, 92. Cambridge University Press, Cambridge, 1989.

MA 69600, CRN: 34038, TOPICS IN COMPLEX GEOMETRY
Instructor: Prof. Yeung, office: Math 712, phone: 49–41942, e-mail: yeung@math.purdue.edu
Time: MWF 12:30
Prerequisite: MA 56200, 53000
Description: The purpose of the course is to discuss some topics of current interests in Kaehler geometry, bring in some analytic techniques and study their applications in complex manifolds and algebraic geometry. The followings are some tentative topics to be discussed. We will try to begin from an elementary level.

1. The use of Bergman kernels in complex geometry.
2. The work of Donaldson, Mabuchi and others on Hitchin-Kobayashi correspondence.
3. Some geometric results related to hypergeometric differential equations.
4. Special, fake and exotic structures on four dimensional manifolds.
5. Some aspects of arithmetics and geometry.

References: 1. Kodaira, K., Morrow, J., *Complex manifolds*
2. Griffiths, P., Harris, J., *Principle of algebraic geometry*
3. Mok, N., *Metric rigidity theorems on locally hermitian symmetric spaces.*

Seminars

Algebra and Algebraic Geometry Seminar, Prof. Abhyankar
Time: Thursday 4:30–6:00

Applied Math Lunch Seminar, Prof. Buzzard
Time: Friday 11:30

Automorphic Forms and Representation Theory Seminar, Prof. Goldberg
Time: Thursdays, 1:30

Bridge to Research Seminar, Prof. Bell
Time: Mondays 4:30

Commutative Algebra Seminar, Profs. Heinzer and Ulrich
Time: Wednesdays 4:30

Computational and Applied Math Seminar, Prof. Shen
Time: Fridays 3:30

Computational Finance Seminar, Profs. Figueroa-Lopez and Viens
Time: Fridays 2:30

Function Theory Seminar, Prof. Eremenko
Time: flexible.

Geometric Analysis Seminar, Prof. Yeung
Time: Monday 3:30

Foundations of Analysis Seminar, Prof. de Branges
Time: Thursday 9:30

Number Theory Seminar, Prof. Goins
Time: Thursday, 3:30

Operator Algebras Seminar, Prof. Dadarlat
Time: Tuesdays, 2:30

PDE Seminar, Prof. Bauman
Time: Thursdays, 3:30

Probability Seminar
Time: Tuesday 10:30

Spectral and Scattering Theory Seminar, Prof. Sá Barreto
Time: Thursday 4:30

Topology Seminar, Prof. Kaufmann
Time: Thursday 3:30

Working Algebraic Geometry Seminar, Profs. Arapura and Matsuki
Time: Wednesday 3:30-5:00