Courses and Seminars of Interest to Graduate Students
offered by the
Mathematics Department
Spring, 2012

MA 53000: CRN:22127 FUNCTIONS OF A COMPLEX VARIABLE I
Instructor: Prof. Buzzard, office: Math 402, phone: 49–41937, e-mail: `buzzard@math.purdue.edu`
Time: MWF 1:30
Prerequisite: MA 54400
Description: Complex numbers and complex-valued functions of one complex variable; differentiation and contour integration; Cauchy's theorem; Taylor and Laurent series; residues; conformal mapping; special topics. More mathematically rigorous than MA 52500.
Text: Stein and Shakarchi *Complex Analysis* , (Princeton Lectures in Analysis II), Princeton University Press

MA 54500: CRN:22178 FUNCTIONS OF SEVERAL VARIABLES AND RELATED TOPIS
Instructor: Prof. Bañuelos, office: Math 428, phone: 49–41977, e-mail: `banuelos@math.purdue.edu`
Time: MWF 11:30
Prerequisite: MA 54400. But depending on need, some topics from MA 54400 will be reviewed.
Description: This course will cover some of the basic tools of analysis that are extremely useful in many areas of mathematics, including PDE's, stochastic analysis, harmonic analysis and complex analysis. Specific topics covered in the course include: "Geometric lemmas" (Vitali, Wiener, etc.) and "geometric decomposition theorems" (Whitney, etc.) and their applications to differentiation theory and to the Hardy–Littlewood maximal function; convolutions; approximations to the identity and their applications to boundary value problems in R^d with L^p-data; the Fourier transform and its basic properties on L^1 and L^2 (including Plancherel's theorem); interpolation theorems for linear operators (Marcinkiewicz, Riesz–Thorin); the basic (extremely elegant and useful) Calderón-Zygmund singular integral theory and some of its applications; the Hardy-Littlewood-Sobolev inequalities for fractional integration and powers of the Laplacian and other elliptic operators; the inequalities of Nash and Sobolev viewed from the point of the heat semigroup in R^d.
Text: No text book is required. The course follows my lecture notes *"Lectures in Analysis"*.
Recommended are:
(1) E. M. Stein, *"Singular Integrals and Differentiability Properties of Functions"*,
(2) L. Grafakos *"Modern Fourier Analysis"*,
(3) E. H. Lieb and M. Loss, *"Analysis"*.

MA 57200: CRN:46826 INTRODUCTION TO ALGEBRAIC TOPOLOGY
Instructor: Prof. R. Kaufmann, office: Math 710, phone: 49–41205, e-mail: `rkaufman@math.purdue.edu`
Time: TTh 9:00-10:15
Description: The course is an introduction to algebraic topology. The focus will be on homology and cohomology theory. This subject is important to topology, but also to many other fields, such as differential, symplectic and algebraic geometry, number theory, mathematical physics, etc.

We will treat the classical simplicial and singular homology and cohomology, but we also plan to cover CW complexes and differential forms.
Text: James R. Munkres, *Elements of Algebraic Topology*

MA 59800: CRN:43028 MATHEMATICAL UPSCALING
Instructor: Prof. Cushman, office: Math 416, phone: 49–48040, e-mail: `jcushman@math.purdue.edu`
Time: TTh 12:00-1:15
Description: One of the primary goals of physical scientists is to develop accurate predictive capabilities on a variety of natural scales and/or between natural scales. To rationally accomplish this, numerous researchers have been studying ways to propagate information over scale hierarchies, both functionally and structurally. In this course we will categorize and then review a number of methods for upscaling over both discrete and continuous hierarchies. Topics to be considered include, but are not limited to, projection operator formalisms, generalized central limit theorems, homogenization, moment methods, continuous time random walks, spectral integral methods, recursive Eulerian closures and diagrammatic perturbation methods.

MA 59800, CRN: 59356, USEFUL ALGEBRA
Instructor: Prof. Abhyankar, office: 600, phone 49–41933, e-mail: `ram@math.purdue.edu`
Time: TTh 3:00-4:15
Description: Many versatile topics from classical concrete algebra are not easily accessible in recent books. More importantly, the connections between the classical and modern treatments of these topics are rarely discussed. The knowledge of these topics is found to be extremely useful for grasping modern abstract algebra, algebraic geometry, as well as their applications to science and engineering. The aim of this course is to bring back in to circulation these topics such as:

- Permutations and Combinations, Progressions and Series, Factorization.
- Partail Fractions, Lagrange Interpolation and lagrange Resolvent.
- Theorems of Newton, Gauss, and Weierstrass, on Polynomials and Power Series.
- Determinants, Resultants, Discriminants, and Jacobians.
- Solving Simultaneous Linear Equations in Several Variables.
- Solving Higher Degree Equations in One and Many Variables.
- Graphical Representations of Equations in One and Many Variables.
- Singularities of Curves and Surfaces.
- Evolution of Polynomials and Power Series into Rings and Fields of Modern Algebra.

The lectures will be expository in nature and so will be accessible to everyone. Thus there are no formal prerequisites and all interested students are welcome. I am planning to write lecture notes for this course which will be distributed in class. I expect these notes to be similar to my former lecture notes entitled *Algebraic Geometry for Scientist and Engineers*. In other words, the porposed notes should do for algebra what the former notes did for algebraic geometry. In the course, undergrad and g rad experience will be distinguished by requiring the grads to do extra work such as oral or written presentation of some advanced topic.
Texts: 1) *Algebraic Geometry for Scientists adn Engineers*, by S. S. Abhyankar, American Mathematical Society.
2) Notes on Useful Algebra by S. S. Abhyankar, to be distributed in class.

MA 59800, CRN: 22196, BRIDGE TO RESEARCH SEMINAR
Instructor: Prof. S. Bell, office: Math 628, phone: 49–41967, e-mail: bell@math.purdue.edu
Time: M 4:30
Description: The seminar has two main goals, both aimed at helping students early in their graduate career find their place in the department. The first is to help students discover what area of mathematics they might be interested in researching, as well as who they might like to work with. The second is to provide students with an opportunity to interact with faculty in a casual setting. This is achieved by having professors from the department give brief talks about their research area at a level that is accessible to those in their first and second year of graduate study.

MA 61100: CRN:34046 METHODS OF APPLIED MATH I
Instructor: Prof. Danielli, office: Math 620, phone: 49–41920, e-mail: `danielli@math.purdue.edu`
Time: NOTE NEW TIME MWF 10:30-11:20
Prerequisite: MA 51100, 54400
Description: The purpose of this course is to present the most fundamental theorems of operator theory and functional analysis, keeping applications in mind. Topics covered will include metric spaces; Banach spaces; analytic and geometric formulations of the Hahn-Banach theorem; the principle of uniform boundedness, the closed graph theorem and the open-mapping theorem; weak topologies; reflexive and separable spaces; Hilbert spaces; compact operators and the Fredholm-Riesz-Schauder theory; spectral theory for self-adjoint operators. Applications to ordinary and partial differential equations, as well as to integral equations, will be discussed. If time permits, we will cover the Hille-Yosida theorem and its applications to evolution equations.
Text: A. Friedman, *Foundations of modern analysis*, Dover
References: H. Brezis, *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, Springer

MA 61500: CRN:43130 NUMERICAL METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS I
Instructor: Prof. Lucier, office: Math 400, phone: 49–41979, e-mail: `lucier@math.purdue.edu`
Time: MWF 9:30-10:20
Prerequisite: MA 51400, 52300
Description: Numerical Methods For Partial Differential Equations I (CS 615) Finite element method for elliptic partial differential equations; weak formulation; finite-dimensional approximations; error bounds; algorithmic issues; solving sparse linear systems; finite element method for parabolic partial differential equations; backward difference and Crank-Nicholson time-stepping; introduction to finite difference methods for elliptic, parabolic, and hyperbolic equations; stability, consistency, and convergence; discrete maximum principles.

MA 64300: CRN:52797 METHODS OF PARTIAL DIFFERENTIAL EQUATIONS II
Instructor: Prof. Garofalo, office: Math 616, phone: 49–41971, e-mail: `garofalo@math.purdue.edu`
Time: TTh 12:00-1:15
Prerequisite: MA 64200
Description: Continuation of MA 64200. Topics to be covered are L_p theory for solutions of elliptic equations, including Moser's estimates, Aleksandrov maximum principle, and the Calderon-Zygmund theory. Introduction to evolution problems for parabolic and hyperbolic equations, including Galerkin approximation and semigroup methods. Applications to nonlinear problems.

MA 66100: CRN:43124 MODERN DIFFERENTIAL GEOMETRY
Instructor: Prof. Donnelly, office: Math 716, phone: 49–41944, e-mail: `hgd@math.purdue.edu`
Time: TTh 3:00-4:15
Prerequisite: MA 56200.
Description: A foundational course in Riemannian geometry. Topics include the Levi–Civita connection, geodesics, normal coordinates, Jacobi fields. Emphasis on curvature and its relation with topology. Familiarity with differentialble manifolds, tensor fields, and differential forms is assumed.
Text: John M. Lee, *Riemannian Manifolds: An Introduction to Curvature*, Springer–Verlag, 1997

MA 68400: CRN:58243 CLASS FIELD THEORY
Instructor: Prof. Liu, office: Math 604, phone: 49–41946, e-mail: `tongliu@math.purdue.edu`
Time: MWF 12:30-1:20
Prerequisite: MA 58400 of the instructors approval.
Description: Ideles, adeles, L-functions, Artin symbol, reciprocity, local and global class fields, Kronecker-Weber Theorem.

MA 69000: CRN:22204 POLYTOPES RINGS AND K-THEORY
Instructor: Prof. Caviglia, office: Math 608, phone: 49–41973, e-mail: `gcavigli@math.purdue.edu`
Time: MWF 1:30–2:20
Description: This course is intended as a continuation of the analogous course offered in Fall 2011. The focus will be on chapter 2 and on part of chapter 3 of the following book by Bruns and Gubeladze: Polytopes Rings and K-Theory [Springer Monographs in Mathematics (2009)]

MA 69000: CRN:22210 Cardinality
Instructor: Prof. de Branges, office: Math 8800 phone: 49–46057, e-mail: `branges@math.purdue.edu`
Time: MWF 9:30–10:20
Description: Cardinality, the comparability of sets, is a fundamental hypothesis of mathematical analysis which is usually stated in the equivaent form of the axiom of choice. An argument due to Cantor shows that the cardinality of a set is less than the cardinality of the class of its subsets. The existence of infinite sets which are not countable results. Such sets are fundamental to mathematical analysis, for example in proving that a Cartesian product of compact sets is compact. The course is offered to graduate students who would like to know more about cardinality (including research issues) than they have learned in preparation for qualifying examinations.

MA 69000: CRN:58253 LINKAGE AND DUALITY
Instructor: Prof. Ulrich, office: Math 618, phone: 49–41972, e-mail: `ulrich@math.purdue.edu`
Time: TTh 1:30–2:45
Description: I plan to cover the theory of canonical modules, local cohomology, and local duality. This will lead into a treatment of linkage (or liaison), which is a method of classifying ideals and projective varieties. If time permits, I will talk about residual intersections and applications to Rees algebras.

The first part of the course (duality) is more basic and can be viewed as a continuation of MA 65000, The second part (linkage) is more specialized. The course is meant for students currently taking MA 65000 and for more senior graduate students that took commutative algebra in the past. The material is accessible to anyone with a good background in commutative algebra.

No text will be used, but good sources for the material of the first part are:

1. W. Bruns and J. Herzog, *Cohen-Macaulay Rings*, Cambridge Univ. Press
2. D. Eisenbud, *Commutative algebra with a view towards algebraic geometry*, Springer

MA 69000: CRN:60225 ABELIAN VARIETIES AND MODULI
Instructor: Prof. Arapura, office: Math 642, phone: 49–41983, e-mail: `dvb@math.purdue.edu`
Time: TTh 3:00-4:15
Description: An abelian variety is a sort of higher dimensional version of an elliptic curve. Over $\mathbb{C}$, it is a complex torus which is also a projective algebraic variety. Just as elliptic curves evolved, historically, from the study of elliptic integrals, abelian varieties came out of the more complicated class of abelian integrals, which in modern terms are periods of 1-forms on Riemann surfaces. Nowadays there are plenty of other good reasons to study them. Simplest of all is that their structure can be understood in fairly explicit terms. But perhaps a more basic reason is that their study is both rich and beautiful, involving a bit of algebra (endomorphism algebras), Fourier analysis (theta functions), topology (Chern classes) and geometry (everything else). My goal in the first part of the class is to go through the basic theory of abelian varieties, primarily over $\mathbb{C}$.

It is likely the some of you are familiar with fact the elliptic curves are parametrized by $\mathbb{C}$ using the j-function. I'll go over this in any case. The story in higher rank is similar but much more subtle. In the second part, I want explain the basic ideas for construction of the moduli (or universal parameter) space of all abelian varieties. These are basic examples of Shimura varieties.

I'll assume only a basic knowledge of algebraic geometry and/or manifold theory and some complex analysis.

References: 1. Birkenhake, Lange, *Complex Abelian Varieties*
2. Griffiths, Harris, *Principles of Algebraic Geometry*
3. Mumford, *Abeliain Varieties*

MA 69200: CRN:52831 APPLIED INVERSE PROBLEMS
Instructor: Prof. de Hoop, office: Math 422, phone: 49–66439, e-mail: `mdehoop@math.purdue.edu`
Time: TTh 10:30-11:45
Description: The subject will be Waves and inverse scattering. The emphasis will be on techniques from computational microlocal and harmonic analysis.

MA 69200: CRN:22211 ADVANCED TOPICS IN SCIENTIFIC COMPUTING AND NUMERICAL ANALYSIS
Instructor: Prof. Shen, office: Math 406, phone: 49–41923, e-mail: `shen@math.purdue.edu`
Time: MWF 2:30-3:20
Prerequisite: A good knowledge on the polynomial approximation results in Sobolev spaces and on the implementation of spectral methods.
Description: A variety of scattered topics will be covered in this course. Among them are:
* Numerical methods for Helmholtz equation, Maxwell equation and MHD
* Numerical approximation for High dimensional PDEs with applications to kinetic equations
* Phase-field model for complex fluids

No textbook is needed.

MA 69300: CRN:52834 COMPUTATIONAL METHODS IN POTENTIAL THEORY AND CONFORMAL MAPPING
Instructor: Prof. Bell, office: Math 628, phone: 49–41967, e-mail: `bell@math.purdue.edu`
Time: MWF 1:30-2:20
Prerequisite: The only prerequisites for this course are MA 53000 and an understanding of L^2 as a Hilbert space.
Description: I will cover some developments in complex analysis arising from the remarkable discovery made in 1978 by N. Kerzman and E. M. Stein that the centuries old Cauchy Transform is nearly a self adjoint operator when viewed as an operator on L^2 of the boundary. This new, but fundamental, result represented a shift in the bedrock of complex analysis. It has allowed the classical objects of potential theory and conformal mapping in the plane to be constructed and analyzed in new and very concrete terms.

Another theme of the course will be applications of quadrature domains in complex analysis. The unit disc is the simplest example of a quadrature domain because the average value of an analytic function with respect to area measure is the value of the function at the origin. A quadrature domain has the property that the average value of an analytic function with respect to area measure is given as a finite complex linear combination of the values of the function and its derivatives at finitely many points. I will show how quadrature domains in the plane can be used to simplify the objects of potential theory, and how they give rise to a new "Riemann Mapping Theorem for multiply connected domains." The results give rise to new methods to compute the classical objects of potential theory and conformal mapping. Among the objects I will define and study this way are the Bergman kernel, the Szegő kernel, the Poisson kernel, and the Green's function, as well as cannonical conformal mappings to representative domains.
References: S. Bell, *The Cauchy transform, potential theory, and conformal mapping*, CRC Press, 1992

MA 69600, CRN: 34038, TOPICS IN COMPLEX GEOMETRY
Instructor: Prof. Yeung, office: Math 712, phone: 49–41942, e-mail: yeung@math.purdue.edu
Time: MWF 11:30–12:20
Description: The followings are some tentative topics to be discussed.

1. Introduction to complex hyperbolicity.
2. Introduction to diophantine geometry.
3. Elliptic surface.
4. Multiplier ideals and modules.
5. Jacobians, abelian varieties and solitons.

We will probably only be able to discuss several interesting aspects of the topics.

Seminars

Algebra and Algebraic Geometry Seminar, Prof. Abhyankar
Time: Thursday 4:30–6:00

Applied Math Lunch Seminar, Prof. Buzzard
Time: Friday 11:30

Automorphic Forms and Representation Theory Seminar, Prof. Goldberg
Time: Thursdays, 1:30

Bridge to Research Seminar, Prof. Bell
Time: Mondays 4:30

Commutative Algebra Seminar, Profs. Heinzer and Ulrich
Time: Wednesdays 4:30

Computational and Applied Math Seminar, Prof. Shen
Time: Fridays 3:30

Computational Finance Seminar, Profs. Figueroa-Lopez and Viens
Time: Thursdays, 6:30 pm – 7:20 pm

Function Theory Seminar, Prof. Eremenko
Time: Wedneday, time varies.

Number Theory Seminar, Prof. Goins
Time: Thursday, 3:30

PDE Seminar, Prof. Bauman
Time: Thursdays, 3:30

Topology Seminar, Profs. Kaufmann and McClure
Time: Thursday 3:30