MA 55400 **LINEAR ALGEBRA** **3 credits**	**Prof. Moh** **Office: MATH 638** **Phone: 4-1930** **Email: ttm**	•Modules over an arbitrary commutative ring. Kernel and image, direct sums and products, the isomorphism theorem and the fundamental theorem of linear algebra. Definition of exact sequences • Modules over principal ideal domains. The examples of a square matrix and f.g. Abelian groups. Main theorem about f.g. modules over a PID. Application in the case of fixed linear operator over a finite dimensional vector space. Torsion factors and elementary factors of a linear operator and how to calculate these. Minimal polynomial. Rational canonical form and Jordan decomposition theorem. Eigenvalues, eigenvectors and characteristic polynomials. CayleyHamilton Theorem. References: T.T.Moh Algebra, pp 196-214 or K.Ho man & R Kunze pp 181-261. ff • Several equivalent definitions of the determinant of a square matrix, including the axioms of determinants, and the Laplace formulas. The Cauchy-Binet formulas for the determinant. The multilinear produces and alternative produce of vector spaces and modules. The tensor product and exterior product of modules and vector spaces. Definitions and main properties of projective and free modules. Dual modules and double duals. Chain complex and cochain complex. Applications to advanced calculus. Definitions of exact sequences and resolutions. Describe Hilbert's Syzygies theorem and global dimension. Projective, injective and flat modules. Ext and Tor. Reference: D. Dummit & R Foote Abstract Algebra pp339-381. 744-763 •Inner product spaces. Gram-Schmidt orthogonalization. Projections and the least square approximations. Definition of the adjoint of a linear operator. Definition and properties of selfadjoint and unitary operators. Definition of normal operators. The spectral theorem for normal operators over C. • Linear Operators on inner product spaces, especially Hilbert space. Sesquilinear forms. Positive forms. Positive linear operators. The applications of positivities of the principal minors to the local minimal problem for n variables in calculus and high dimensional ellipsoids. Singular value decomposition theorem and its applications • Bilinear forms. Symmetric bilinear forms. Matrix congruence. Sylvester's law of inertia. Signatures. Applications to physics and geometry. Lorantz group SO(1, 3). Reference K.Homan & R Kunze pp 270-385 There will be weekly homeworks, a mid-term and a final exam. The main reference books are K.Homan & R Kunze and my lecture notes ff which will be provided to students free.

MA 57200 **INTRODUCTION IN ALGEBRAIC TOPOLOGY** **3 credits**	**Prof. McClure** **Office: MATH 714** **Phone: 4-2719** **Email: mcclure**	A common difficulty that students have with algebraic topology is building intuition. Fulton's book Algebraic Topology, An Introduction deals with this issue by starting with a concrete low-dimensional situation, namely line integrals defined in open subsets of the plane. He then shows how the more sophisticated parts of the subject grow naturally from this beginning. Fulton's book is also useful for seeing connections between algebraic topology and other areas such as algebraic geometry and complex analysis (Fulton is an algebraic geometer). I will begin by using Fulton's book for the first five weeks. The rest of the semester will be from Massey's textbook A Basic Introduction to Algebraic Topology, which fits well with Fulton's point of view. Prerequisite: basic point-set topology, up to compactness and connectedness. Some knowledge of the fundamental group is also desirable.
MA 59800 **HOMOLOGICAL ALGEBRA AND DERIVED CATEGORIES** **3 credits**	**Prof. R. Kaufmann** **Office: MATH 710** **Phone: 4-1205** **Email: rkaufman**	We will treat the classical constructions of homological algebra including Tor and Ext.Then we will go on to treat them in the more modern framework of derived categories. Along the way we will encounter several categorical constructions. At the end we plan to briefly talk about ensuing subjects like triangulated categories, exceptional collections and model categories. The course will be based on the books of Weibel and Gelfand-Manin.
MA 59800 **INTRODUCTION TO GAMMA CONVERGENCE** **3 credits**	**Prof. Yip** **Office: MATH 432** **Phone: 4-1941** **Email: yip**	This is an introductory course on Gamma Convergence which studies the convergence of variational functionals. It is an extremely versatile technique in understanding the limit properties of functionals which demonstrate small scale oscillatory behaviors and/or exhibit the appearance of singular (defect) structures. These phenomena arise in many applications such as homogenization, image processing, and phase transitions in materials science. Some of the topics to be covered include: (1) homogenization of integral functionals; (2) segmentation and free boundary value problems; (3) phase transitions; (4) derivation of continuum limits from discrete problems. (5) gradient flow and dynamical problems. This course is intended for students with interests in analysis and applications.

		Textbooks: No official textbooks. But I will basically follow: (1) Gamma Convergence for Beginners, Andrea Braides (available in library); (2) Topics on Concentration Phenomena and Problems with Multiple Scales, Anrea Braides-Valeria Chiado Piat (eds.) (available online through Purdue library page) (3) selected journal papers. Prerequisites: 544 (real analysis) and 523 (partial differential equations, preferred)
MA 59800 **SCATTERING THEORY AND RESONANCES** **3 credits**	**Prof. Stefanov** **Office: MATH 448** **Phone: 6-7330** **Email: stefanov**	We will start with an introduction to Scattering Theory which studies systems in infinite domains and their behavior as the distance and/or the time get large. Typical examples are the perturbed wave or the Schrodinger equation in the whole space or the wave equation outside a compact obstacle. We will sketch the proofs of the most basic inverse scattering problems in obstacle and potential scattering and their relationship with analogous boundary value problems. We will also list and discuss some open inverse scattering problems. The second part of the course will be about resonances (scattering poles). They are an analog of the eigenvalues of a self-adjoint Hamiltonian but there are essential differences - they are not real, and even though the Hamiltonian is self-adjoint, they are better understood with tools from non-selfadjoint spectral theory. (Quantum) resonances are a fundamental object in Quantum mechanics since those with small imaginary parts are easily observable. The main topics I will cover are Weyl type of bounds for their asymptotic distribution, relation between resonances and geometry, respectively the classical mechanical system, tunneling, etc. The course will be based on lecture notes freely available online by Johannes Sjostrand http://www.math.polytechnique.fr/~sjoestrand/CoursgbgWeb.pdf), Maciej Zworski (http://math.berkeley.edu/~zworski/res.pdf) and Richard Melrose (http://www-math.mit.edu/~rbm/lecstn.ps). I will also use some recent research articles. The proofs of the more advanced results will be merely sketched but I will try to explain the ideas. Strong background in PDEs and Functional analysis is required. Familiarity with Semiclassical Microlocal Analysis would be useful as well.

MA 64300 **METHODS OF PARTIAL DIFFERENTIAL EQUATIONS II** **3 credits**	**Prof. Danielli** **Office: MATH 620** **Phone: 4-1920** **Email: danielli**	This course will be a continuation of MA 64200, but only in the sense that we will continue to explore some of the most pervasive ideas used in the modern theory of elliptic and parabolic PDE's . The course will have a self-contained character, and can be attended also by students who have not previously taken MA 64200. Topics to be covered are: 1. The analysis of strong solutions to linear second-order elliptic equations. These are solutions to (non-divergence form) equations which possess second derivatives in a weak sense. The analysis consists of two parts: (a) the so called \$L^p\$-theory, which constitutes an analogue of the Schauder estimates for classical solutions; (b) the Alexandrov maximum principle, a priori bounds, and pointwise estimates of Holder and Harnack type. 2.The almost-everywhere regularity of minimal hypersurfaces, à la De Giorgi; The Dirichlet problem for minimal surfaces; The Bernstein problem. 3. Estimates for harmonic measure and Green's function; Fatou type theorems in non-smooth domains. 4. The maximum principle and the theory of weak solutions for linear parabolic PDE's. The relevant tools from harmonic analysis, potential theory, and geometric measure theory will be developed concurrently. Texts: 1. D. Gilbarg and N. S. Trudinger, Elliptic Partial Differential Equations of Second Order. 2. E. Giusti, Minimal Surfaces and Functions of Bounded Variations. 3. C. E. Kenig, Harmonic Analysis Techniques for Second Order Elliptic Boundary Value Problems. 4. G. M. Lieberman, Second Order Parabolic Differential Equations.
MA 69000 **MODULAR FORMS** **3 credits**	**Prof. De Branges** **Office: MATH 800** **Phone: 4-6057** **Email:branges**	A proposed proof of the Riemann hypothesis requires information about the zeta functions constructed from modular forms, originally constructed by Heche as functions analytic in the upper half-plane. A new construction of these functions is made in Fourier analysis on skew-fields in which they appear as kernels for a generalization of the Laplace transformation. A new interpretation of the Lapalace transformation is given as a spectral analysis of a Radon transformation which show that the transformation is maximal dissipative. Students are expected to have passed qualifying examinations but are otherwise required to have no special knowledge other than an interest in mathematics.

MA 69000 **REPRESENTATION THEORY AND AUTOMORPHIC FORMS** **3 credits**	**Prof. Shahidi** **Office: MATH 650** **Phone: 4-1917** **Email: shahidi**	This will cover the remaining aspects of representations of real groups which is needed in automorphic forms, followed by its applications to automorphic forms and number theory, through GL(2) and conjectures of Selberg and Ramanujan on eigenvalues of the Laplacian and Hecke operators, as well as other number theoretic problems. We will also discuss automorphic forms on other groups and give a survey of recent results on Langlands functoriality, including a hint of recent progress for classical groups as well as GSpin.
MA 69200 **MATHEMATICAL BIOLOGY** **3 credits**	**Prof. Feng** **Office: MATH 414** **Phone: 4-1915** **Email: zfeng**	Description: This special topic course focuses on recent advances in modeling studies for biological systems including both mathematical methods and modeling approaches and frameworks. The mathematical contents consist of difference and differential equations and elements of stochastic processes. The topics to be covered include dynamical systems theory motivated in terms of its relationship to biological theory, deterministic models of population processes, epidemiology, immunology, ecology, structured population models, nonlinear dynamics, and stochastic simulations. Bio-mathematical research projects (in small group) may be carried out. References: 1.) Brauer and Castillo-Chavez, Mathematical Moldes in Population Biology and Epidemiology (optional); 2.) Kot, Elements of Mathematical Ecology (optional); 3.) Thieme, Mathematics in Population Biology (optional) Text: Class notes, Handouts and Articles
MA 69400 **ROUGH PATHS THEORY** **3 credits**	**Prof. Baudoin** **Office: MATH 438** **Phone: 4-1406** **Email: fbaudoin**	This class will be an introduction to the theory of rough paths that was developed by T. Lyons in the 1990's and that nowadays is an extremely active field of research. The theory of rough paths allows to define integrals of differential forms against irregular paths and differential equations controlled by very irregular paths. This theory makes use of an extension of the notion of iterated integrals of the paths, whose algebraic properties appear to be fundamental. Stochastic processes give natural class of paths for which such integrals or differential equations are required, and this theory may be used for many types of stochastic processes. The following topics shall be covered: 1. Young's integration theory; 2. Young's dierential equations; 3. Rough paths topology; 4. The continuity theorem of Lyons;

		5. Stochastic dierential equations driven by Gaussian processes. Lecture notes will be posted on my blog and my webpage.
MA 69600 **GROUPS ACTIONS, TORIC VARIETIES, AND BIRATIONAL GEOMETRY** **3 credits**	**Prof. Wlodarczyk** **Office: 604** **Phone: 6-7414** **Email: wlodar**	Prerequisites. Basic knowledge about algebraic geometry (like R.Hartshorne 'Algebraic Geometry' Chapter I or similar). The purpose of this course is to give a survey on various techniques used in birational geometry and its interactions with invariant theory and toric geometry. We will introduce algebraic group actions , good and geometric quotients, C^*-actions, Bialynicki-Birula decomposition, reductive groups, geometric invariant theory, Luna's slice theorem, birational cobordisms (techniques inspired by topological cobordisms), and elements of Mori theory. In the course we introduce and briefly discuss the theory of toric varieties as the illustration of the above mentioned techniques with particular emphasis on Mori theory , Morelli cobordisms and C^* -actions and the theory of valuations. One of the main goals will be the the sketch of a proof of the Weak Factorization Theorem which states that any birational map between smooth projective varieties is a composition of blow-ups and blow-downs along smooth centers. The seminar should deal with the following: • We prove the classical Bialynicki-Birula decomosition theorem (for C^*-actions) • We introduce birational cobordisms, and GIT for torus actions and show the relation with birational factorization • Introduce the reductive groups and the classical notion of categorical, good and geometric quotients X//G for G reductive. We extend the results on GIT for the reductive group actions . • We prove the linear reductivity of some classical groups in characteristic zero, and give the proof of Hilbert's 14th problem for reductive groups. • We discuss the classical Luna's etale and Luna's slice Lemma for Torus and Reductive group actions. • We show the Hilbert-Mumford criterion for stability for the reductive groups. • We introduce toric varieties and illustrate the above concepts in the toric setting. The focus of this course is to give an intuition about the interplay of different areas of algebraic geometry.

		Main texts: M. Brion. Introduction to actions of algebraic groups. P.E. Newstead. Geometric Invariant Theory. M. Bukstedt. Notes on Invariant group Theory J.Wlodarczyk Algebraic Morse Theory and Factorization of Birational Maps. J.Wisniewski Toric Mori Theory and Fano Manifolds. Pdf. **Additional texts**: Bialynicki-Birula **Some theorems on group actions**. Igor Dolgachev **Lectures on Invariant Theory** Jerzy Konarski **The B–B decomposition via Sumihiro Theorem** Tadao Oda **Convex bodies and Toric Varieties** Kenji Matsuki **Introduction to Mori Theory** V.L. Popov and E. B. Vinberg **Invariant Theory** in Algebraic Geometry 4. T.A. Springer **Linear Algebraic Groups**