QUALIFYING EXAMINATION

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MATH 519 - Prof. DasGupta

(20) 1. A group of n people are lined up in a row at random. Let S denote a randomly selected nonempty subset of these n people, selected from the $2^n - 1$ possible nonempty subsets of the full set of n people.

Find the probability that the members of S occupy consecutive positions in the line-up.

(20) 2. Suppose $X_1, X_2, \cdots, X_k$ are k IID Poisson random variables with mean 1, and $n_1, n_2, \cdots, n_k$ are k nonnegative integers.

Characterize all k-tuples $(n_1, n_2, \cdots, n_k)$ such that $\sum_{i=1}^{k} n_i X_i$ has also a Poisson distribution.

(20) 3. Take two IID Uniform $[0, 1]$ random variables X, Y.

Let $U = X + Y,\ V = XY$.

Find an expression for $E(U|V = v)$.

(20) 4. Let $Z_1, Z_2, \cdots, Z_n$ be n IID $N(0, 1)$ variables. We now define

$$\begin{aligned} X_i &= Z_i \quad \text{if} \quad |Z_i| > 1 \\ &= 0 \quad \text{if} \quad |Z_i| \leq 1. \end{aligned}$$

Consider now the random variable T_n

$$T_n = \sum_{i=1}^{n} X_i.$$

Approximately calculate $P(T_n > 10)$ when $n = 50$.

(20) 5. A couple have agreed on having children until they have at least 2 boys and at least 2 girls. Assume that childbirths are mutually independent and that at each birth, a male or a female child are equally likely.

What is the most likely number of children this couple will have?