Introduction to Partial Differential Equations
Instructor: Professor Harold Donnelly
Course Number: MA 52300
Credits: Three
Time: 10:30–11:20 AM MWF

Description

First order quasi-linear equations, characteristics, classification, and canonical form for linear equations, equations of mathematical physics, study of Laplace, wave, and heat equations, connection with differential geometry, Cauchy-Kovalevsky theorem.

Abstract Algebra II
Instructor: Professor Bernd Ulrich
Course Number: MA 55800
Credits: Three
Time: 4:30–5:20 PM MWF

Introduction in Algebraic Topology
Instructor: Professor Ralph Kaufmann
Course Number: MA 57200
Credits: Three
Time: 8:30–9:20 AM MWF

Description

The course is an introduction to algebraic topology. The focus will be on homology and cohomology theory which are a basic tool in many subjects. It is fundamental for topology, but also important for many other fields, such as differential, symplectic and algebraic geometry, number theory, mathematical physics, data science etc.

We will treat the classical simplicial and singular homology and cohomology, but we also plan to cover CW complexes and differential forms and more advanced topics as time permits.

The basic text for the course will be Elements of Algebraic Topology by James R. Munkres with additions from other sources and the lecture to update the material to a more modern presentation. These will be made available.

Introduction to Model Theory
Instructor: Professor Saugata Basu
Course Number: MA 59800CIMT
Credits: Three
Time: 12:00–1:15 PM TTh

Description

The course will be an introduction to model theory. We will cover the basic theory including: definitions of structures, languages and theories, model–completeness and quantifier elimination, ultrafilters and compactness theorem, types, categoricity, and stability. I will try to emphasize certain key examples, like o-minimal structures, and hopefully get to the point of discussing some more modern aspects, like connections with number theory and combinatorics. An outcome of (and motivation behind) the course is to be in a position to understand the recent works of Hrushovski and Loeser on stably dominated types and its application in the study of non-archimedean geometry (such as the properties of Berkovich analytic spaces).

I will assume no prerequisite in logic and fill in any background in set theory that might be needed.

Textbook:

1. A course in model theory by K. Trent and M. Ziegler, Cambridge ASL Series.
2. A course on basic model theory, by H. Sarbadhikari and S. M. Srivastava, Springer.
3. Non-Archimedean Tame Topology and Stably Dominated Types (AM-192) (Annals of Mathematics Studies), E. Hrushovski and F. Loeser.

Mathematical Aspects of Neural Networks
Instructor: Professor Greg Buzzard
Course Number: MA 59800CMNN
Credits: Three
Time: 2:30–3:20 PM MWF

Description

Neural networks have dramatically changed the capabilities of computers to perform tasks such as image classification and processing, image generation, voice recognition, and game playing, and many others. I will give an overview of the development of neural networks, describe the primary architectures and training methods, and describe some of the mathematical results associated with neural networks. Required work will include programming projects, oral presentations on research papers, and a final group project that includes a written paper and a neural network implementation. Some prior programming experience is required, as well as solid understanding of both linear algebra and probability at the level of a good undergraduate course.

Numerical Methods for PDEs
Instructor: Professor Xiangxiong Zhang
Course Number: MA 61500
Credits: Three
Time: 1:30–2:45 PM TTh

Description

The lectures will start with finite difference methods for the Poisson equation. The main focus of this course will be various aspects (accuracy, stability and convergence) of finite difference methods for time dependent problems including wave equations and parabolic equations. Linear system solvers (including the conjugate gradient method and the multigrid method) and ODE solvers such as Runge-Kutta method will also be discussed. If time permits, the finite element method will be briefly introduced. Homework and the final exam will consist of both analysis problems and coding by Matlab. Sample Matlab codes will be provided thus prior knowledge on coding is not required. Recommended prerequisites include linear partial differential equations, linear algebra and Fourier analysis, which will be reviewed during the lectures.

Calculus of Variations
Instructor: Professor Donatella Danielli–Garofalo
Course Number: MA 64400
Credits: Three
Time: 10:30–11:20 AM MWF

Description

The calculus of variations is one of the classical subjects in mathematics. Besides its links with other branches of mathematics (such as geometry, optimal control theory, and differential equations), it is widely used in physics, engineering, economics, and biology. The essence of the calculus of variations is to identify necessary and sufficient conditions that guarantee the existence of minimizers for integral functionals of the type

$$\mathcal{F}(u;\Omega) = \int_\Omega F(x,u,Du)\,dx.$$

In this course we will focus on the so–called direct methods, which consist in proving the existence of the minimum of $\mathcal{F}$ directly, rather than resorting to its Euler equation (that is, an associated PDE or system of PDEs). The central idea is to consider $\mathcal{F}$ as a real–valued mapping on the space of functions taking on $\partial\Omega$ given boundary values, and applying to it a generalization of Weierstrass' theorem on the existence of the minimum of a continuous function. One of the main issues in this approach is to identify the Sobolev spaces as the proper function spaces for compactness results to hold. In turn, the solution of the existence problem in the Sobolev class opens up a series of questions about the regularity of the minimizers. This was a long–standing open problem, until the way to its solution (in a non-direct fashion, since it involves the Euler equation) was opened by the celebrated De Giorgi–Nash–Moser result concerning the Hölder continuity of solutions to uniformly elliptic PDEs in divergence form with bounded and measurable coefficients. A first step towards the use of direct methods in the regularity issue came from a 1982 paper by Giaquinta and Giusti, who proved the Hölder continuity of quasi-minima, that is functions u for which

$$\mathcal{F}(u; K) \leq Q\mathcal{F}(u + \phi; K)$$

for every ϕ with compact support $K \subset \Omega$. The notion of quasi-minima reduces of course to the one of minimum when $Q = 1$, but it is substantially more general, since it includes solutions of linear and nonlinear elliptic equations and systems. The course will follow the path outlined above, and it will combine the classical approach to the subject with its latest developments. In particular, we will analyze some beautiful examples, such as the obstacle problem, whose study culminated in the theory of free boundary problems.

Prerequisites: The course will be essentially self-contained. The only prerequisites are familiarity with the Lebesgue integral and the first properties of L^p spaces, as well as a basic knowledge of Sobolev spaces.

Introduction to Algebraic Geometry II

Instructor: Professor Jaroslaw Wlodarczyk

Course Number: MA 69000AG

Credits: Three

Time: 3:00–4:15 PM TTh

Description

The course in Algebraic Geometry is a continuation of the fall 2018 AG course and is based upon Hartshorne'es Algebraic Geometry: Chapters 2 and 3 and Mumford's Red book, and others. The important part of the class will be solving problems from Hartshorne's book and others.

The tentative list of the topics includes but is not limited to:

- Projective morphisms
- Ample divisors
- Flatness
- Derived functors
- Coherent and Quasicoherent modules
- Cohomology of Sheave
- Formal schemes

Textbook:
Algebraic Geometry by Robin Hartshorne
The Red Book of Varieteies and schemes by David Mumford
Basic Algebraic Geometry (Part 1 and 2) by Igor Shafarevich

Homework:
Problem solving is vital for this class. I will actively seek groups of volunteers to report their solutions in the problem sessions.

Regarding Commutative Algebra: It is very useful to have basic working knowledge of commutative algebra, at the level of Introduction to Commutative Algebra by Atiyah and Macdonald, before plunging into Hartshorne's book.

Exams: No exam.

Representation Theory of Real Lie Groups
Instructor: Professor Freydoon Shahidi
Course Number: MA 69000ARTLG
Credits: Three
Time: 9:30 AM-10:20 PM MWF

Description

Representation theory of Lie groups is of significance not only by itself, but for many applications that it has in other subjects and disciplines including physics. The course is aimed to cover some of the fundamental concepts of representation theory of real reductive groups, mainly their infinite dimensional representation theory which may be considered as a continuation of the present course offered by Basu. The topics include:Reductive groups and basic notions of their representation theory, a brief survey of representations of compact Lie groups, universal enveloping algebra and smooth vectors, discrete series, induced representations, admissible representations and some discussions of Langlands classification of their irreducible ones. Examples of reductive groups are classical groups and they will be used to provide examples throughout the course.

Prerequisite: Some knowledge of manifolds, tangent spaces, Lie algebras and Lie groups.

Textbook:
We will follow Anthony Knapp's orange book: Representation Theory of Semi-simple Groups, Princeton University Press, 1986.

Reference for compact groups and prerequisites: T. Brocker and T. Dieke: Representations of Compact Lie Groups, GTM98, Springer, 2003

The Theory of Multiplicities and Mixed Multiplicities

Instructor: Professor Bernd Ulrich

Course Number: MA 69000MULT

Credits: Three

Time: 3:30–4:20 PM MWF

Description

Multiplicities are an important tool in commutative algebra and algebraic geometry, especially in connection with enumerative questions. They have applications in intersection theory, equisingularity theory, and the study of integral closures, for instance. In addition to the classical Hilbert-Samuel multiplicity, we will treat more general notions, such as mixed multiplicities, generalized multiplicities, and intersection numbers.

The course serves in some sense as a continuation of MA 65000. However, it should be accessible to anybody with basic knowledge in commutative algebra.

Prerequisites: Basic knowledge about commutative rings (such as the material of MA 557).

Quantitative Geometric and Functional Inequalities

Instructor: Dr. Emanuel Indrei

Course Number: MA 69300QGFI

Credits: Three

Time: 8:30–9:20 AM MWF

Description

In recent years there has been much research devoted to the study of functional and geometric inequalities which quantify proximity to optimizers. For instance, the isoperimetric inequality states that given a set $E \subset \mathbb{R}^n$ such that $|E| = |B_1|$,

$$P(E) \geq P(B_1),$$

where B_1 is the unit ball in $\mathbb{R}^n$, $P(E)$ denotes the perimeter of E, and $|E|$ denotes the Lebesgue measure of E; equality holds if and only if E is a ball. Such a result is classical and there has been an effort to understand sharp perturbative results: if $\delta(E) = \frac{P(E)}{P(B_1)} - 1$, then does $\delta(E) << 1$ imply that E is close to B_1 (up to a translation)? The difficulty of such questions involves understanding the appropriate conditions for which

(i) the inequality makes sense, i.e., identifying the necessary regularity assumptions to define the perimeter;

(ii) what types of functionals quantify proximity to a ball;

(iii) whether such quantifications are optimal.

These issues have been classical problems in the calculus of variations and well-known metrics may not yield a suitable quantification. For instance if one considers the Hausdorff metric, then a ball with a thin long neck attached to it will produce a large distance to an optimizer although $\delta(E)$ could be made arbitrarily small. Given two sets A an B, let

$$A \Delta B = (A \setminus B) \cup (B \setminus A)$$

denote the symmetric difference. The following inequality holds for sets of finite perimeter that satisfy $|E| = |B_1|$:

$$\inf_{x \in \mathbb{R}^n} |E \Delta (x + B_1)|^2 \leq c_n \delta(E),$$

where the exponent 2 is sharp in every dimension. There are several proofs involving symmetrization techniques, optimal transport theory, and partial differential equations. There are also several extensions to anisotropic settings in which the minimizer is a convex set instead of a ball and one can also allow the ambient space to be a convex cone (instead of $\mathbb{R}^n$). Analogous quantitative results hold for the polygonal isoperimetric inequality, the classical Sobolev inequality, the logarithmic Sobolev inequality, and the Brunn-Minkowski inequality. In this course, we will study such inequalities and applications.

Course grade: The course grade is based on an in-class presentation.

Hilbert Spaces of Entire Functions (Spectral Theory)

Instructor: Professor Louis de Branges de Bourcia

Course Number: MA 69400ST

Credits: Three

Time: 9:30–10:20 AM MWF

Description

The purpose of this course is to recruit doctoral students for the spectral theory of differential equations using the theory of Hilbert Spaces of Entire Functions.

The text for the course is available at my webpage under "papers". I will follow the material given in my research proposal on the Riemann Hypothesis also available at my webpage under "papers". My presentation will be computational starting with polynomials orthogonal with respect to a measure. Then I will proceed to computations of the spectral theory in the cases when it is possible using hypergeometrics series. This leads to the interesting special cases of measures which are periodic of period 2π. Again computations will be presented. The course should lie in the capacity of all graduate students in mathematics. But students are preferred was have passed qualifying examinations for the doctoral program. This course is possible but not intended for students who have already chosen thesis topics.

Topics in Complex Geometry
Instructor: Professor Sai Kee Yeung
Course Number: MA 69600A
Credits: Three
Time: 9:30–10:20 AM MWF

Description

Here are some tentative topics to be discussed, which may be adjusted as the class proceeds.

1) Introduction to L^2 estimates.
2) Introduction to L^2 cohomology.
3) Introduction to O minimality.
4) Introduction to Andre-Oort conjecture.
5) Introduction to zeta functions.

Prerequisites: 562, 525

Reference: I would provide reference as the class proceeds.

Modular Curves and Surfaces
Instructor: Professor Donu Arapura
Course Number: MA 69600MCS
Credits: Three
Time: 12:00–1:15 PM TTh

Description

The subject lies in the intersection of geometry, broadly speaking, and number theory. So it should be of interest to students in these areas. A smooth complex algebraic curve can be given explicitly either by equations or via the uniformization theorem: as $\mathbb{P}^1$, a quotient of $\mathbb{C}$ or the upper half plane $\mathbb{H}$ by a group Γ of holomorphic automorphisms. To connect these two points of view, we need to be able to describe the ring of (classical) theta functions/automorphic forms, i.e. functions which are almost invariant under Γ. The generators and relations in this ring give the equations. This can be done fairly explicitly for quotients of $\mathbb{C}$ (elliptic curves) or of $\mathbb{H}$ by $\Gamma \subset SL_2(\mathbb{Z})$ (modular curves). We'll review the theory of elliptic curves if necessary, but will focus mostly on the second case. Modular curves are very interesting for lots of different reasons. Among other things, they are moduli spaces of elliptic curves with extra structure. This interpretation yields models over $\overline{\mathbb{Q}}$. The first half of the class will be devoted to this beautiful story.

In the second half, we will look at some higher dimensional generalizations called Shimura varieties. Rather than emphasizing the general theory, which is pretty technical, we will focus on low dimensional examples, such as Hilbert or Picard modular surfaces, which can handled by more explicit methods coming from algebraic surface theory.

Part of my motivation for doing this is to teach basic tools from algebraic curve and surface theory (e.g. Riemann-Roch, vanishing theorems) in a situation where these are used in serious way.

I won't assume too much background; some knowledge of algebraic geometry or complex analytic geometry should be enough. I'll try to fill in the rest as we need. Besides the standard texts in these areas, more specific references, that we will follow to some extent, are:

1. Diamond, Schurman, A first course in modular forms.
2. Milne, Introduction to Shimura varieties
3. Shimura, Introduction to the arithmetic theory of automorphic forms
4. Van der Geer, Hilbert modular surfaces