Introduction to Partial Differential Equations
Instructor: Professor Changyou Wang
Course Number: MA 52300
Credits: Three
Time: 12:00–1:15 pm TTh

Description

Study of basic qualitative properties of solutions to the Laplace, the wave and the heat equations. First order quasi-linear and nonlinear equations and their applications to physical and social sciences. The Cauchy-Kovalevsky theorem. Characteristics, classification and canonical forms of linear equations. Equations of mathematical physics.

Probability Theory I
Instructor: Professor Emanuel Indrei
Course Number: MA 53800
Credits: Three
Time: 8:30–9:20 am MWF

Description

This course consists of a mathematically rigorous, measure-theoretic introduction to probability spaces, random variables, independence, weak and strong laws of large numbers, conditional expectations, and martingales.

Recommended Textbook: Probability: Theory and Examples (Cambridge Series in Statistical and Probabilistic Mathematics) (4) by Rick Durrett.

Functions of Several Variables and Related Topics
Instructor: Professor Rodrigo Banuelos
Course Number: MA 54500
Credits: Three
Time: 1:30 pm MWF

Description

This course will cover some of the basic tools of analysis that are extremely useful in many areas of mathematics, including PDE's, stochastic analysis, harmonic analysis and complex analysis. Specific topics covered in the course include: "Geometric lemmas" (Vitali, Wiener, etc.) and "geometric decomposition theorems" (Whitney, etc.) and their applications to differentiation theory and to the Hardy–Littlewood maximal function; convolutions; approximations to the identity and their applications to boundary value problems in R^d with L^p-data; the Fourier transform and its basic properties on L^1 and L^2 (including Plancherel's theorem); interpolation theorems for linear operators (Marcinkiewicz, Riesz–Thorin); the basic (extremely elegant and useful) Calderón-Zygmund singular integral theory and some of its applications; the Hardy-Littlewood-Sobolev inequalities for fractional integration and powers of the Laplacian and other elliptic operators; the inequalities of Nash and Sobolev viewed from the point of the heat semigroup in R^d.

Text Books: No text book is required. The course follows my lecture notes *"Lectures in Analysis"*. Recommended are: (1) E. M. Stein, *"Singular Integrals and Differentiability Properties of Functions"*, (2), L. Grafakos *"Modern Fourier Analysis"*, (3) E. H. Lieb and M. Loss, *"Analysis"*.

Prequisites: Math 544. But depending on need, some topics from 544 will be reviewed.

Introduction to Functional Analysis

Instructor: Professor Marius Dadarlat

Course Number: MA 54600

Credits: Three

Time: 2:30pm-3:20 pm MWF

Description

1. Banach spaces
2. Hilbert spaces
3. Linear Operators and functionals
4. The Hahn-Banach Theorem
5. Duality
6. The Open Mapping Theorem
7. The Uniform Boundedness Principle
8. Weak Topologies
9. Spectra of operators
10. Compact operators
11. Banach algebras and C*-algebras
12. Riesz calculus
13. Fredholm index
14. Gelfand transform
15. Spectral theorem for normal operators
16. Unbounded Operators
17. Applications: Differential operators, Peter-Weyl theorem

Prerequisites: Familiarity with basic measure theory

Grading:

Attendance 35%,

HW 40%,

Final Exam 25% (a take-home no collaboration exam).

No specific textbook is required. These topics are covered by most books on functional analysis. A good reference is: Gert Pedersen, *Analysis Now*, (Graduate Texts in Mathematics) *Corrected Edition*!

Linear Algebra
Instructor: Professor Tzuong-Tsieng Moh
Course Number: MA 55400
Credits: Three
Time: 9:30–10:20 AM MWF

Description

- Modules over an arbitrary commutative ring. Kernel and image, direct sums and products, The isomorphism theorem and the fundamental theorem of linear algebra. Definition of exact sequences,
- Modules over principal ideal domains. The examples of a square matrix and f.g. abelian groups. Main theorem about f.g. modules over a PID. Application in the case of fixed linear operator over a finite dimensional vector space. Torsion factors and elementary factors of a linear operator and how to calculate these. Minimal polynomial. Rational canonical form and Jordan decomposition theorem. Eigenvalues, eigenvectors and characteristic polynomials. Cayley–Hamilton Theorem. Perron-Frobenius Theorem for Non-negative Matrices and Applications (including the Google search engine).

  References: T.T. Moh Algebra, pp. 196–214 or K. Hoffman & R. Kunze pp. 181–261.
- Several equivalent definitions of the determinant of a square matrix, including the axioms of determinants, and the Laplace formulas. The Cauchy-Binet formulas for the determinant. The multilinear produces and alternative produce of vector spaces and modules. The tensor product and exterior product of modules and vector spaces. Definitions and main properties of projective and free modules. Dual modules and double duals. Chain complex and cochain complex. Applications to advanced calculus. Definitions of exact sequences and resolutions. Describe Hilbert's Syzygies theorem and global dimension. Projective, injective and flat modules. Ext and Tor.

  Reference: D. Dummit & R. Foote Abstract Algebra pp. 339–381, 744–763
- Inner product spaces. Gram-Schmidt orthogonalization. Projections and the least square approximations. Definition of the adjoint of a linear operator. Definition and properties of selfadjoint and unitary operators. Definition of normal operators. The spectral theorem for normal operators over C.
- Linear Operators on inner product spaces, especially Hilbert space. Sesquilinear forms. Positive forms. Positive linear operators. The applications of positivities of the principal minors to the local minimal problem for n variables in calculus and high dimensional ellipsoids. Singular value decomposition theorem and its applications.
- Bilinear forms. Symmetric bilinear forms. Matrix congrence. Sylvester's law of inertia. Signatures. Applications to physics and geometry. Lorantz group $SO(1,3)$.

  Reference K. Hoffman & R. Kunze pp. 270–385

There will be weekly homeworks, a mid-term and a final exam. The main reference books are K. Hoffman & R. Kunze and my lecture notes which will be provided to students free.

Abstract Algebra II

Instructor: Professor Bernd Ulrich
Course Number: MA 55800
Credits: Three
Time: 3:30–4:20 PM MWF

Description

This is an introductory course in commutative algebra and homological algebra. The course is a continuation of MA 557, but should be accessible to anybody with basic knowledge in commutative algebra (localization, Noetherian and Artinian modules, associated primes, Krull dimension, integral extensions).

The topics of this semester will include: The functors Ext and Tor, structure of injective modules, flatness, completion, dimension theory, regular sequences, depth and Cohen-Macaulayness.

Prerequisites: Some basic knowledge of commutative algebra.

Texts: No specific text will be used, but possible references are:

- J. Rotman, An introduction to homological algebra, Springer
- H. Matsumura, Commutative ring theory, Cambridge University Press
- W. Bruns and J. Herzog, Cohen-Macaulay rings, Cambridge University Press
- D. Eisenbud, Commutative algebra with a view toward algebraic geometry, Springer.

Introduction in Algebraic Topology

Instructor: Professor Ralph Kaufmann
Course Number: MA 57200
Credits: Three
Time: 9:00–10:15 AM TTh

Description

The course is an introduction to algebraic topology. The focus will be on homology and cohomology theory. This subject is important to topology, but also to many other fields, such as differential, symplectic and algebraic geometry, number theory, mathematical physics, etc. It is a basic tool in many subjects. We will treat the classical simplicial and singular homology and cohomology, but we also plan to cover CW complexes and differential forms.

The basic text for the course will be Elements of Algebraic Topology by James R. Munkres with some exceptions.

Curves, Surfaces and Abelian Varieties
Instructor: Professor Donu Arapura
Course Number: MA 59800
Credits: Three
Time: 12:00–1:15 PM TTh

Description

Description: This will be a sequel to my course from the fall, in the sense that we will make free use of stuff we learned such as sheaves and cohomology, but the emphasis will be different. Rather than focusing on general techniques, the emphasis will be more on the actual geometry of the special classes of varieties mentioned in the title.

To help focus things, it might be good to have specific goal, even if we never reach it. In the 1980's Falting got a Fields medal for proving the following theorem: Any curve of genus at least two over a number field has at most a finite number of rational points. OK, this isn't the goal, because it's too hard. However, there is an older purely geometric analogue due to Arakelov and Parshin, that we could try for.

Introduction to The Basic Theory of Elliptic Curves
Instructor: Professor Kenji Matsuki
Course Number: MA 59800
Credits: Three
Time: 10:30–12:00 MWF

Description

The purpose of this course is to introduce the beginning graduate students to the basic theory of elliptic curves, assuming the minimum amount of prerequisite. (Actually my lecture notes were originally prepared for the course aimed at the undergraduate/beginning graduate students at Kyoto University, Japan.) The subject of elliptic curves sits at the intersection of analysis, topology and number theory, i.e., almost all the areas of mathematics. As such, it has been the center of intensive studies classically and recently, ranging from the old problem of the congruent numbers, of computing the elliptic integral, to the proof of the Fermat's Last Theorem, to name a few. We try to give a series of lectures which introduce the students to this fascinating subject at an elementary level with little background material required.

Some of the main topics covered are the Mordell-Weil theorem stating that the rational points on an elliptic curve form a finitely generated Abelian group, the topological and analytic properties of an elliptic curve via the Weierstrass p-function, the classification of the elliptic curves by the j-invariant, and the Weil conjecture counting the number of points on an elliptic curve over finite fields:

- Mordell-Weil theorem,
- Elliptic curves over C (the analytic theory),
- The j-invariant,

- Weil conjecture for elliptic curves. GradingScheme: I will give several report problems along the way, and the

  nal grade will be determined by the report submitted at the end of the semester. Textbooks:
- The arithmetic of elliptic curves by Joseph H. Silverman, Graduate Texts in Mathematics 106, Springer
- Elliptic curves by Dale Husemoller, Graduate Texts in Mathematics 111, Springer
- Elliptic curves by Anthony W. Knapp, Mathematical Notes 40, Princeton University Press
- Introduction to elliptic curves and modular forms by Neal Koblitz, Gradu- ate Texts in Mathematics 97, Springer Prerequisites:
- basic knowledge of complex analysis of one variable,
- basic knowledge of algebra at the level of 553 and 554
- willingness to work hard :) (the most important prerequisite)

Random Walks in Random Environments

Instructor: Professor Jonathan Peterson

Course Number: MA 59800

Credits: Three

Time: 10:30–11:20 AM MWF

Description

This course will give an introduction to random walks in random environments (RWRE) — a simple model for random motion in a non-homogeneous environment. While the model of RWRE seems to be a minor modification of the classical simple random walk model, the analysis turns out to be much more difficult and often leads to very surprising conclusions (such as directional transience with sublinear speed and non-Gaussian limiting distributions). RWRE are currently a very active area of research within probability theory, and this course will provide students with an understanding of the tools and techniques that have been developed for the study of RWRE. Many of these methods also have connections to other areas of probability such as stochastic homogenization, polymer models, and self-interacting random walks.

The primary reference for the course will be lecture notes by Ofer Zeitouni. Topics to be covered during the semester include

1. One-dimensional RWRE
    1. Recurrence/transience and a law of large numbers
    2. Limiting distributions
    1. Recurrent RWRE (Sinai's model)
    2. Transient RWRE ? including non-Gaussian limiting distributions
    3. Large deviations
2. Multi-dimensional RWRE
    1. General results
    1. Kalikow's 0-1 Law

2. Sznitman and Zerner's approach to a law of large numbers
2. Sznitman's ballisticity conditions (and recent refinements)
3. Specific models of RWRE
1. Balanced environments
2. Dirichlet environments
3. Biased RW on percolation clusters

Background material: Basic understanding of Markov chains, ergodic theory, and martingales.

Prerequisites: 532, 538.

Eisenstein Series and L-Function

Instructor: Professor Freydoon Shahidi
Course Number: MA 59800ES
Credits: Three
Time: MWF 9:30 AM

Description

I am planning to continue my present course as it is at least a two semester topics. The course will therefore cover the theory of Eisenstein series and L-functions. I will go over whatever needed to be covered about intertwining operators, one of the central objects in the study of automorphic forms on reductive algebraic groups, either through trace formula or L-functions, as well as a study of generic spectrum. I will then discuss the analytic properties of Eisenstein series, study their constant and non-constant terms and conclude from them the analytic properties of certain class of L-functions, those appearing in the constant terms of Eisenstein series. I will then conclude with discussing their connections with functoriality, number theory and representation theory.

This course is a continuation of the course Professor Shahidi is giving in Fall 2016. It will cover the theory of Eisenstein series and L-functions. It will go over whatever is needed to be covered about intertwining operators, one of the central objects in the study of automorphic forms on reductive algebraic groups, either through trace formula or L-functions, as well as a study of generic spectrum. It will also cover the analytic properties of Eisenstein series, study their constant and non-constant terms and conclude from them the analytic properties of certain class of L-functions, those appearing in the constant terms of Eisenstein series. It will conclude by discussing their connections with functoriality, number theory and representation theory.

References:

Borel's Corvallis notes: Automorphic L-functions, AMS Proceedings in Pure Math., No. 33, Vol. 2.

F. Shahidi, Eisenstein Series and L-functions, AMS Colloquium Pub., Vol. 58, 2010.

F. Shahidi, An overview of Eisenstein series, in "p-adic Representations, Theta-correspondence and the Langlands-Shahidi Theory", Lecture Series of Modern Number Theory, Science Press, Beijing, 2013.

C. Moeglin and J-L Waldspurger, Spectral Decomposition and Eisenstein Series, Cambridge Tracts in Mathematics 113, Cambridge University Press, 1995.

Introduction to p-adic Galois Representation

Instructor: Professor Tong Liu
Course Number: MA 59800PGR
Credits: Three
Time: MWF 1:30-2:20 PM

Description

This course provides the rudiments of theory of p-adic Galois representations. I plan to start with basic propeties of p-adic representations then cover the following topics: l-adic representations and Grothendiecks l-adic monodromy Theorem, C-representations and Sen?s theory, semi-stable representations and Fontaine's theory. If time allows, we will discuss some topics of integral p-adic Hodge theory and Fontaine-Mazur conjecture.

References: Jean–Marc Fontaine and Yi Ouyang's book: Theory of p-adic Galois representations

Prerequisite: MA58400

Numerical Methods for Partial Differential Equations

Instructor: Professor Xiangxiong Zhang
Course Number: MA 615
Credits: Three
Time: 1:30–2:45 PM TTh

Description

The lectures will start with finite difference methods for the Poisson equation. The main focus of this course will be various aspects (accuracy, stability and convergence) of finite difference methods for time dependent problems including wave equations and parabolic equations. Linear system solvers such as the conjugate gradient method and the multigrid method will be discussed. If time permits, the finite element method will be briefly introduced. Homework and the final exam will consist of both analysis problems and coding by Matlab. Recommended prerequisites include MA 514 and MA 511 (or equivalent/similar courses).

Introduction to Representation Theory of Finite and Classical Lie Groups and Lie Algebras

Instructor: Professor Saugata Basu
Course Number: MA 690
Credits: Three
Time: 3:00–4:15 PM TTh

Description

The course will cover the basics of representation theory of finite groups (and in particular, that of the symmetric group), as well as classical Lie groups and algebras over $\mathbb{C}$. Towards the end of the course we will explore certain applications of the theory. This could include applications to the study of topological complexity of symmetric varieties, theory of representational stability and

FI-modules, as well as “geometric complexity theory” which is an algebro-geometric approach to some classical problems in the theory of computational complexity.

We will use as our main text the book by C. Procesi—“Lie groups: an approach through invariants and representations”.

Topics in Algebra: Linkage, Residual Intersection, and Rees Algebras

Instructor: Professor Bernd Ulrich
Course Number: MA 690
Credits: Three
Time: 4:30–5:20 PM MWF

Description

The course is an introduction to linkage theory. Linkage (or liaison) is a method for classifying ideals and projective varieties. The course will give an introduction to linkage theory and its generalization, residual intersection. Residual intersections are used widely, for instance in intersection theory and in the study of Rees algebras. The course will treat some of these applications.

The course is meant for students currently taking MA 65000 and for more senior graduate students that took commutative algebra in the past. However, the material is accessible to anyone with a good background in commutative algebra. No text will be used.

Spectral Element Method

Instructor: Professor Steven Dong
Course Number: MA 692T
Credits: Three
Time: 9:00–10:15 AM TTh

Description

Spectral element method combines the geometric flexibility of finite elements and the high accuracy of spectral methods. It can deal with computational domains with arbitrarily complex geometry, and at the same time achieve global high-order accuracy for smooth solutions. This course aims to provide a gentle introduction to this technique. The construction of expansion bases functions, local elemental operations, and global operations in one and higher dimensions, as well as the applications in several areas such as fluid dynamics and solid mechanics, will be covered. The students will have the opportunity to work through several projects during the course of the semester, upon which the final course grade will be based. Spectral element method is one of the most important numerical techniques in practice, and the subject material is expected to benefit graduate students from a number of fields.

Introduction Topological to K-theory

Instructor: Professor Thomas Sinclair

Course Number: MA 69300KT

Credits: Three

Time: MWF 11:30 AM–12:20 PM

Description

The purpose of the course is to offer an introduction to complex topological K-theory and the K-theory of Banach algebras. Topological K-theory was developed by Atiyah and Hirzebruch based on earlier work of Grothendieck and Bott. Topological K-theory provides a remarkably powerful framework for some of the most celebrated achievements in twentieth century mathematics such as Bott's periodicity theorem and the Atiyah-Singer index theorem. Topological K-theory was subsequently generalized to the theory of C*-algebras (often referred to as "noncommutative topological spaces") and Banach algebras where it is now the main tool for classification. The course is designed to be essentially self-contained and will have few prerequisites other than a knowledge of basic algebraic topology. Topics covered will include:

Topological K-theory: the structure of complex vector bundles; K-theory and generalized cohomology theories; Bott periodicity; Characteristic classes; DeRham cohomology and the Chern character.

Banach algebra K-theory: Banach algebra basics; Swan's theorem; Bott periodicity; the index map; the six term exact sequence.

No exams will be given. Grades will be based on attendance and a short presentation.

Lectures will be pulled from a variety of source materials. A few good resources:

Atiyah, "K-Theory"

Hatcher, "Vector Bundles and K-Theory"

Blackadar, "K-Theory for Operator Algebras"

Rordam, Larsen, and Laustsen, "An Introduction to K-Theory for C*-algebras"

Introduction to Free Boundary Problems of Obstacle Type

Instructor: Professor Arshak Petrosyan

Course Number: MA 69400

Credits: Three

Time: 12:30–1:20 PM MWF

Description

Free boundary problems are boundary value problems for partial differential equations (PDEs) which are defined in a domain with an apriori unknown part of its boundary; this part is accordingly named a free boundary. A further quantitative condition must be then provided at the free boundary to exclude indeterminacy. Typical examples include interfaces between different phases or different types of media, moving boundaries, shocks and discontinuities, etc.

This course will serve as an introduction to the theory of regularity of free boundaries on the example of so-called obstacle-type problems, including the classical obstacle problem, a problem from potential theory, and the Signorini problem.

We are going to discuss classical as well as more recent methods in such problems, including the optimal regularity of solutions, several types of monotonicity formulas (Alt-Caffarelli-Friedman, Almgren, Weiss, Monneau), classification of global solutions, criteria for the regularity of the free boundary, structure of the singular set.

Prerequisite: MA52300, MA54400 absolute minimum; MA64200, MA64300 desirable;

Text: A. Petrosyan, H. Shahgholian, N. Uraltseva, Regularity of free boundaries in obstacle-type problems, Graduate Studies in Mathematics 136, American Mathematical Society, Providence, RI, 2012.

Topics in Complex Geometry

Instructor: Professor Sai Kee Yeung

Course Number: MA 69600

Credits: Three

Time: 8:30–9:30 AM MWF

Description

Here are some tentative topics to be discussed, which may be adjusted as the class proceeds.

1. Introduction to Hodge theory, non-abelian Hodge Theory.
2. Harmonic maps and rigidity.
3. Monge-Ampere equation and related problems.
4. Complex ball quotients and special algebraic surfaces.
5. Introduction to questions related to Andre-Oort Conjecture.
6. Complex hyperbolicity.
7. Introduction to Gelfond-Schneider techniques and diophantine geometry.

Reference: Reference as the class proceeds.

Prerequisite: 562, 525