Score:___________

Numerical Analysis Qualifying Exam, January 2019

Note:

1. **No books and no notes in this test. Calculators are allowed.**
2. **Show intermediate steps of your work. no credit otherwise.**

1. (15 points)
   The exact value of x is given by $(2^{13} + 2^{-11} + 2^{-12} + 2^{-13})$. Determine the absolute error $|x - fl(x)|$. Assume single precision for this problem, and $fl(x)$ denotes the floating-point number corresponding to x.

2. The following table provides the values of function $f(x)$ and its derivatives on a set of points:

x	$f(x)$	$f'(x)$	$f''(x)$
0	1	14	-1
1	6	5	8
-1	-1	2	4

   (a) (10 points)
   Determine $f[0, 1]$, $f[0, 0]$ and $f[0, -1, -1]$.

   (b) (10 points)
   Let $p(x)$ denote the polynomial of the lowest degree that interpolates f on the nodes $\{1, 1, 0, 1, 0\}$. Determine the value $p(2)$.

3. (15 points)
   Let $f(x)$ and $g(x)$ denote two functions that are sufficiently differentiable, and $F(x) = x + f(x)g(x)$. It is known that $f(x)$ has a simple root r. Find the conditions on the function g so that the iteration $x_{n+1} = F(x_n)$ will converge to r at least cubically if started near r.

4. (15 points)
   Determine the coefficients A_i $(i = 0, 1, 2)$ in the explicit Adams method

$$y_{n+1} - y_n = A_0 f_n + A_1 f_{n-1} + A_2 f_{n-2}$$

   for solving $\frac{dy}{dt} = f(t, y)$, where $f_n = f(t_n, y_n)$. Assume the step size is h.

5. (15 points)
   Determine the Gaussian quadrature formula of the form

$$\int_{-1}^{1} (1 - x^2) f(x) dx \approx A_0 f(x_0) + A_1 f(x_1), \quad x_0 \leqslant x_1.$$

6. Function $f(x)$ has continuous first derivative and it has a simple root r. A root-finding method results in the following error relation

$$e_{n+1} = \frac{1}{f'(\xi_n)} e_n^3$$

where x_n is the approximation of r at step n, $e_n = x_n - r$, and ξ_n is some value between x_n and r.

(a) (15 points)
Show that this method converges to r if the initial guess is sufficiently close to r.

(b) (5 points)
What is the order of convergence of this method?