## INTRODUCTION IN ALGEBRAIC TOPOLOGY

Instructor: Prof. James McClure (mcclure@math.purdue.edu, 4-42719) Course Number 57200 CRN: 46826 Credits: Three Time: 4:30p-5:20p MWF

## Description

A common difficulty that students have with algebraic topology is building intuition. Fulton's book Algebraic Topology, An Introduction deals with this issue by starting with a concrete lowdimensional situation, namely line integrals defined in open subsets of the plane. He then shows how the more sophisticated parts of the subject grow naturally from this beginning. Fulton's book is also useful for seeing connections between algebraic topology and other areas such as algebraic geometry and complex analysis (Fulton is an algebraic geometer). I will begin by using Fulton's book for the first five weeks. The rest of the semester will be from Massey's textbook A Basic Course in Algebraic Topology, which fits well with Fulton's point of view. Prerequisite: basic point-set topology, up to compactness and connectedness. Some knowledge of the fundamental group is also desirable.

Text: Fulton, Algebraic Topology, An Introduction Massey, A Basic Course in Algebraic Topology (WARNING: Massey has a different book with a similar title) Massey's book will be available at Copymat.

#### ALGEBRAIC NUMBER THEORY

Instructor: Prof. Freydoon Shahidi (shahidi@math.purdue.edu, 4-1917) Course Number: MA 58400 CRN: 66365 Credits: Three Time: 9:30 A.M.-10:20 A.M. MWF

#### Description

Dedekind domains, norm, discriminant, different, finiteness of class numbers, Dirichlet unit theorem, quadratic and cyclotomic extensions, quadratic reciprocity, decomposition and inertia groups, completions and local fields, unramified, tamely and wildly ramified extensions MA553 and 554 are the only prerequisites.

Text: I will mainly use my own notes. One recommended book is: G. Janusz, Algebraic Number fields

Reference: S. Lang, Algebraic Number Theory J. Neukirch, Algebraic Number Theory

# MATHEMATICS FOR POROUS MEDIA PHYSICS

Instructor: Prof. John Cushman (jcushman@math.purdue.edu, 4-8040) Course Number: MA 59800/EAPS591 CRN: 65499 Credits: Three Time: 1:30 p.m.-2:45 p.m. TTh

#### Description

This course will provide the student with a number of mathematical tools; many of which have been developed specifically for porous media science. Most of these will be employed using real-world

porous media problems. Tools to be considered include various homogenization methods, real-space renormalization groups, stochastic perturbation methods, sub-structural continuum theories, and fractional and other nonlocal-pde concepts of relevance to long-rang correlations and rare events. Practical problems to be considered include dispersive mixing in chromatography and geophysical science; quasi-static electro hydrodynamics related to porous electrodes, clays and photovoltaics; and swelling colloidal systems of relevance to drug delivery and biological tissues.

Text: None

References: None

## INTRODUCTION TO NONCOMMUTATIVE GEOMETRY

Instructor: Prof. Marius Dadarlat (mdd@math.purdue.edu, 4-1940) Course Number: MA 59800 CRN: 64952 Credits: Three Time: 1:30 p.m.-2:45 p.m. TTh

### Description

We aim for a friendly introduction based on examples to the ideas of noncommutative geometry. Topics will include: (0) Review of de-Rham cohomology (1) Hochschild (co)homology and noncommutative differential forms (2) Cyclic (co)homology (3) Review of K-theory and K-homology (4) Characteristic numbers and the Connes-Chern character (5) spectral triples and noncommutative Riemannian manifolds

References: (1) A. Connes, Noncommutative Geometry, Academic Press, San Diego, CA, 1994, 661 p. (2) A. Connes, Non commutative differential geometry, Publ. Math. IHES no. 62 (1985), 41-144 (both available on line: http://www.alainconnes.org/en/bibliography.php)

# INTRODUCTION TO GEOMETRIC GROUP THEORY

Instructor: Prof. Ben McReynolds (dmcreyno@math.purdue.edu, 4-1938) Course Number: MA 59800 CRN: 10706 Credits: Three Time: 10:30 a.m.-11:45 a.m. TTh

### Description

The title of the course in "Introduction to geometric group theory". I can provide a blurb:

The course plans to cover three broad topics:

(1) Basics. Cayley graphs, word metrics, quasi-isometries, word functions, group actions, etc. (2) Hyperbolic groups. Definitions, examples, basic results, motivational questions and philosophies. (3) Analytic properties. Amenability and Property (T). Basics examples, basic results, applications.

If time permits, further topics will be covered. Additionally, we may have to cover some basic constructions and results in group theory of a more preliminary nature.

NUMERICAL SIMULATION IN APPLIED GEOPHYSICS. FROM THE MESOSCALE TO THE MACROSCALE Instructor: Prof. Juan Santos (santos@math.purdue.edu, 4-XXXX) Course Number: MA59800 CRN: 65307 Credits: Three Time: 4:30 pm.-5:45 p.m. TTh

#### Description

Wave propagation is a common technique used in hydrocarbon exploration geophysics, mining and reservoir characterization and production, among other fields. Local variations in the fluid and solid matrix properties, fine layering, frac- tures and craks at the mesoscale (on the order of centimeters) are common in the earth?s crust and induce attenuation, dispersion and anisotropy of the seismic waves observed at the macroscale. These effects are caused by equi- libration of wave-induced fluid pressure gradients via a slow-wave diffusion process that can be analyzed using numerical experiments. Numerical rock physics offers an alternative to laboratory measurements, being inexpensive and informative, allowing to inspect the physical process of wave propagation using alternative models of the rock and fluid properties. This approach has applications in many fields. In the Petroleum Industry, to analyze the seismic response of unconventional hydrocarbon reservoirs; in Foods Science using ultrasound to monitor the state of foods, such as fruit ripeness and degree of freezing; in Medicine to study how porosity increases in human bones affect velocities and attenuation of ultrasonic waves.

References: E. B. Becker, G. F. Carey and J. T. Oden, Finite Elements, an Introduction, Volume I, Prentice Hall, 1981. M. A. Biot, Theory of propagation of elastic waves in a fluid-saturated porous solid. I. Low frequency range, J. Acoust. Soc. Am., 28, 168 (1956). M. A. Biot, Theory of propagation of elastic waves in a fluid-saturated porous solid. II. High frequency range, J. Acoust. Soc. Am., 28, 179 (1956). Biot, M. A., Mechanics of deformation and acoustic propagation in porous media, J. Appl. Physics 33 4, 1482-1498, 1962. T. Bourbie and O. Coussy and B. Zinszner, Acoustics of Porous Media, Editions Technip, Paris, (1987). S. C. Brenner and L. R. Scott, The Mathematical Theory of Finite Element Methods, Springer, New York, 1994. Carcione, J.M., 2007. Wave fields in real media: wave propagation in anisotropic, anelastic, porous and electromagnetic media, in Handbook of Geophysical Exploration, 2nd edn, Vol. 38, 515pp., eds Helbig, K. & Treitel, S., Elsevier, Oxford. Carcione, J. M., Santos, J. E. and Picotti, S., Anisotropic poroelasticity and wave-induced fluid flow. Harmonic finite-element simulation, Geophys. J. Internat., 186, 1245-1254, 2011. P. G. Ciarlet, The Finite Element Method for Elliptic Problems, North-Holland, 1980 Schoenberg, M., and Douma, J., Elastic wave propagation in media with parallel fractures and aligned cracks, Geophys. Prosp., 36, 571-590, 1988. J. E. Santos, J. Douglas, Jr., J. Corbero, and O. M. Lovera A model for wave propagation in a porous medium saturated by a two-phase fluid, Journal of the Acoustical Society of America, (87), 1990, 1439?1448. J. E. Santos Introduction to the Theory of Poroelasticity, Technical Report, Purdue University. J. E. Santos, J. M. Corbero, and J. Douglas, Jr. Static and dynamic behaviour of a porous solid saturated by a two-phase fluid, Journal of the Acoustical Society of America (87), 1990, 1428?1438. Santos, J. E., Rubino, J. G., and Ravazzoli, C. L., A numerical upscaling procedure to estimate effective bulk and shear moduli in heterogeneous fluid saturated porous media, Comput. Methods Appl. Mech. Engrg., 198, 2067-2077, 2009. J. E. Santos S. Picotti and J. M.Carcione Evaluation of the stiffness tensor of a fractured medium with harmonic experiments, Computer Methods in Applied Mechanics and Engineering, (247-248), 2012, 130-145. J. E. Santos S. Picotti and J. M.Carcione Evaluation of the stiffness tensor of a fractured medium with harmonic experiments, Computer Methods in Applied Mechanics and Engineering, (247-248), 2012, 130-145. J. E. White and N. G. Mikhaylova and F. M. Lyakhovitskiy, Low-frequency seismic waves in fluid-saturated layered rocks, Izvestija Academy of Siences USSR, Physics of Solid Earth, 10, 1975, 654-659.

# SEVERAL COMPLEX VARIABLES Instructor: Prof. Laszlo Lempert (lempert@math.purdue.edu, 4-1952) Course Number: MA 63100 CRN: 10707 Credits: Three

Time: 10:30 A.M.-11:20 A.M. MWF

## Description

Power series, holomorphic functions, representation by integrals, extension of functions, holomorphically convex domains. Differential forms and the inhomogeneous Cauchy–Riemann equations. Cousin problem and cohomology groups. Holomorphic maps. Local theory of analytic sets (Weierstrass preparation theorem and consequences). Complex manifolds. Siegel's theorem on meromorphic functions. Prerequisite: MA 53000 + passing of qualifiers

Text: None.

Reference: L. Hormander, An introduction to complex analysis in several variables, (3rd edition) North Holland J.-P. Demailly, Complex analytic and differential geometry (free electronic text)

## INTRODUCTION TO THE THEORY OF ABELIAN VARIETIES

Instructor: Prof. Kenji Matsuki (kmatsuki@math.purdue.edu, 4-1970)

Course Number: MA 59800 CRN: 64949

Credits: Three

Time: 10:30 A.M.-11:20 A.M. MWF

## Description

The theory of elliptic curves is one beautiful place where almost all the branches of mathematics, analysis, algebra, and geometry, converge for its study, and then from that convergence, many new subjects emerge like a big bang. The subject of abelian varieties is one such. In the simplest terms, an elliptic curve is a compact complex torus of dimension 1, equipped with an obvious group structure inherited from that of  $\mathbb{C}$  (when we think of the torus as the quotient  $\mathbb{C}/\Gamma$ by a lattice  $\Gamma$ ). An elliptic curve is also a complete variety of dimension 1 equipped with a group structure compatible with the underlying algebraic structure (when we think of it as the cubic curve in  $\mathbb{P}^2$  and the group structure induced via the chord-tangent law). Therefore, it is only natural to think of the higher dimensional analogues: a compact complex torus of higher dimension, and a complete variety of higher dimension equipped with a group structure compatible with the underlying algebraic structure. It is then remarkable that incredibly rich mathematics comes out of this rather naive looking generalization to higher dimensions. The goal of this course is invite the student to have a glimpse of this rich mathematics under the course titled "Introduction to the theory of abelian varieties". We will follow the textbook "Complex Abelian Varieties" by Biekenhake and Lange. The book is quite well-written, to the extent that a reader needs no teacher to learn the subject from the book ... my intention is to summarize the essential points of the book in the lectures so that you can fill in the detail by reading the book. I would like to go over the subject in a slow-paced and leisurely manner so that the students actually feel they understand the materials rather than swallow them. One of the prerequisites is some basic knowledge of complex manifolds, but almost no knowledge of algebraic geometry is needed (No Hartshorne !) Actually the textbook is user-friendly to spell out all the details. Hence, even if you do not know the words in advance at some point of the course, you can learn the definitions and meanings at the site as we proceed.

Text: "Complex Abelian Varieties" by Biekenhake and Lange.

Reference: Abelian varieties, theta functions, and the Fourier transform" by A. Polishchuk, Abelian varieties" by D. Mumford

#### SQUARE SUMMABLE POWER SERIES

Instructor: Prof. Louis deBranges (branges@math.purdue.edu, 4-6057) Course Number: MA 69000 CRN: XXXXX Credits: Three Time: 9:30-10:30 A.M. MWF

#### Description

This introduction to complex analysis applies the metric topology of Hilbert spaces to treat convergence of power series. Factorization of analytic functions is treated in conjunction with a construction of invariant subspaces. This formulation of complex analysis is applied in the proof of the Bieberbach conjecture which supplies estimates for the Riemann mapping theorem. No previous knowledge of complex analysis is required.

# TOPICS IN COMPLEX GEOMETRY

Instructor: Prof. Sai-Kee Yeung (yeung@math.purdueedu, 4-41942) Course Number: MA69600 CRN: 64482 Credits: Three Time: 1:30 p.m.-2:30 p.m. MWF

## Description

Here are some tentative topics to be discussed. 1. Topics of classical complex analysis and complex geometry, such as hypergeometric equations, triangle groups, Eichler cohomology etc. 2. Some partial differential equations in complex geometry, such as questions related Monge-Ampere equations and K"ahler-Ricci flow. 3. Some results in diophantine geometry, such as Roth's Theorem and Vojta's proof of Faltings' Theorem.

Text: The lecturer would provide reference as the class proceeds.