QUALIFYING EXAMINATION

JANUARY 2000

MATH 519 - Prof. Sellke

Problems are worth 20 points each. A table of the standard normal distribution is attached.

1. Brand A lightbulbs have exponential lifetimes with mean one year. Brand B lightbulbs have exponential lifetimes with mean two years. Brand C lightbulbs have exponential lifetimes with mean three years.

   A box contains three A bulbs, two B bulbs, and one C bulb. A bulb is chosen at random and screwed into a socket. If the bulb is still working after three years, what is the probability that it is the C bulb?

2. Roll a die 100 times. Let X be the sum of the even numbers rolled. Let Y be the sum of the odd numbers rolled. Calculate the correlation of X and Y.

3. Suppose that Bulgarian lightbulbs have independent lifetimes that are uniformly distributed between 0 years and 1 year. Bulb #1 is screwed into a socket today. In the future, bulb #n will be immediately replaced upon burnout with bulb #(n+1).

   Calculate the probability that the bulb occupying the socket exactly 10 years from now is bulb #18.

4. Let X be a unit exponential random variable, with density $f(x) = e^{-x}I\{x > 0\}$. Let Z be standard normal and independent of X. Let $T = X + Z$. Given that $T = 0$ (exactly, <u>or</u> to 10 significant digits, whichever you prefer), find the approximate numerical conditional probability that $X > 2$.

5. Flies enter a room according to a Poisson process with rate 3 per minute. Each time a fly enters the room, the length of time that it stays is uniformly distributed between 0 minutes and 1 minute. Different "sojourn times" are independent.

   What is the probability that there are exactly 2 flies in the room at a given time?