MA 514 Numerical Analysis
Qualifying Exam
January 2020

Name ______________________

PUID ______________________

INSTRUCTIONS

1. Fill in the information requested above.
2. Make sure you have a complete test. There are **7** pages (including this cover page) and **6** problems.
3. **For problem 1, clearly circle your choice. No partial credit will be given. For problems 2 to 6, you have to show your work in the space provided to receive partial credit.**
4. No books and no notes in this test. Calculators are allowed.
5. This is an **120** minute exam.

Question	Points	Score
1	10	
2	20	
3	15	
4	15	
5	20	
6	20	
Total:	100	

1. (10 points) Which of the following statement is true?

A. The number π can be represented exactly in a finite floating point system (i.e., some finite integer base β and precision t).

B. The explicit midpoint method $Y = y_i + \frac{h}{2} f(y_i)$, $y_{i+1} = y_i + hf(Y)$ to solve the ODE $y' = f(y)$ with $f(y) = \lambda y$ is A-stable.

C. The fixed point iteration $x_{k+1} = \phi(x_k)$ converges quadratically if $\phi'(x^*) = 0$, $\phi''(x^*) \neq 0$, where x^* is the fixed point of $\phi(x)$.

D. Evaluating $\sin(x)$ near $x = \pi/2$ is an ill-conditioned problem.

E. None of the above.

2. (20 points) Suppose $f(x)$ is smooth enough, x^* is a root of $f(x)$ with multiplicity $m \geq 2$ (m is an integer), i.e.,

$$f(x) = (x - x^*)^m g(x), \quad g(x^*) \neq 0.$$

a) Show that the Newton's method

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

converges linearly.

b) Show that the iteration method

$$x_{k+1} = x_k - m\frac{f(x_k)}{f'(x_k)}$$

converges at least quadratically.

3. (15 points) For some function f, you have the following table of extended divided differences

i	x_i	$f[\cdot]$	$f[\cdot,\cdot]$	$f[\cdot,\cdot,\cdot]$	$f[\cdot,\cdot,\cdot,\cdot]$
0	5	$f[x_0]$			
1	5	$f[x_1]$	$f[x_0,x_1]$		
2	6	4	5	-3	
3	4	2	$f[x_2,x_3]$	$f[x_1,x_2,x_3]$	$f[x_0,x_1,x_2,x_3]$

Using the data in the table, find the polynomial $p_2(x)$ of degree at most 2 that satisfies

$$p_2(5) = f(5), \quad p_2'(5) = f'(5), \quad p_2(6) = f(6).$$

4. (15 points) Find the best least squares polynomial approximation of degree 1 to the function $f(x) = e^{-x}$ on the interval $[0, 2]$. (Recall that the first two Legendre polynomials defined on the interval $[-1, 1]$ are $\phi_0(x) = 1$, $\phi_1(x) = x$.)

5. (20 points) Determine the abscissae x_0, x_1 and weights a_0, a_1 in the Gaussian quadrature:

$$\int_0^1 \frac{1}{\sqrt{x}} f(x)\,\mathrm{d}x \approx a_0 f(x_0) + a_1 f(x_1).$$

6. (20 points) Determine the coefficients a, b, c in the two-step Adams-Moulton formula

$$y_{i+1} = y_i + h(af_{i+1} + bf_i + cf_{i-1}),$$

for solving the equation $y' = f(y)$, where h is the time step, y_i is the approximate solution at t_i, and $f_i = f(y_i)$. What is the order of this method? Justify your answer.