**Introduction to Partial Differential Equations**
Instructor: Professor Daniel Phillips
Course Number: MA 52300
Credits: Three
Time: 9:30–10:20 AM MWF

**Description**

Development of qualitative properties for solutions to the Laplace, the wave and the heat equations and methods for representing solutions. First order quasi–linear and nonlinear equations and their applications. The Cauchy–Kovalevsky theorem. Characteristics, classification and canonical forms of linear equations. Equations of mathematical physics.

Textbook: Partial Differential Equations: 2nd Edition (Graduate Studies in Mathematics) by Lawrence C. Evans

**Ordinary Differential Equations and Dynamical Systems**
Instructor: Professor Nung Kwan Yip
Course Number: MA 54300
Credits: Three
Time: 12:00–1:15 PM TTh

**Description**

This is a beginning graduate level course on ordinary differential equations. It covers basic results for linear systems, local theory for nonlinear systems (existence and uniqueness, dependence on parameters, flows and linearization, stable manifold theorem) and their global theory (global existence, limit sets and periodic orbits, Poincare maps). Some further topics include bifurcations, averaging techniques and applications to mechanics and population dynamics.

Prerequisites: one undergraduate course in each of the following topics:
linear algebra (for example, MA 265, 351)
differential equation (for example, MA 266, 366)
analysis (for example, MA 341, 440, 504) or instructor's consent.

Textbook:
James D. Meiss: Differential Dynamical Systems (available online from Purdue)

**Introduction to Functional Analysis**
Instructor: Professor Thomas Sinclair
Course Number: MA 54600
Credits: Three
Time: 11:30 AM-12:20 PM MWF

## Description

This course will serve as a general introduction to functional analysis. Topics will include: basic theory of Hilbert spaces and Banach spaces; weak and weak* topologies; locally convex topologies; bounded operators on Hilbert space; the Spectral Theorem; and Banach algebras. Depending on interest, several applications will discussed along the way such as Haar measure, the Peter–Weyl theorem, the theory of distributions, Tauberian theorems, and fixed point theorems.

No textbook will be required. The recommended textbook will be Functional Analysis by Peter D. Lax.

**Abstract Algebra II**
Instructor: Professor Bernd Ulrich
Course Number: MA 55800
Credits: Three
Time: 3:30–4:20 PM MWF

## Description

This is an introductory course in commutative algebra and homological algebra. The course is a continuation of MA 55700, but should be accessible to anybody with basic knowledge in commutative algebra (localization, Noetherian and Artinian modules, associated primes, Krull dimension, integral extensions).

The topics of this semester will include: The functors Ext and Tor, structure of injective modules, flatness, completion, dimension theory, regular sequences, depth and Cohen–Macaulayness.

Prerequisites: Some basic knowledge of commutative algebra.

Texts: No specific text will be used, but possible references are:

- J. Rotman, An introduction to homological algebra, Springer
- H. Matsumura, Commutative ring theory, Cambridge University Press
- W. Bruns and J. Herzog, Cohen-Macaulay rings, Cambridge University Press
- D. Eisenbud, Commutative algebra with a view toward algebraic geometry, Springer.

**Basic Algebraic Geometry II**
Instructor: Professor Donu Arapura
Course Number: MA 59800ADA
Credits: Three
Time: 1:30–2:45 PM TTh

**Description**

This is intended as a follow up to Prof. Matsuki's algebraic geometry course from the fall. In particular, I won't assume anything that he hasn't already covered, and may likely review things that he has. (If for some reason you didn't take his class, but are interesting in taking this, then talk to me.) Like it's predecessor, this is intended for students who have some interest in learning algebraic geometry, but whose primary interest may lie elsewhere. So I will mostly stick to the classical setting of quasiprojective varieties, and cover things such as Bertini theorems, blow ups, incidence varieties, Grassmanians and flag varieties, algebraic groups and actions. I won't rule out doing fancier stuff such as schemes and cohomology, but that really depends on how much everyone understands. One way for me to gauge this is by having students present solutions to problems and other material during class. And that's something that I will insist upon. I won't follow any book as such, but will recommend the ones listed below as references and as sources of problems.

1. D. Eisenbud, Commutative algebra. With a view toward algebraic geometry.
2. J. Harris, Algebraic geometry. A first course.
3. R. Hartshorne, Algebraic Geometry
4. I. Shafarevich, Basic Algebraic Geometry I, II

**Introduction to Geometric Measure Theory**
Instructor: Professor Monica Torres
Course Number: MA 59800GMT
Credits: Three
Time: 10:30–11:20 AM MWF

**Description**

Geometric Measure Theory is widely applied to many areas of Analysis and Partial Differential Equations. This class will be an introduction to Geometric Measure Theory and the topics that will be covered include:

- Radon measures
- Hausforff measures
- Besicovitch's covering theorem, differentiation of measures
- Rademacher's theorem
- Rectifiable sets and blow–ups of Radon measures
- Area formula
- Sets of finite perimeter

- Existence of minimizers in geometric variational problems (i.e. minimal surface)
- Coarea formula, approximation of sets of finite perimeter
- Isoperimetric inequality
- Reduced boundary and De Giorgi's structure theorem
- Regularity of minimizers, monotonicity formula
- Properties of the "excess"
- Lipschitz continuity of local perimeter minimizers
- $C^{1,\alpha}$ regularity of local perimeter minimizers
- Analysis of singularities, Federer's dimension reduction argument
- Fine properties of functions of bounded variation (BV )
- Traces of BV functions and the Gauss–Green formula

We will use the following books:

(1) *Sets of finite perimeter and geometric variational problems:* An introduction to geometric measure theory, by Francesco Maggi, Cambridge studies in advanced mathematics (135), 2012.

(2) *Minimal surfaces and functions of bounded variation*, by Enrico Giusti, Monographs in Mathematics (80), Boston, 1984.

(3) *Measure theory and fine properties of functions*, by L.C. Evans and R. Gariepy, Studies in Advanced Mathematics, CRC Press, 1992.

Prerequisites:

- Readers require only basic measure theory (as in MA544 or equivalent).

## Finite Element Methods for Partial Differential Equations

Instructor: Professor Zhiqiang Cai
Course Number: MA/CS 61500
Credits: Three
Time: 12:00–1:15 PM TTh

### Description

The finite element method is the most widely used numerical technique in computational science and engineering. This course covers the basic mathematical theory of the finite element method for partial differential equations (PDEs) including variational formulations of PDEs and construction of continuous finite element spaces. Adaptive finite element method as well as fast iterative solvers such as multigrid and domain decomposition for algebraic systems resulting from discretization will also be presented.

The main textbook of this course is the book by Brenner and Scott entitled "The Mathematical Theory of Finite Element Methods".

Prerequisite: MA/CS 514 or equivalent or consent of instructor.

**Several Complex Variables**
Instructor: Professor Laszlo Lempert
Course Number: MA 63100
Credits: Three
Time: 10:30–11:20 AM MWF

**Description**

Power series and holomorphic functions of several complex variables, representation by integrals, extension of functions, holomorphic convexity, pseudoconvexity. The inhomogeneous Cauchy-Riemann equations. Local theory of holomorphic functions and of analytic sets (Weierstrass preparation theorem and consequences). Complex manifolds, meromorphic functions, analytic cohomology.

**Methods of Partial Differential Equations II**
Instructor: Professor Donatella Danielli–Garofalo
Course Number: MA 64300
Credits: Three
Time: 11:30–12:20 AM MWF

**Description**

This course will be a continuation of MA 64200, but only to the extent that we will continue to explore some of the most pervasive ideas used in the modern theory of elliptic and parabolic PDE's. The course will have a self-contained character, and can be attended also by students who have not previously taken MA 64200. Topics to be covered are:

1. The analysis of strong solutions to linear second-order elliptic equations. These are solutions to (non-divergence form) equations which possess second derivatives in a weak sense. The analysis consists of two parts: (a) the so called $L^p$–theory, which constitutes an analogue of the Schauder estimates for classical solutions; (b) the Alexandrov maximum principle, a priori bounds, and pointwise estimates of Holder and Harnack type.
2. The almost–everywhere regularity of minimal hypersurfaces, à la De Giorgi; The Dirichlet problem for minimal surfaces; The Bernstein problem.
3. Estimates for harmonic measure and Green's function; Fatou type theorems in non-smooth domains.
4. The maximum principle and the theory of weak solutions for linear parabolic PDE's. The relevant tools from harmonic analysis, potential theory, and geometric measure theory will be developed concurrently.

References:

1. D. Gilbarg and N. S. Trudinger, Elliptic Partial Differential Equations of Second Order.
2. E. Giusti, Minimal Surfaces and Functions of Bounded Variations.
3. C. E. Kenig, Harmonic Analysis Techniques for Second Order Elliptic Boundary Value Problems.

4. G. M. Lieberman, Second Order Parabolic Differential Equations.

**Class Field Theory**
Instructor: Professor Freydoon Shahidi
Course Number: MA 68400
Credits: Three
Time: 9:30–10:20 AM MWF

**Description**

Class field theory is that of understanding abelian extensions of local and global fields. It is a crowning achievement of number theory in the 20th century and the main motivating object for the Langlands program. We will treat the subject by mainly concerntrating on number fields.

Syllabus: Ideles, adeles, $L$–functions, first and second inequalities, Artin symbol, reciprocity, local and global class fields, Kronecker–Weber theorem.

I will generally follow my notes which is now posted on my bio page. Other sources are:

1. S. Lang, "Algebraic Number Theory", Addison Wesley, 1970.
2. J. Cassels and A. Frolich, "Algebraic Number Theory", Thompson book company, 1967

Prerequisite: MA 584 or the instructor's approval.

**Abelian Variety**
Instructor: Professor Tong Liu
Course Number: MA 69000AV
Credits: Three
Time: 1:30–2:20 PM MWF

**Description**

Abelian variety is high dimensional generalization of elliptic curve. But it has much more richer structure than that of elliptic curve. Abelian variety is one of core objects to study in algebraic geometry and arithmetic geometry, and it connects almost all the areas in arithmetic geometry. For example, the moduli space of abelian varieties (with level structure) is Shimura variety, which is one of key objects to understand in Langlands program. In this course, we introduce rudiments of Abelian variety, which include the following topics: Abelian variety over C, isogeny, group scheme, torsion subgroup and finite group scheme, dual abelian variety, polarization, Tate module, endomorphism of abelian variety, Weil paring and Rosati involution.

**Topics in Algebra: Differential Methods in Commutative Algebra**
Instructor: Professor Bernd Ulrich
Course Number: MA 69000CA
Credits: Three
Time: 4:30–5:20 PM MWF

**Description**

We will review some duality theory and then talk about basic properties of modules of differentials. We will see how differentials can be used to describe singular loci and ramification loci.

We will also treat differents and complementary modules. Applications include several classical theorems: the theorem on the purity of the branch locus, the Briancon–Skoda theorem on integral closures of ideals, and Tate's formula for the socle of complete intersections.

The course is meant for students currently taking MA 65000 and for more senior graduate students that took commutative algebra in the past. However, the material is accessible to anyone with a good background in commutative algebra. No text will be used, but some of the material can be found in:

E. Kunz, Residues and duality for projective algebraic varieties, University Lecture Series 47, AMS, 2009.

J. Lipman and A. Sathaye, Jacobian ideals and a theorem of Briancon–Skoda, Michigan Math. J. 28 (1981), 199–222.

**Introduction to Kinetic Theory**
Instructor: Professor Jingwei Hu
Course Number: MA 69200SP
Credits: Three
Time: 1:30–2:45 PM TTh

**Description**

In multiscale modeling hierarchy, kinetic theory serves as a basic building block that bridges atomistic and continuum models. It describes the non-equilibrium dynamics of a gas or system comprised of a large number of particles using a probability density function. On one hand, kinetic descriptions are more efficient (requiring fewer degrees of freedom) than molecular dynamics; on the other hand, they provide rich information at the mesoscopic level when the well–known fluid mechanics laws of Navier–Stokes and Fourier become inadequate, and have proved to be reliable in many fields such as rarefied gas/plasma dynamics, radiative transfer, semiconductor modeling, or even social and biological sciences. This course will constitute an introduction to the kinetic theory with the focus on the Boltzmann equation and related kinetic models. We will discuss basic mathematical theory, numerical methods, and various applications.

Specific topics will include: derivation and properties of the Boltzmann equation (collision integral, conservation laws, H–theorem, etc.), connection to molecular dynamics, connection to fluid equations, Chapman–Enskog expansion, moment closures, deterministic numerical methods (e.g., discrete–velocity methods, spectral methods, fast summation methods), stochastic numerical methods (e.g., direct simulation Monte Carlo methods), asymptotic–preserving schemes (multiscale methods coupling kinetic and fluid equations). Other possible topics depending on the interests of the audience: Vlasov and Fokker–Planck–Landau equations for plasmas, quantum Boltzmann equation for bosons and fermions, inelastic Boltzmann equation for granular media, multi-species Boltzmann equation for gaseous mixtures, semiconductor Boltzmann equation for electron transport, kinetic models for collective behavior of swarming and flocking, etc. No textbook is required. The material will be based on the lecture notes by the instructor, and some papers and book chapters. No exams. There might be some homework assignments. Students are expected to present course–related material in class or work on small research projects.

**References:**


- C. Cercignani. *The Boltzmann Equation and Its Applications*. Springer–Verlag, 1988.
- S. Chapman and T. Cowling. *The Mathematical Theory of Non-Uniform Gases*. Cambridge University Press, third edition, 1991.
- C. Cercignani, R. Illner, and M. Pulvirenti. *The Mathematical Theory of Dilute Gases*. Springer–Verlag, 1994.
- C. Cercignani. *Rarefied Gas Dynamics. From Basic Concepts to Actual Calculations*. Cambridge University Press, 2000.
- C. Villani. A review of mathematical topics in collisional kinetic theory. In S. Friedlander and D. Serre, editors, *Handbook of Mathematical Fluid Mechanics*, volume I, pages 71–305. North–Holland, 2002.
- S. Harris. *An Introduction to the Theory of the Boltzmann Equation*. Dover Publications, 2004.


**Topics In Complex Geometry**

Instructor: Professor Sai Kee Yeung

Course Number: MA 69600A

Credits: Three

Time: 12:30–1:20 PM MWF

**Description**

Here are some tentative topics to be discussed, which may be adjusted as the class proceeds.

1. Introduction to comparison theorems and basic tools in geometry.
2. Introduction to Gelfond-Schneider techniques and diophantine geometry.
3. Introduction to complex two ball quotients and complex surfaces.
4. Introduction to zeta functions.
5. Recent studies on Kaehler-Einstein metrics and complex geometry
6. Reference: I would provide reference as the class proceeds.

Prerequisite: MA 562, 525

**Intersection Theory and applications**

Instructor: Professor Deepam Patel

Course Number: MA 69600D

Credits: Three

Time: 3:00–4:15 PM TTH

**Description**

This course will be an introduction to intersection theory and some applications. In the firrst part of the course we will de?ne chow groups and study their basic structure such as existence of ring structure (i.e. intersection product), functoriality, and gysin maps. Instead of proving abstract foundations, we shall focus mostly on explicit computations including applications to enumerative geometry. Depending on audience interest we might also discuss excess intersections.

The second part of the course will be determined by audience interest. One option is to discuss cycles class maps, the theorems of Mumford and Roitman on zero cycles, and the Bloch–Srinivas decomposition theorem. Following that, we can discuss more recent results of Voisin and Totaro on applications of these methods to the study of stable rationality. Another option is to discuss conjectures of Beilinson, Bloch and Deligne relating algebraic cycles to special values of $L$–functions.