QUALIFYING EXAMINATION

AUGUST 2001

MATH 519 - Prof. Davis

(15) 1. Let (X, Y) have joint density f given by $f(x, y) = 2$ if $0 < x < 1$ and $x < y < 1$, $f(x, y) = 0$ elsewhere. Define (U, V) as follows: A fair coin is tossed. If the coin is heads, $U = X$ and $V = Y$. If the coin is tails, $U = Y$ and $V = X$. Give the joint density of (U, V).

(15) 2. A penny and a dime are tossed together. This is repeated until both are heads, after which no more tosses are made. Find the expected number of times the penny comes up heads.

(20) 3. A Poisson process has rate $\lambda = 1$. Let N be the number of integers k such that k is one of $0, 1, 2, \ldots, 99$ and there is at least one observation in $[k, k + 2]$. (So for example if the process has hits at .7 and 25.8 and no other hits in $[0, 101]$, then $N = 3$).

Find EN and $E(N^2)$.

(25) 4. (a) Find a probability density function $f(x, y)$ such that $f(x, y) = 0$ unless $0 < x < 1$ and $x < y < 1$ and such that if (X, Y) has joint density $f(x, y)$ then X is uniform $(0, 1)$.

(b) Show there is no probability density function $g(x, y)$ such that $g(x, y) = 0$ unless $0 < x < 1$ and $x < y < 1$ and such that if (Z, W) have joint density $g(x, y)$ then both Z and W are uniform on $(0, 1)$.

(25) 5. Three balls are drawn at random with replacement from an urn containing 10,000 balls numbered from 1 to 10,000.

(a) Find the probability that the median of the three numbers on the balls is 5000. (The median is the middle number: 5,3,4 has median 4 as does 5,4,4).

(b) Is the probability that the average of the three numbers drawn equals 5000 larger, smaller, or the same as the answer to (a)? Prove your answer.